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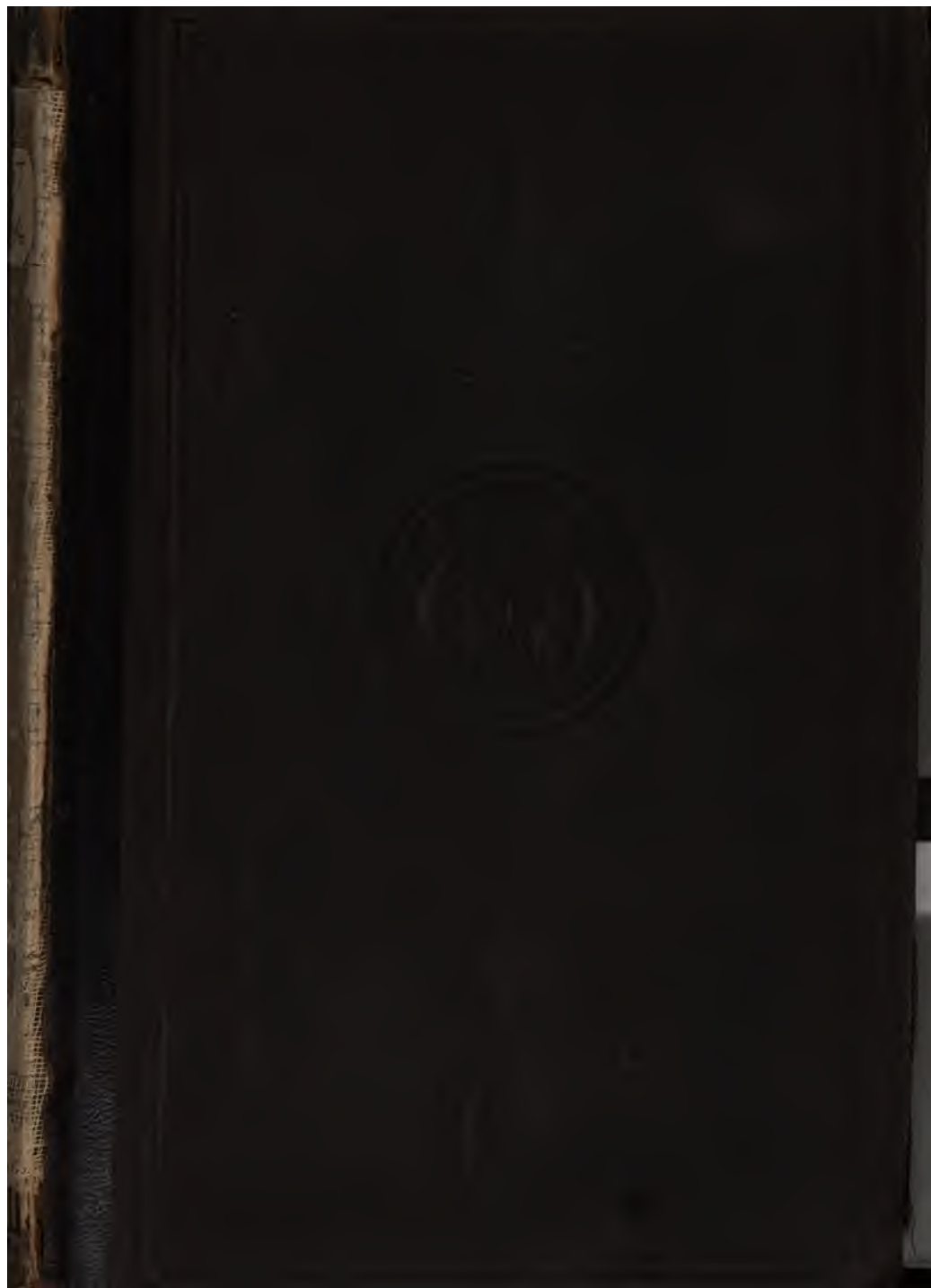
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THE

FRANKLIN

WRITTEN ARITHMETIC

With Examples for Oral Practice

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PREFACE.

THE Franklin Written Arithmetic contains a full course of arithmetical instruction and drill for pupils in the common schools. The definitions and principles are thoroughly illustrated and explained, so that the learner may work intelligently; while the range of applications is broad and varied enough to afford him good preparation for ordinary business affairs.

Topics of a merely theoretical interest, antiquated or curious matter, and puzzling problems, are omitted altogether; while parts of the subject not very necessary to the greater number of pupils are given in the Appendix.

To avoid a multiplicity of rules, decimals and integers have been treated together whenever that could easily be done. For the same purpose, the various problems in Percentage have all been referred to a few fundamental principles, stated and illustrated at the outset.

The Metric System has been treated in a way to indicate the most practical course to pursue in teaching it.

The topics that follow Simple Interest in this book, as in most arithmetics, can wisely be deferred till the last years of the common-school course.

In the arrangement of work it will be noticed that Oral Exercises precede Examples for the Slate. For convenience in the class-room, the latter are numbered consecutively through the whole section, with the exception of four pages of typical examples (pages 17, 25, 35, and 48), which are lettered. The Oral Examples are designated by letters. All answers to Examples for the Slate, except those to Illustrative and Typical Examples, are omitted from the body of the book. Miscellaneous Examples are given in great number and variety, and each section is supplemented by a set of questions for review.

A special feature of the book is the Drill Exercises. In general character these are like those previously published by Walton and Cogswell in their Book of Problems; and they have been introduced in this book by consent of Mr. Cogswell. They give a large number of miscellaneous examples, with answers, on all the topics treated in the Arithmetic; and the teacher will be spared the trouble of selecting from other books examples for class drill. The fact that these exercises have been extensively imitated in books published of late shows the high estimation in which they are held by teachers and authors.

TABLE OF CONTENTS.

	PAGE		PAGE
READING AND WRITING NUMBERS	1	WEIGHTS AND MEASURES	136
ADDITION.....	12	COMPOUND NUMBERS	146
SUBTRACTION	20	Reduction.....	147
MULTIPLICATION.....	28	Fundamental Operations.....	152
DIVISION	38	Longitude and Time	158
UNITED STATES MONEY.....	64	Mensuration of Surfaces and Solids.....	160
Addition, Subtraction, Multi- plication, and Division	65	METRIC SYSTEM	172
Coins and Paper Currency....	67	PERCENTAGE	185
Accounts and Bills	68	Profit and Loss.....	192
FACTORS	75	Commission	194
Symbols of Operation.....	79	Stocks, Dividends, and Brok- erage	197
Cancellation.....	80	Insurance	199
Greatest Common Factor.....	82	Taxes	201
MULTIPLES	84	Customs or Duties	204
Least Common Multiple.....	85	Simple Interest.....	209
COMMON FRACTIONS	88	Accurate Interest.....	215
Reduction.....	90	Partial Payments	216
Addition	94	Problems in Interest	220
Subtraction	97	Present Worth and Discount	224
Multiplication	99	Bank Discount	226
Division.....	105	Commercial Discount	230
To find the Whole when a Part is given.....	111	Compound Interest	231
To find what Fraction one Number is of another.....	113	Average of Payments	234
Aliquot Parts	114	Average of Accounts	238
DECIMAL FRACTIONS	124	Exchange.....	241
Reading and Writing 8, 124, 300		Bonds.....	245
Reduction	125	RATIO AND PROPORTION	254
Addition.....15, 17, 19		Partnership	263
Subtraction	24, 25, 27	POWERS AND ROOTS	265
Multiplication.....34, 35, 36, 130		Square Root	267
Division...43, 44, 45, 48, 49, 132		Cube Root	272
DRILL EXERCISES	57, 59, 63, 73, 123, 135, 171, 253	MENSURATION	277
GENERAL REVIEWS	50, 117, 165, 205, 247, 293	APPENDIX	299
QUESTIONS FOR REVIEW	11, 56, 73, 115, 135, 168, 250, 264, 294		
MISCELLANEOUS EXAMPLES ...27, 37, 52, 74, 118, 119, 166, 206, 229, 248, 295, 315			

ARITHMETIC.

SECTION I.

READING AND WRITING NUMBERS.

ARTICLE 1. A collection of single things or ones is a **number**. By common usage *one* is also called a number.

2. A knowledge of numbers is Arithmetic.

3. Some numbers have simple names. These are *one, two, three, four, five, six, seven, eight, nine, ten*; also *a hundred, a thousand, a million*, etc. All other numbers have compound names. (See Appendix, p. 299.)

Exercises.

1. Name the numbers in regular order, or count, from *one* to *fifty*; from *fifty* to *one*.

2. Count to a hundred by twos; by fives; by tens. Count from a hundred downward by twos; by fives; by tens.

3. Name the number that is made up of two tens and five ones; of one ten and seven ones; of one ten and a one; of one ten and two ones; of six tens and six; of eight tens and five; of nine tens; of nine tens and nine; of ten tens.

4. Name the number that is made up of one hundred, one ten, and a one; of two hundreds, seven tens, and three ones; of six hundreds and three tens; of five hundreds and four ones; of four hundreds, three tens, and three ones; of nine hundreds, nine tens, and nine ones.

4. From their names we see that small numbers are reckoned by *ones*, larger numbers by *tens*, and still larger numbers by *hundreds*, as far as ten hundred. To ten hundred we give the simple name *thousand*.

Above a thousand numbers are reckoned by thousands, by tens of thousands, and by hundreds of thousands, up to ten hundred thousands, or a thousand thousands, which we call *a million*.

Above a million numbers are reckoned by millions, by tens of millions, and by hundreds of millions, up to a thousand millions, which we call *a billion*.

Above a billion numbers are reckoned by billions, by tens of billions, and by hundreds of billions, up to a thousand billions, which we call *a trillion*. And so we go on with higher numbers as far as we choose.

5. One, Ten, a Hundred, a Thousand, Ten-thousand, a Hundred-thousand, a Million, etc., are called **units**, because they are used in reckoning or measuring other numbers.*

6. To distinguish these units, we call *one* a unit of the **first order**, *ten* a unit of the **second order**, *a hundred* a unit of the **third order**, *a thousand* a unit of the **fourth order**, *ten thousand* a unit of the **fifth order**, and so on. When we speak of a unit without mentioning the order, we usually mean a unit of the first order, or one.

7. These units form a scale; and because ten units of any order make a unit of the next higher order, the scale is called a **scale of tens**, or a **decimal scale**.

* A *unit* is a fixed quantity of any kind used to measure other quantities of the same kind. Thus, a foot, a yard, a meter, are units, being fixed lengths used to measure other lengths; a pound, an ounce, a dollar, a cent, an hour, a second, are units, used to reckon or measure weight, value, or time. The word *unit* is also much used as a name for *one*, and *units* for *ones*.

8. A system of numbers whose successive units form a scale of tens is a *decimal system of numbers*. The system of numbers in common use is a decimal system.

9. Table of Units of the Different Orders.

Ten ones (or units)	. . .	make a Ten,
Ten tens	. . .	make a Hundred,
Ten hundreds	. . .	make a Thousand,
Ten thousands	. . .	make a Ten-thousand,
Ten ten-thousands	. . .	make a Hundred-thousand,
Ten hundred-thousands	. .	make a Million,

and so on.

10. It will be convenient to remember that

A thousand ones (or units)	. .	are a Thousand,
A thousand thousands	. .	are a Million,
A thousand millions	. .	are a Billion,
A thousand billions	. .	are a Trillion,

and so on.

11. Exercises.

5. Count by hundreds to a thousand; to two thousand; to two thousand five hundred.

6. Count by thousands to ten thousand; by tens of thousands to a hundred thousand; by hundreds of thousands to a million.

7. Count by millions to ten million; by tens of millions to a hundred million; by hundreds of millions to a billion.

8. How many units of each order are there in twenty-five? seventeen? eleven? ninety? ninety-nine? four hundred? five hundred forty-four? one thousand eight hundred?

NOTE. This kind of exercise may be extended at the discretion of the teacher.

Writing Numbers.

12. Besides being expressed in words, numbers are expressed by writing the signs 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, which are called *figures*. These signs are also called Arabic numerals, because they were first made known to Europeans by the Arabs.*

13. The first of these signs, 0, is called *zero*, or *cipher*, and is used to stand for *no number*; the others are used to stand for the first nine numbers, and take their names, thus :

1,	2,	3,	4,	5,	6,	7,	8,	9.
<i>one,</i>	<i>two,</i>	<i>three,</i>	<i>four,</i>	<i>five,</i>	<i>six,</i>	<i>seven,</i>	<i>eight,</i>	<i>nine.</i>

14. Numbers higher than nine have no single signs for themselves, but are expressed by writing side by side two or more of the figures above.

15. Tens are expressed by writing a figure to tell how many tens, and then writing a zero at the right of it. The tens' figure is then said to stand in the *second place*, the first or units' place being filled by a zero. Thus, we write

Ten (*one ten*), 10. Forty (*four tens*), 40. Seventy (*seven tens*), 70.
 Twenty (*two tens*), 20. Fifty (*five tens*), 50. Eighty (*eight tens*), 80.
 Thirty (*three tens*), 30. Sixty (*six tens*), 60. Ninety (*nine tens*), 90.

16. Numbers that are made up of tens and ones are expressed by writing a figure in the *second place* for the tens, and a figure in the *first place* for the ones. Thus,

Eleven (ten and one), 11. Twenty-one (two tens and one), 21.
 Twelve (ten and two), 12. Twenty-two (two tens and two), 22.

Let the teacher dictate numbers between ten and a hundred for the pupil to write.

* For an account of the Roman numerals, which were displaced by the Arabic, see Appendix, p. 299.

17. Hundreds are expressed by writing a figure in the *third place*, the second and first places being filled by zeros. Thus,

One hundred, 100.	Four hundred, 400.	Seven hundred, 700.
Two hundred, 200.	Five hundred, 500.	Eight hundred, 800.
Three hundred, 300.	Six hundred, 600.	Nine hundred, 900.

18. Numbers made up of hundreds, tens, and ones are expressed by writing a figure in the third place for the hundreds, a figure in the second place for the tens, and a figure in the first place for the ones. Thus,

Four hundred eighty-three (4 hundreds, 8 tens, 3 ones), 483.
 Nine hundred sixty (9 hundreds, 6 tens, no ones), 960.
 Nine hundred six (9 hundreds, no tens, 6 ones), 906.

Let the teacher dictate numbers between a hundred and a thousand for the pupil to write.

19. Thousands, tens of thousands, and hundreds of thousands are expressed by writing figures in the fourth, fifth, and sixth places respectively. The figures in these three places taken together form the Thousands' group; while the figures in the hundreds', tens', and units' places taken together form the Units' group. These groups are usually separated by a comma. Thus,

One thousand, . . .	1,000.	Three thousand, . . .	3,000.
Ten thousand, . . .	10,000.	Twenty thousand, . . .	20,000.
A hundred thousand, . . .	100,000.	Five hundred thousand, . . .	500,000.
Five hundred twenty-three thousand,			523,000.
Six hundred eight thousand, seven hundred twenty-eight, . . .			608,728.

20. Exercises.

Write in figures:

9. Four thousand.
10. Four thousand four hundred.
11. Four thousand forty.
12. Four thousand four.

13. Eight thousand, four hundred twenty-two.
14. Three hundred fifty-six thousand, ~~eight~~ hundred ninety.
15. Sixty thousand, sixty-five.
16. ~~Eighteen~~ hundred seventy-eight.

Let the teacher dictate other numbers, to a million, for the pupil to write.

21. The examples above given illustrate the principle on which all numbers are written, and which is this: *Units of any order are expressed by writing a figure in the place corresponding to that order.*

If the units of any orders are wanting in the number, the corresponding places are filled by zeros.

22. The general method of writing numbers on the principle above stated is shown by the following

TABLE.

etc.	Hundred-trillions.	Hundred-billions.	Hundred-millions.	Hundred-thousands.	Hundreds.	<i>Names of the Units.</i>		
	Ten-trillions.	Ten-billions.	Ten-millions.	Ten-thousands.	Tens.			
	Trillions.	Billions.	Millions.	Thousands.	Units.	<i>Places.</i>		
	15th.	12th.	9th.	6th.	3d.	2d.	1st.	
	4	2	0	3	5	6	7	2
	8	9	3	0	0	7	2	
	0	7	4	8				
	,	,	,	,				
	5th group.	4th group.	3d group.	2d group.	1st group.	<i>Groups.</i>		
etc.	Trillions.	Billions.	Millions.	Thousands.	Units.			

NOTE. For the names of higher numbers, see Appendix, page 300.

23. This table shows that the figures used to express a number fall into groups of three figures each. The first group expresses simple units, tens, and hundreds; the second, units, tens, and hundreds of thousands; the third, units, tens, and hundreds of millions; and so on.

These groups are called the *units'* group, the *thousands'* group, the *millions'* group, etc., from the lowest order of *units* which they express.

NOTE. The units themselves ~~are~~ grouped as the figures are. (Arts. 4 and 10.)

24. In writing large numbers it will be found convenient to think chiefly of the groups as above described. Thus, let it be required to write the number *Forty-nine billion, three hundred seven million, seventy thousand, six hundred forty-three*. The groups are

49 billion, 307 million, 70 thousand, 643.

and the number itself is written

49,307,070,643.

25. Exercises.

I. Beginning with the *units'* group, repeat the names of the groups to trillions; repeat the names from trillions to units.

II. Write the groups of figures required to express the following numbers, with the names of the groups:

17. Forty-six thousand, five hundred twenty.

18. Four hundred six thousand, five hundred two.

19. One million, one thousand, one hundred ten.

26. Exercises.

I. Write in figures the following numbers:

20. Eighty-five million, five hundred three thousand, seven.

21. Nine hundred six million, two hundred eighteen thousand, twenty-eight.

22. Three billion, thirty-seven million, nine hundred thousand, two hundred.

23. Eighteen billion, four.

24. Forty million, seven hundred thousand.

25. Thirty-seven trillion, ninety-nine billion, nine million.

II. Write in figures as many of the numbers named on page 62 as the teacher may indicate.

Reading Numbers.

27. Let it be required to read the number 53869214. To prepare this expression for reading, we begin at the right, and point off three figures for the units' group, three more for the thousands' group, leaving two for the millions' group, thus :

53,869,214.

Now beginning at the left, we name the number expressed by each group, adding the name of the group, thus :

Fifty-three million, eight hundred sixty-nine thousand, two hundred fourteen [units].

NOTE. The name of the units' group is usually omitted in reading.

28. Exercises.

I. Read the following :

- | | | |
|---------------|-----------------|-------------------|
| (26.) 361. | (30.) 9000200. | (34.) 3670980347. |
| (27.) 3261. | (31.) 86320029. | (35.) 9008007006. |
| (28.) 9301. | (32.) 81402020. | (36.) 7676767676. |
| (29.) 654327. | (33.) 89743208. | (37.) 90002000. |

II. Read across the page as many of the numbers expressed on page 60 as the teacher may indicate.

29. It is frequently convenient to separate a number into parts, each part containing only the units of a single order. Thus, the number 734 may be separated into 7 hundreds, 3 tens, and 4 units. Such parts are called the *terms* of a number.

Decimal Fractions.

30. As a hundred is made up of ten equal parts, each of which is a *ten*, and as a ten is made up of ten equal

parts, each of which is *one*, so we may consider one to be made up of ten equal parts, each of which is *a tenth*; a tenth to be made up of ten equal parts, each of which is *a hundredth*; a hundredth to be made up of ten equal parts, each of which is *a thousandth*; and so on.

Now a hundred is written 100; the tenth part of a hundred (ten) is written 10, *the figure 1 being moved one place to the right*; and the tenth part of ten (one) is written 1, *the figure 1 being moved one place further to the right*; so, following the same plan, the tenth part of one (one tenth) is written 0.1; the tenth part of a tenth (one hundredth) is written 0.01; the tenth part of a hundredth (one thousandth) is written 0.001; and so on.

Tenths, hundredths, thousandths, etc., are *fractional units*, or fractions; and, as they form a decimal scale (Art. 7), collections of such units are called *decimal fractions*.

31. The dot put at the right of the units' place is called the *decimal point*.

32. The relations of these fractional units to the higher units are shown by the following table, which may be extended both ways as far as we please:

A thousand	1000.
A hundred	100.
Ten	10.
One	1.
A tenth	0.1
A hundredth	0.01
A thousandth	0.001

33. We see then that decimal fractions may be written on the principle stated in Art. 21. Thus, we write

Two tenths 0.2	Three thousandths 0.003
Five hundredths 0.05	Thirty-two thousandths 0.032
Twenty-five hundredths 0.25	Three hundred sixteen thousandths 0.316

34. The method of writing decimal fractions is shown by the following table, which is merely an extension of the table given in Art. 22.

TABLE.

Names of the Units.	Units.	DECIMAL POINT.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	etc.
Places.	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	etc.		
Figures.	0	7	0	8	9	6	3	4	3	2	.	.

NOTE. In writing decimal fractions it is well to fill the units' place with a zero when there is no other figure to be written there.

35. To read a decimal fraction, *name the number expressed by the figures, and then add the name of the units expressed by the right-hand figure.*

Thus, 0.0739 is read "seven hundred thirty-nine *ten-thousandths*." See Appendix, p. 300.

When a whole number and a decimal fraction are written together, *read first the whole number and then the fraction.*

Thus, 56076.028 is read "fifty-six thousand seventy-six, and twenty-eight thousandths."

36. Exercises.

I. Read the following:

(38.) 0.7	(43.) 0.072	(48.) 2548.
(39.) 0.03	(44.) 0.0806	(49.) 254.8
(40.) 0.25	(45.) 5.05	(50.) 25.48
(41.) 0.83	(46.) 4.056	(51.) 2.548
(42.) 0.005	(47.) 7.0056	(52.) 0.02548

II. Write in figures the following numbers :

- (53.) Seven tenths. (58.) 7 units and 5 thousandths.
 (54.) Seven hundredths. (59.) 25 units and 49 ten-thousandths.
 (55.) Seven thousandths. sandths.
 (56.) Twenty-five hundredths. (60.) 306 hundred-thousandths.
 (57.) Thirty-nine thousandths. (61.) 5047 hundred-thousandths.

Let the teacher dictate other numbers between units and millionths for the pupil to write.

37. Questions for Review.

What is a number? How are numbers reckoned? (Art. 4.) What general name do you give to one, a ten, a hundred, a thousand, etc.? How do you distinguish the different units? What kind of a scale do they form? What system of numbers is in common use? Why is it so called?

What is the meaning of the word *thirteen*? *eleven*? *twelve*? *twenty*? *thirty-seven*? (Appendix, page 299.)

How many units make a thousand? How many thousands make a million? How many millions make a billion?

What is the use of figures? How are numbers higher than nine written? On what principle are all numbers written? (Art. 21.) What is the use of zeros?

How are the figures used to express a number grouped? Name the first five groups. How do you write large numbers? (Art. 24.) Illustrate. How do you read a number? (Art. 27.) Illustrate. What are the terms of a number? Name the terms of the number 6725.

What is the largest number that can be expressed by one figure? by two figures? by three figures?

What is the least number of figures that will express units? thousands? millions?

In 100, how many tens? how many units? In 15000, how many hundreds? units? tens? In 18462, how many tens, and how many units remain? how many hundreds, and how many units remain?

What is the effect of placing zeros at the right of an expression for whole numbers? at the left?

SECTION II.

ADDITION.

38. If you have 5 cents and 3 cents and 2 cents, and count them together, how many cents do you find there are? Counting them together, you find there are 10 cents.

39. The process of counting numbers together is *addition*.

40. The result found by addition is the *sum* or *amount* of the numbers added. Thus, the sum of 5 and 3 and 2 is 10.

41. The addition of numbers is indicated by the sign +, which is read *plus*.

The sign = indicates equality, and is read *equals*, or *is equal to*. Thus, the expression $5 + 3 + 2 = 10$ means that the sum of 5 and 3 and 2 is 10, and is read "five plus three plus two equals ten."

42. Oral Exercises.

I. Name the sums of the pairs of numbers expressed below till you can give them rapidly at sight:

a.	b.	c.	d.	e.	f.	g.	h.	i.	j.	k.	l.
4	4	6	4	5	3	2	3	4	7	3	6
2	3	3	4	2	6	3	4	6	2	3	2
6	7	8	4	8	7	5	3	6	9	5	2
8	4	8	8	3	8	4	8	7	2	9	7
8	5	3	3	9	6	4	6	6	2	9	7
4	8	5	7	6	4	7	1	6	9	4	5

a.	b.	c.	d.	e.	f.	g.	h.	i.	j.	k.	l.
2	9	3	8	5	7	8	2	9	8	2	7
8	7	9	6	6	1	7	2	5	9	5	6
7	8	9	1	2	5	4	5	8	6	8	1
7	2	3	8	4	3	9	5	5	9	1	7
3	2	5	7	4	9	7	6	9	9	5	8
2	6	7	3	5	9	9	5	1	8	9	2

II. Count to a hundred,

- m. By twos, beginning with 2; with 1.
- n. By threes, beginning with 2.
- o. By fours, beginning with 3; with 2.
- p. By fives, beginning with 4; with 3; with 2; with 1.
- q. By sixes, beginning with 5; with 4.
- r. By sevens, beginning with 6.
- s. By eights, beginning with 7; with 6.
- t. By nines, beginning with 8.

III. Add the numbers expressed in the following columns:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
2	9	4	4	6	60	600	6000
3	6	2	3	3	30	300	3000
5	3	6	7	8	80	600	9000
7	2	8	4	8	80	800	8000
6	5	8	5	3	30	400	6000
3	8	4	8	5	50	500	5000
9	4	2	9	3	30	300	3000
1	7	8	7	9	90	700	9000

Begin at the bottom and add upward, naming only the results. Thus, in the first column, say 1, 10, 13, 19, 26, 31, 34, 36; sum, 36. Now, to see if you are right, begin at the

top and add downward. Thus, 2, 5, 10, 17, 23, 26, 35, 36; sum again, 36. Practise exercises of this kind till you can add with great rapidity.

For further drill of this sort the teacher is referred to exercises on pages 59 and 61.

Examples for the Slate.

43. ILLUSTRATIVE EXAMPLE I. What is the sum of 413, 102, and 134?

WRITTEN WORK. *Explanation.* — To find the sum of large numbers like these, we add the units, the tens, and the hundreds separately; hence, for convenience, we write the numbers so that units of the same order may be expressed in the same column. (Art. 6.)

413	
102	
134	
<u>Sum, 649</u>	

Drawing a line beneath, and adding the units (thus, 4, 6, 9), we find there are 9 units, which we write under the line in the units' place. Adding the tens (thus, 3, 4), we find there are 4 tens, which we write under the line in the tens' place. Adding the hundreds (thus, 1, 2, 6), we find there are 6 hundreds, which we write under the line in the hundreds' place. The sum is, then, 6 hundreds 4 tens and 9 units, or 649.

44. ILLUSTRATIVE EXAMPLE II. What is the sum of 960, 748, 932, and 867?

WRITTEN WORK. *Explanation.* — Writing the numbers as before, and adding the units (thus, 7, 9, 17), we find there are 17 units, which are equal to 1 ten and 7 units. We write the 7 units in the units' place, but keep the 1 ten to add with the tens expressed in the next column. Adding the tens (thus, 1, 7, 10, 14, 20), we find there are 20 tens, which are equal to 2 hundreds and no tens. We write 0 in the tens' place, to show there are no tens in the sum, but keep the 2 hundreds to add with the hundreds expressed in the next column. Adding the hundreds (thus, 2, 10, 19, 26, 35), we find there are 35 hundreds, which are equal to 3 thousands and 5 hundreds. We write 5 in the hundreds' and 3 in the thousands' place. The sum is, then, 3 thousands 5 hundreds 0 tens 7 units, or 3507.

960	
748	
932	
867	
<u>Sum, 3507</u>	

Keeping a number and adding it with the numbers expressed in the next column is called *carrying*.

In working examples, use as few words as possible. Thus, in the above example, say merely, "7, 9, 17; * 1, 7, 10, 14, 20; 2, 10, 19, 26, 35; sum, 3507."

1. Add together 6234, 785, and 5861.
2. Add together 582, 2, 49, and 124.
3. How many are 2356, 8004, and 987?
4. Find the sum of 70639, 600, and 7000.
5. Add 76, 33, 92, 53, 305, 78, 8, and 19.
6. What is $213 + 819 + 37 + 66$?

Addition of Decimals.

45. ILLUSTRATIVE EXAMPLE III. What is the sum of 425.37, 433.126, 0.076, 442.09, 0.6, and 0.319?

WRITTEN WORK.

425.37
433.126
0.076
442.09
0.6
0.319
—

Sum, 1301.581

Explanation.—Writing the numbers so that units of the same order may be expressed in the same column, we begin with the units of the lowest order (in this case thousandths) to add, and proceed in the manner already explained, briefly thus: thousandths, 9, 15, 21; write 1, carry 2; hundredths, 2, 3, 12, 19, 21, 28; write 8, carry 2; tenths, 2, 5, 11, 12, 15; write 5, carry 1; units, 1, 3, 6, 11; write 1, carry 1; tens, 1, 5, 8, 10; write 0, carry 1; hundreds, 1, 5, 9, 13; write 13; sum, 1301.581.

7. Add together 90.7, 43.68, 0.045, and 0.812.
8. Add together 0.005, 2.864, 0.9, and 0.25.
9. Add together forty-two thousandths, one hundred seventeen thousandths, thirteen and twenty-two hundredths, seven and five hundredths.

* Do not stop to say "write 7 and carry 1," but do it.

46. From the preceding examples we may derive the following

Rule for Addition.

1. *Write the numbers to be added so that units of the same order may be expressed in the same column. Draw a line beneath.*

2. *Add the units of each order separately, beginning with those of the lowest order.*

3. *When the sum of the units of any order is less than ten, write it under the line in its proper place; when ten or more, write only the units of the sum, and carry the tens to the numbers expressed in the next column.*

4. *Write the whole sum of the last addition.*

Proof.

Repeat the work, adding downward instead of upward.

Adding two or more columns at once.

47. Accountants often add at once the numbers expressed in two, three, or more columns. The following example will illustrate the method:

WRITTEN WORK.	<i>Explanation.</i> —Beginning with 29, add to it first
35	the 4 tens and then the 2 units of 42; then to the
72	sum the 8 tens and the 7 units of 87; and so on.
53	Naming the results merely, say 29, 69, 71; 151,
34	158; 188, 192; 242, 245; 315, 317; 347, 352. Add-
87	ing downwards, say 35, 105, 107; 157, 160; 190,
42	194; 274, 281; 321, 323; 343, 352.
29	After practice it will be found unnecessary
<hr/>	to name all the results; and it is by omitting
Sum, 352	to name them that great rapidity is acquired.

NOTE. The examples on the opposite page embrace the chief varieties in form of examples in Addition. After performing these, and before taking the Applications on page 18, pupils will usually need additional practice in similar work. Examples for such practice will be found on pages 59–63.

48. Examples in Addition.

- a.* Add 5274, 206, 87, and 428. *Ans.* 5995.
b. Add 132, 3618, 7, and 53. *Ans.* 3810.
c. Find the sum of 8972, 980, 5607, and 89. *Ans.* 15648.
d. What is the sum of 34, 4800, 147, and 675? *Ans.* 5656.
e. How many are 346, 4682, 64, and 798? *Ans.* 5890.
f. What is the amount of 6079, 416, 346, and five thousand one hundred sixty-four? *Ans.* 12005.
g. Two thousand eight hundred twenty-one + nine hundred nine + 376 + 43 equals what number? *Ans.* 4149.
h. Six thousand two hundred ten + eight thousand eight + 4743 + 259 = what number? *Ans.* 19220.
i. Five thousand fifty plus 9782 plus seven thousand seven hundred seventy plus 842 are how many? *Ans.* 23444.
j. Six hundred two plus 7524 plus six thousand twenty plus 78 plus 4 are how many? *Ans.* 14228.
k. How many miles are 467 miles, 1349 miles, nine hundred seven miles, and sixty-four miles? *Ans.* 2787 miles.
l. How many dollars are 7419 dollars, 864 dollars, four thousand twenty-five dollars, and ninety dollars? *Ans.* 12398 dollars.
m. Add the numbers expressed by figures in the examples *f*, *g*, and *h*. *Ans.* 12262.
n. Add the numbers expressed by words in examples *i*, *g*, and *h*. *Ans.* 23112.

Examples with Decimals.

- o.* What is the sum of 7.62, 14.2, 120.5, 9.08, 0.875, and 2.125? *Ans.* 154.4.
p. Find the sum of twenty-three thousandths, five hundredths, ninety-seven hundredths, seven and eight tenths, fifteen and forty-one hundredths. *Ans.* 24.253.

For drill exercises, see pages 59 – 63.

49. Applications.

10. A farmer raised 169 bushels of potatoes in one field, 262 bushels in another, 58 bushels in another, and 1827 in another. How many bushels of potatoes did he raise in all?

11. My cow Mabel gave 1388 pounds of milk in April, 1456 pounds in May, 1440 in June, 1317 in July, and 1175 in August. How many pounds did she give in all?

12. I paid \$2400* for my farm, \$155 for a horse, \$26 for a cart, \$86 for a mowing-machine, \$10 for a horse-rake, and \$108 for a yoke of oxen. What did I pay for all?

13. A merchant buys 5 bales of cloth, the first containing 768 yards; the second, 754 yards; the third, 698 yards; the fourth, 702 yards; and the fifth, 1003 yards. How many yards are there in all?

14. A planter sold 6 bales of cotton, weighing as follows: the first, 495 pounds; the second, 509 pounds; the third, 508 pounds; the fourth, 498 pounds; the fifth, 526 pounds; and the sixth, 487 pounds. What was the whole weight?

15. A merchant bought at one time 324 barrels of flour for \$2430; at another, 260 barrels for \$2080; at another, 500 barrels for \$3000; and at another, 107 barrels for \$749. How many barrels did he buy? How much did he pay in all?

16. A steamship sailed 203 miles on Monday, 243 miles on Tuesday, 214 miles on Wednesday, 226 miles on Thursday, 239 miles on Friday, 241 miles on Saturday, and 238 miles on Sunday. How many miles did she sail in the week?

17. In St. Joseph's District, Michigan, there were at one time 335530 peach-trees on 2953 acres of land, 57519 pear-trees on 758 acres, 9786 plum-trees on 502 acres, 17654 cherry-trees on 125 acres, 195995 apple-trees on 2958 acres, and 4988 quince-trees on 33 acres. How many acres of land were occupied by these fruit-trees? How many fruit-trees were there in all?

* \$ is the sign for dollars.

18. The distance from Boston to Albany is 202 miles; from Albany to Buffalo, 297 miles; from Buffalo to Toledo, 296 miles; and from Toledo to Chicago, 243 miles. What is the distance from Boston to Chicago?

19. A man dying left by his will \$42000 to his wife; \$14000 to his daughter; \$3500 in cash, and other property worth \$13650, to his son; \$750 to each of his two nieces; and the remainder of his property, worth \$2627, to his brother. What was the value of the whole property?

50. Examples with Decimals.

20. Add together 3.07, 0.096, 8.431, and 0.7.

21. What is the sum of \$875.16, \$538.12, and \$400.875?

22. What is the sum of 0.08 of a mile, 0.39 of a mile, and 4.7 miles?

23. A man paid \$15 for a coat, \$8.50 for a hat, \$6.75 for a pair of boots, and \$3.45 for other articles. How much did he pay in all?

24. A surveyor measures four fields, and finds in the first 1.625 acres; in the second, 7.316 acres; in the third, 12.776 acres; and in the fourth, 17.306 acres. How many acres in all?

25. How far does a man travel who walks 5.5 miles before breakfast, 17.25 miles between breakfast and dinner, 12 miles between dinner and supper, and 0.875 of a mile after supper?

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
\$75.46	\$754.60	\$2.40	\$476.48	\$47.84	\$780.00
18.72	42.87	74.09	207.42	98.76	15.85
9.47	5.30	53.67	77.99	3.69	119.68
15.08	106.84	184.76	4.44	0.49	45.45
11.80	70.00	66.67	0.85	9.84	99.99
4.55	14.76	407.99	109.98	55.00	710.00
7.67	107.34	31.08	3.17	46.50	84.37
17.38	21.95	213.67	6.51	27.75	76.85

a.	b.	c.	d.	e.	f.	g.	h.	i.
5	4	7	6	13	12	13	15	13
4	3	2	4	8	5	9	8	7
—	—	—	—	—	—	—	—	—
4	5	7	6	12	17	12	14	16
2	2	6	3	9	9	3	9	7
—	—	—	—	—	—	—	—	—
9	7	9	8	15	14	11	12	16
7	5	4	7	6	7	6	8	9
—	—	—	—	—	—	—	—	—
8	9	8	7	11	11	14	11	12
5	8	2	3	8	4	8	9	4
—	—	—	—	—	—	—	—	—
9	6	9	8	13	16	15	12	11
3	5	6	6	6	8	7	3	3
—	—	—	—	—	—	—	—	—
9	8	8	9	11	11	14	17	15
2	3	4	5	5	2	6	8	9
—	—	—	—	—	—	—	—	—

II. Subtract (that is, count downward)

- j.* By 2's from 50; from 49.
- k.* By 3's from 50.
- l.* By 4's from 50; from 49.
- m.* By 5's from 50; from 49; 48; 47; 46.
- n.* By 6's from 100; from 99.
- o.* By 7's from 100.
- p.* By 8's from 100; from 99.
- q.* By 9's from 100.

III. What is $80 - 30$? $50 - 20$? $90 - 50$? $150 - 70$?
 $700 - 400$? $1200 - 300$? $1600 - 900$? $1500 - 800$?

IV. From 100 subtract 25, 35, 85, 67, 39, 48, 73, 44, 78,
 60, 51, 72, 13, 64, 57, 36, 53, 62, 46, 17, 77, 24, 87, 75.

For additional oral drill, see pages 59-63.

Examples for the Slate.

58. ILLUSTRATIVE EXAMPLE I. If 147 trees are taken from a nursery of 489 trees, how many trees will be left?

WRITTEN WORK.

Minuend, 489

Subtrahend, 147

Remainder, 342

Explanation.—To find how many will be left, we take 147 of the number 489 away. In subtracting a large number like this we take away the units, the tens, and the hundreds separately; hence, for convenience, we write the minuend and the subtrahend as in the margin, so that units of the same order shall be expressed in the same column. Drawing a line beneath, and beginning with the units, we subtract thus: 7 units taken from 9 units leave 2 units, which we write under the line in the units' place; 4 tens taken from 8 tens leave 4 tens, which we write under the line in the tens' place; 1 hundred taken from 4 hundreds leaves 3 hundreds, which we write under the line in the hundreds' place; and we have for the whole remainder 3 hundreds 4 tens and 2 units, or 342. *Answer*, 342 trees.

1. If a man having 375 oranges in a box should sell 234 of them, how many would be left?

2. I had a farm of 493 acres, and sold a part containing 172 acres. How many acres had I left?

59. ILLUSTRATIVE EXAMPLE II. If a minuend is 7592 and the subtrahend 3674, what is the remainder?

WRITTEN WORK.

(6) (15) (8) (12)

Min. 7 5 9 2

Sub. 3 6 7 4

Rem. 3 9 1 8

Explanation.—We write these numbers and subtract as before. As we have but 2 units in the minuend, we cannot now take the 4 units away, so we change one of the 9 tens (leaving 8 tens) to units. This 1 ten equals 10 units. We add the 10 units to the 2 units, making 12 units. Subtracting 4 units from the 12 units, we find 8 units left, which we write as part of the remainder. Subtracting 7 tens from the 8 tens we now have, we find 1 ten left, which we write. As we have but 5 hundreds in the minuend, we cannot now take 6 hundreds away, so we change one of the 7 thousands (leaving 6 thousands) to hundreds, and add the 10 hundreds thus obtained to the 5 hundreds, making 15

hundreds. Subtracting 6 hundreds from 15 hundreds, we find 9 hundreds left, which we write. Subtracting 3 thousands from the 6 thousands we now have, we find 3 thousands left, which we write; and we have for the whole remainder, 3 thousands 9 hundreds 1 ten and 8 units, or 3918.

This explanation may be given briefly thus: 4 from 12 leaves 8, 7 from 8 leaves 1, 6 from 15 leaves 9, 3 from 6 leaves 3; remainder, 3918. In actual work, however, *all* explanation should be omitted. Do not stop to say "4 from 12 leaves 8," etc., but *do* the work, naming only results as you write them, thus: "8, 1, 9, 3; remainder, 3918." In this way you will learn to work rapidly.

What are the remainders in the following examples?

(3.)	(4.)	(5.)	(6.)
849	321	8642	3089
<u>278</u>	<u>219</u>	<u>370</u>	<u>2435</u>

7. If I had \$685 in a bank and withdrew \$328, how many dollars remained?

8. How old was a person in 1876 who was born in 1798?

60. ILLUSTRATIVE EXAMPLE III. If a farm is bought for \$965 and sold for \$2000, how much is gained?

WRITTEN WORK.

$$\begin{array}{r}
 (1) \ (9) \ (9) \ (10) \\
 \$ \ 2 \ 0 \ 0 \ 0 \\
 \quad 9 \ 6 \ 5 \\
 \hline
 \text{Ans. } \$ \ 1 \ 0 \ 3 \ 5
 \end{array}$$

Explanation. — To find how much is gained, we take away a part of \$2000 equal to \$965.

As we have no units, no tens, and no hundreds in the minuend, we change one of the thousands (leaving 1 thousand) to 10 hundreds; then change one of the 10 hundreds (leaving 9 hundreds) to 10 tens; and one of the 10 tens (leaving 9 tens) to 10 units. 2000 is thus changed to 1 thousand 9 hundreds 9 tens and 10 units, from which taking 9 hundreds 6 tens and 5 units, we have for the remainder 1035. *Ans.* \$1035.

9. From 2000 years take 1028 years.

10. From 3000 oxen take 229 oxen.
11. How many more birds are there in a flock of 960 birds than in one of 487 birds?

Subtraction of Decimals.

61. ILLUSTRATIVE EXAMPLE IV. What is the difference between 20.69 and 8.745?

WRITTEN WORK. *Explanation.* — Writing these numbers so that units of the same order shall be expressed in the same column, and beginning with the units of the lowest order (in this case thousandths) to subtract, we have for the remainder 11.945.

20.69	
8.745	
11.945	

12. Take 20.5 from 199.
13. From \$27.68 take \$15.96.
14. Find the difference between one thousand and one thousandth.

62. From the examples above explained we may derive the following

Rule for Subtraction.

1. *Write the minuend and underneath write the subtrahend, so that units of the same order may be expressed in the same column. Draw a line beneath.*
2. *Begin with the units of the lowest order to subtract, and proceed to the highest, writing each remainder under the line in its proper place.*
3. *If any term of the minuend is less than the corresponding term of the subtrahend, add ten to it and then subtract; but consider that the next term of the minuend has been diminished by one.*

Proof.

Add the remainder to the subtrahend: the sum ought to equal the minuend.

63. Examples in Subtraction.

- a. From 7282 subtract 4815. *Ans.* 2467.
 b. Take 3084 from 6231. *Ans.* 3147.
 c. How many are 64037 less 5908 ? *Ans.* 58129.
 d. Subtract 807605 from 1740932. *Ans.* 933327.
 e. What number taken from 71287 will leave 40089 ?
Ans. 31198.
 f. How many more than 94736 is 104083 ? *Ans.* 9347.
 g. Find the difference between 86045 and 708406.
Ans. 622361.
 h. $2684753 - 764287 =$ how many ? *Ans.* 1920466.
 i. From four hundred twenty thousand six hundred eighty-three, take two hundred fifty-nine thousand seventy-five.
Ans. 161608.
 j. Take eight hundred ten thousand twenty-three from one million sixty thousand forty-one. *Ans.* 250018.
 k. 1001001 minus 909199 equals what ? *Ans.* 91802.
 l. Subtract the sum of the numbers in example c from the sum of the numbers in example d. *Ans.* 2478592.
 m. Find the difference between the amount of the numbers in example a and the amount of the numbers in example b.
Ans. 2782.

Examples with Decimals.

- n. From \$17.60 take \$5.25. *Ans.* \$12.35.
 o. From 426.17 take 11.723. *Ans.* 414.447.
 p. Subtract three hundred sixty-four thousandths from one.
Ans. 0.636.
 q. What must be added to 0.0476 to make 1 ?
Ans. 0.9524.

NOTE. The examples on this page embrace the chief varieties in form of examples in Subtraction. After performing these, and before taking the Applications on page 26, pupils will usually need additional practice in similar work. Examples for such practice will be found on pages 59-63.

64. Applications.

15. A farmer who raised 948 bushels of corn sold all but 198 bushels. How much did he sell?

16. The year's earnings of a family were \$1172. If their expenses were \$875, what was saved?

17. A and B together own 5740 acres of land. If B owns 2964 acres, how much does A own?

18. Mount Washington is 6234 feet high, which is 2286 feet higher than Vesuvius. How high is Vesuvius?

19. The several items of an account amount to \$9867.62; of this amount \$7985.75 has been paid. Find the balance.

20. Franklin was born in 1706, and died in 1790. What was his age at the time of his death?

21. The difference between A's and B's estates is \$1463; B's, which is the greater, is worth \$7638. What is A's worth?

22. In one week a grain elevator received 984560 bushels of grain; of this 769386 bushels were delivered. How much remained in the elevator?

23. The sailing distance from New York to Queenstown is 2890 miles. If a Cunard steamer has run 1368 miles on her course from New York, how far has she still to run?

The population of the city of New York was 60489 in the year 1800; 96373 in 1810; 123706 in 1820; 202589 in 1830; 312710 in 1840; 515547 in 1850; 813669 in 1860; and 942292 in 1870. What was the increase in population

24. From 1800 to 1810?

28. From 1840 to 1850?

25. From 1810 to 1820?

29. From 1850 to 1860?

26. From 1820 to 1830?

30. From 1860 to 1870?

27. From 1830 to 1840?

31. From 1800 to 1870?

32. The population of London in 1871 was 3266987. How many times may you subtract from this a population equal to that of New York in 1870?

33. The equatorial diameter of the earth is 41847194 feet, and the polar diameter 41707308 feet. What is the difference?

65. Examples with Decimals.

34. A person having 205.6 acres of land, sold 10.75 acres. How many acres had he left ?

35. What is the difference between 0.7 and 0.385 ?

36. How many thousandths must you add to 0.485 to make 1. ?

(37.) $86.67 - 9.8 = ?$ (40.) $641.34 - 56.345 = ?$

(38.) $7561.2 - 9.6456 = ?$ (41.) $101.1 - 90.014 = ?$

(39.) $961.62 - 54.645 = ?$ (42.) $970.2 - 86.37 = ?$

66. Miscellaneous Examples.

43. James Fry has in his possession \$172; he owes \$28 to A, \$36 to B, and \$19 to C. After paying his debts, what will remain ?

44. In a certain mill 2415 persons were employed, of whom 581 were natives, 1119 were foreigners, and the rest unknown. How many were unknown ?

45. I have \$462 in the savings-bank, and \$2180 in government bonds. How much more must I have that I may purchase a house worth \$4700 ?

46. A man gave to his son \$3575, to his daughter \$4680, and to his nephew \$2495 less than to his daughter. His whole property was worth \$30500; what sum remained ?

47. Two persons who are 250 miles apart, travel towards each other, one 36 miles, the other 52 miles a day. How far apart will they be at the end of one day ?

48. If the same persons travel away from each other, how far apart will they be at the end of one day ?

49. From 9460 subtract 5466; from the result subtract 1284; to this add 3989, and from this subtract 5987.

50. A man bought a lot of land for \$1296, and built upon it a house costing \$7364. If he sold the property for \$10000, how much did he make ?

SECTION IV.

MULTIPLICATION.

67. Unite three 7's into one number.

7 This may be done by adding them together thus : 7, 14, 21.
 7 By this process we find that three 7's are 21. In the same way
 7 we can find that seven 6's are 42, eight 9's are 72, eight 7's
 — are 56, and in fact all the results which we commit to memory
 21 when we learn the Multiplication Table.

68. The process of uniting two or more equal numbers into one number is *multiplication*.

69. One of the equal numbers to be united is the *multiplicand*.

70. The number that tells how many equal numbers are to be united is the *multiplier*.

71. The result obtained by multiplication is the *product*.

72. The multiplicand and multiplier are called *factors* (*makers*) of the product.

In the example "three 7's are 21," which is the multiplicand? the multiplier? the product? Name two factors of 21.

73. The multiplication of numbers is indicated by the sign \times . Thus, the expression $50 \times 4 = 200$ means that four 50's are 200; and is read "50 multiplied by 4 equals 200."

74. Oral Exercises.

Turn to page 58, and multiply the numbers expressed

a. In column h by 3.

e. In column q by 7.

b. In column j by 4.

f. In column r by 8.

c. In column k by 5.

g. In column v by 9.

d. In column o by 6.

h. In column v by 12.

75. Compare the product of five 4's with that of four 5's: are they equal or unequal? Compare the products $3 \times 4 \times 6$, $4 \times 3 \times 6$, and $6 \times 4 \times 3$: are they equal or unequal? From examples like these we learn this general principle:

The product of two or more factors is the same, whatever the order in which the factors are taken.

To multiply mentally Numbers greater than 10.

[At the option of the Teacher.]

76. ILLUSTRATIVE EXAMPLE. At \$34 each, what will 4 cows cost?

Solution. — At \$34 each, 4 cows will cost 4 times \$34. Four 30's are 120, and four 4's are 16, which, added to 120, make 136. *Ans.* \$136.

i. If 9 men can build a wall in 25 days, how long would it take 1 man to do it?

j. How many gallons of water in 5 hogsheads of 67 gallons each?

k. At \$8 a month, what is the amount of a soldier's pension for 1 year? for 9 years?

l. How many are three 27's? four 16's? eight times 84?

For additional practice, multiply each number expressed in A, page 58, by such numbers from 1 to 12 as the teacher may select.

See also oral exercises in multiplication, pages 59 and 63.

Examples for the Slate.

77. ILLUSTRATIVE EXAMPLE I. If a steamship goes 258 miles each day, how far does she go in 6 days?

Explanation. — If the steamship goes 258 miles each day, in 6 days she will go 6 times 258 miles. We have then to multiply 258 by 6. Writing the multiplicand and the multiplier as in the margin, we multiply the units, tens, and hundreds separately, beginning with the units.

WRITTEN WORK.	
Multiplicand,	258
Multiplier,	6
Product,	1548

Six 8's are 48. The 48 units are equal to 4 tens and 8 units. We write 8 under the line in the units' place, and carry 4 tens to the product of tens.

Six 5's are 30. The 30 tens with the 4 tens carried are 34 tens, or 3 hundreds and 4 tens. We write 4 under the line in the tens' place, and carry 3 hundreds to the product of hundreds.

Six 2's are 12. The 12 hundreds with the 3 hundreds carried are 15 hundreds, or 1 thousand and 5 hundreds. We write, under the line, 5 in the hundreds' place and 1 in the thousands' place. The entire product is 1548. *Ans.* 1548 miles.

For the sake of rapid working, use as few words as possible. Thus, in the example above say "forty-eight; thirty, thirty-four; twelve, fifteen": while saying "forty-eight," write 8; while saying "thirty-four," write 4; and while saying "fifteen," write 5 and 1.

1. How many pounds of flour are there in 5 barrels, each containing 196 pounds?

2. How many pounds of cheese are there in 6 cheeses of 172 pounds each?

3. If a person earns \$313 every year for 7 years, how many dollars does he earn?

4. What will 9 pianos cost at \$475 each?

5. From Chicago to Peoria is 160 miles; how far does a man travel who goes from Chicago to Peoria and back 8 times?

6. If a person by working 11 hours a day can do a piece of work in 37 days, how many days will it take him if he works 1 hour a day?

7. There are 5280 feet in a mile. How many feet long is a telegraph-wire that connects Boston with Reading, 12 miles distant?

78. ILLUSTRATIVE EXAMPLE II. If 1 barrel of flour costs \$8, what will 427 barrels cost?

Solution. — If 1 barrel of flour costs \$8, 427 barrels will cost 427 times \$8. But 427 times \$8 is the same as 8 times \$427 (Art. 75), which is \$3416. *Ans.* \$3416.

8. What will 732 quarts of milk cost at 7 cents a quart?
9. What must I pay for 324 sheep at \$9 apiece?
10. When coal is \$6 a ton, what must I pay for 476 tons?
11. At 4 cents a mile, what must I pay for riding 1289 miles?
12. If 294 persons gave \$8 apiece for a charitable object, how much did all give?
13. What must I pay for 626 car-fares at 5 cents apiece, and for 87 car-fares at 9 cents apiece?
14. Multiply 267 by 2; by 3; by 4; and add the products.
15. Multiply 628 by 5; by 6; by 7; and add the products.
16. Multiply 3401 by 8; by 9; and add the products.
17. Multiply 90021 by 10; by 11; and add the products.
18. Multiply 66285 by 12; by 8; and add the products.
19. Multiply 89079 by 7; by 12; and add the products.

For additional examples in multiplication by one term only, see pages 59 and 63.

79. ILLUSTRATIVE EXAMPLE III. Multiply 12 by 10; 12 by 100; 12 by 1000.

WRITTEN WORK.			<i>Explanation.</i> — 10 twelves equal 12 tens (Art. 75), or 120; 100 twelves equal 12 hundreds, or 1200; and 1000 twelves equal 12 thousands, or 12000.
12	12	12	
10	100	1000	
120	1200	12000	

In multiplying by 10, 100, 1000, etc., the written work may be omitted, and the product immediately found by *annexing to the multiplicand as many zeros as there are in the multiplier.*

20. What will 10 bushels of potatoes cost at 65 cents a bushel?
21. At \$100 a share, what will 100 shares in a whip company cost?
22. Multiply \$75 by 10; by 100; and add the products.

23. Multiply 5872 by 10; by 1000; and add the products.
 24. Multiply 684 by 10; by 100; by 10000; by 1000; and add the products.
 25. Multiply 3682 by 10000; by 10; by 1000; by 100; and add the products.

80. ILLUSTRATIVE EXAMPLE IV. Multiply 4520 by 300.

WRITTEN WORK. *Explanation.* — Here 4520 equals 452×10 , and 300 equals 3×100 ; hence 4520×300 is the same as $452 \times 10 \times 3 \times 100$, or, since the order of the factors may be changed (Art. 75), the same as $452 \times 3 \times 10 \times 100$. We shall, therefore, find the product if we multiply 452 by 3 and annex three zeros (Art. 79).

$$\begin{array}{r} 4520 \\ \quad 300 \\ \hline 1356000 \end{array}$$

When the multiplicand and multiplier, or either of them, have zeros at the right hand, *the zeros may be disregarded in multiplying, but there must be annexed to the product as many zeros as were disregarded.*

26. I have 600 acres in my farm. What is it worth at \$250 an acre?
 27. How many strawberry plants are there in 400 rows, if there are 280 plants in each row?
 28. What is the product of 1870×90 ? Of 1870 by 900? Of 1870 by 9000?
 29. If 268000 is the multiplicand and 80 the multiplier, what is the product?
 30. Multiply 596 by 3 and by 40, and add the products.
 31. Multiply 984 by 8 and by 60, and add the products.
 32. Multiply 647 by 9 and by 20, and add the products.
 33. Multiply 379 by 5 and by 80, and add the products.
 34. Multiply 4837 by 2, by 30, and by 500, and add the products.
 35. Multiply 2802 by 8, by 70, and by 900, and add the products.

81. ILLUSTRATIVE EXAMPLE V. Multiply 625 by 39.

WRITTEN WORK.

$$\begin{array}{r} 625 \\ 39 \\ \hline 5625 = \text{product by } 9. \\ 1875 = \text{product by } 30. \\ \hline 24375 = \text{product by } 39. \end{array}$$

Explanation.—We shall find the product of 625×39 if we multiply 625 by 9 and then by 30, and add the results. We first find the product by 9, which is 5625, and write it under the line. The product of 625×30 is the same as $625 \times 3 \times 10$. To find this we multiply 625 by 3, obtaining 1875, but instead of annexing a zero (Art. 79), we write the result as 1875 *tens*. We then add the partial products.

NOTE. Compare this process with that of Examples 30 to 33, in the last Article.

36. How many are 34×25 ?

37. Multiply 49 by 98; then multiply 98 by 49. Are these products equal? Why?

38. What is the product of 2842 multiplied by 28?

39. Multiply 3684 by 36 and by 64, and add the products.

40. Multiply 625 by 339; by 705; by 7005.

WRITTEN WORK.

$$\begin{array}{r} 625 \\ 339 \\ \hline 5625 = \text{product by } 9. \\ 1875 = \text{product by } 30. \\ \hline 1875 = \text{product by } 300. \\ \hline 211875 = \text{product by } 339. \end{array}$$

WRITTEN WORK.

$$\begin{array}{r} 625 \\ 705 \\ \hline 3125 = \text{product by } 5. \\ 4375 = \text{product by } 700. \\ \hline 440625 = \text{product by } 705. \end{array}$$

The explanation of this work is left for the pupil. (See Art. 81.)

41. How many are 743×657 ?

42. Multiply 237 by 195; 195 by 237.

43. Multiply 4387 by 235; 235 by 4387.

44. Multiply 7608 by 504; 504 by 7608.

45. Multiply 760500 by 307000.

46. Multiply 907200 by 420900.

Multiplication of Decimals.**82. ILLUSTRATIVE EXAMPLE VI. Multiply 108.67 by 48.****WRITTEN WORK.**

108.67

 48

86936 = prod. by 8.

43468 = prod. by 40.

 5216.16 = prod. by 48.

Explanation. — 10867 hundredths multiplied by 8 is 86936 hundredths. 10867 hundredths $\times 40$ is the same as 10867 hundredths $\times 4 \times 10$. Now 10867 hundredths $\times 4$ is 43468 hundredths; to express this product multiplied by 10 we write the figures one place to the left. Adding the partial products we have 521616 hundredths (5216.16) for the entire product.

Here, as in the preceding examples, we see that *the product is of the same order of units as the multiplicand.*

47. Multiply 8.648 by 5; 432.5 by 21.

48. Multiply 7.0909 by 6; 0.0005 by 18.

49. Multiply 0.625 and 0.375 each by 24, and add the results.

83. From the preceding examples may be derived the following

Rule for Multiplication.

1. *Write the multiplicand and underneath write the multiplier. Draw a line beneath.*

2. *If the multiplier consists of one term only, multiply each term of the multiplicand by the multiplier, beginning with the term of the lowest order, and carrying as in addition.*

3. *If the multiplier consists of more than one term, multiply by each term of the multiplier separately, writing the partial products so that units of the same order shall be expressed in the same column.*

4. *Add the partial products thus obtained, and the result will be the entire product.*

Proof.

Multiply the multiplier by the multiplicand: the two products ought to be equal.

For contractions in multiplication, see Appendix, page 300.

84. Examples in Multiplication.

- a. Multiply 4687 by 8. *Ans.* 37496.
- b. Find the product of 50875 by 7. *Ans.* 356125.
- c. Multiply 5872 by 10, also by 1000, and add the products. *Ans.* 5930720.
- d. Multiply 8756 by 300; by 500; by 7000; and add the results. *Ans.* 68296800.
- e. What is the product of 39700 by 9000? *Ans.* 357300000.
- f. $37406 \times 43 =$ what number? *Ans.* 1608458.
- g. For multiplicand take 46059, for multiplier 76, and find the product. *Ans.* 3500484.
- h. How many are 309 times 46057? *Ans.* 14231613.
- i. Multiply thirty-seven thousand twenty-eight by 508. *Ans.* 18810224.
- j. The multiplier being 987, the multiplicand six thousand four hundred sixteen, required the product. *Ans.* 6332592.
- k. What is the product of 908060 by five thousand four hundred? *Ans.* 4903524000.
- l. One factor being 718151, the other seven hundred, what is the product? *Ans.* 502705700.
- m. At 147 dollars per acre, how much will 385 acres of land cost? *Ans.* \$ 56595.
- n. There are 24 hours in a day. How many hours in 476 days? *Ans.* 11424.

Examples with Decimals.

- o. Multiply 40.27 by 87. *Ans.* 3503.49.
- p. Multiply thirty-one thousandths by 25. *Ans.* 0.775.

NOTE. The examples on this page embrace the chief varieties in form of examples in Multiplication. Examples for additional practice will be found on pages 59 - 63.

85. Applications.

50. At \$45 a month for labor, what will a man earn in a year? In 5 years?

51. If a man saves \$17 a month, what will he save in 25 years?

52. If a sewing-machine can set 690 stitches in a minute, how many stitches can it set in 60 minutes or an hour? In a day of 12 hours? In 6 working days or a week? In 52 weeks or a year?

53. The first House of Representatives of the United States consisted of 65 members; if each member represented 30000 inhabitants, how many inhabitants were represented?

54. In a certain mill, material for 65000 dresses is made in a week. Allowing 18 yards for a dress, how many yards are made in a week? In a year?

55. The cotton crop in Texas in one year was 450000 bales. Allowing 400 pounds to a bale, how many pounds were raised?

56. In a day there are 24 hours, in an hour 60 minutes, in a minute 60 seconds. How many seconds in a day?

57. Light, according to Foucault, travels at the rate of 185172 miles in a second. If it passes from the sun to the earth in 8 minutes 13 seconds (or 493 seconds), what is the distance from the sun to the earth?

86. Examples with Decimals.

58. It took Mary 3.25 hours to learn a piece of music, and Olive 5 times as long. How many hours was Olive in learning it?

59. Mr. Green has 5.175 acres of land and buys 7 times as much of his neighbor. How many acres does he buy of his neighbor?

60. What will 38 barrels of flour cost at \$11.75 a barrel?

61. Mr. Gage sold 175 tons of refined bar-iron at \$45.50 a ton. What did he receive for it?

62. Multiply 5.4328 by 62.

87. Miscellaneous Examples.

63. I have four bins, containing severally 63 bushels, 54 bushels, 37 bushels, and 29 bushels. If there are 60 pounds of corn in a bushel, how many pounds of corn will they all hold?

64. What is the height of an iceberg which is 375 feet above the surface of the water and 7 times as many feet below?

65. Myron walks 847 steps of 2 feet each in going to school. How many more steps must he take to walk a mile, or 5280 feet?

66. What do I save a year, my income being \$1600 a year, and my expenses \$24 a week, 52 weeks making the year?

67. Mr. Fiske receives a salary of \$1500 a year, pays \$130 for clothing, \$275 for other expenses, also \$6 a week for his board. How much money has he left at the end of the year?

68. If 768 be one factor, and $861 - 237$ the other factor, what is the product?

69. Smith & Co. consume 74 tons of coal in a year. How much more did they pay for their coal in 1864, when coal was \$14 a ton, than in 1877, when it was \$7 a ton?

70. If in one yard of cloth there are 580 fibres of warp and 432 of filling, and each fibre of warp contains 32 strands, and each of filling 48, how many strands are there in the yard?

71. One house is valued at \$6750, and another at three times as much. How much will pay for both houses?

72. Mr. Gould had \$2500 with which he bought 17 acres of land at \$42 an acre, a house for \$1500, 2 cows at \$45 apiece, and a horse for \$75. How much money had he left?

73. Mr. Bodwell paid for labor and use of oxen on his land, the following sums: \$135, \$128, and \$90; he also paid \$64 for fertilizers and \$10 for seed, and raised on the land 23 tons of hay which he sold at \$25 a ton. What was his gain above his expenses?

74. Add 284, 1752, 45, and 846; subtract 2731 from the sum; multiply the remainder by 208; and find the difference between the product and 40801.

SECTION V.

DIVISION.

88. Mr. Rice has 24 bushels of sand to bring from the beach. If he brings 8 bushels at each load, how many loads must he bring?

He must bring as many loads as there are 8's in 24. We have already seen by multiplication that *three* 8's are 24, so we know that he must bring 3 loads.

If a cheese weighing 54 pounds be divided equally among 6 persons, how many pounds will each receive?

Each person will receive one of the 6 equal parts into which the 54 pounds is to be divided. We have seen by multiplication that 6 *nines* are 54; hence one of the 6 equal parts of 54 is 9, and each person will receive 9 pounds.

It will be noticed in the first example that we find how many equal numbers, one of which is given, there are in another number (that is, how many times one number is contained in another); and in the second that we find one of the equal parts of a number.

89. The process of finding how many times one number is contained in another or of finding one of the equal parts of a number is *division*.

90. The number to be divided is the *dividend*.

91. The number by which we divide is the *divisor*.

92. The result obtained by division is the *quotient*.

NOTE I. When the divisor is one of the given equal numbers, the quotient will tell *how many* such numbers there are in the dividend.

NOTE II. When the divisor tells how many equal parts the dividend is to be separated into, the quotient will tell *how great* one of those equal parts is.

NOTE III. By comparing the first process with multiplication (Arts. 69–72), we see that the *product* and *multiplcand* are given, and the *multiplier* is to be found. By comparing the second process with multiplication, we see that the *product* and *multiplier* are given, and the *multiplcand* is to be found.

In either case the product and one of the factors are given, and the other factor is required.

93. If Mr. Rice has 31 bushels of sand to bring from the beach, and can bring but 8 bushels at a load, how many full loads can he bring and how many bushels will then remain?

The part of the dividend left after the equal numbers have been taken away is the *remainder*.

In the example above, which is the dividend? the divisor? What is the remainder?

94. The division of numbers is indicated by the sign \div . Thus, the expression $24 \div 8 = 3$ means that the quotient obtained by dividing 24 by 8 is 3, and is read “24 divided by 8 equals 3.” The sign $:$ is also used for division. Thus, $24 : 8 = 3$.

Sometimes the dividend is expressed above a line and the divisor below, in place of the dots. Thus, $\frac{24}{8} = 3$. This expression is called the *fractional form* of indicating division, and is read “24 divided by 8 equals 3,” or “1 eighth of 24 equals 3.”

95. When a thing or a number is divided into 2 equal parts, the parts are called *halves*; when divided into 3 equal parts, the parts are called *thirds*; when into 4 equal parts, the parts are called *fourths*; and so on.

What is one of the parts called when a number is divided into 5 equal parts? 6? 7? 8? 10? 20? 100? 1000?

96. Table for Oral Practice in Division.

1.	4	7	2	6	3	8	5	7	11	9	10	12
2.	16	15	21	14	20	13	22	17	23	18	19	24
3.	25	29	35	33	28	31	26	34	27	30	32	36
4.	39	46	38	42	37	43	47	40	45	41	44	48
5.	58	49	55	51	54	59	52	57	50	53	56	60
6.	62	71	70	66	61	64	69	63	68	65	67	72
7.	76	79	77	81	78	80	75	73	82	74	83	84
8.	90	88	91	87	89	94	85	93	86	95	92	96
9.	99	104	100	98	97	102	106	101	107	103	105	108
10.	110	118	111	109	117	112	115	114	119	113	116	120
11.	124	130	123	125	129	121	128	122	131	126	127	132
12.	134	142	135	140	133	139	136	143	137	141	138	144

97. Oral Exercises upon the Table.

Beginning at the left of the table above, divide by 2 each number expressed in the first two lines, naming quotients and remainders at sight. In the first line the numbers to be divided are 4, 7, 2, 6, 3, 8, 5, etc. The results will be given as follows: "2; 3 and 1 over; 1; 3; 1 and 1 over; 4," etc. *

Divide in the same manner the numbers expressed in either *

- | | |
|--------------------------------------|-----------------------------------|
| a. Of the first 3 lines by 3. | f. Of lines 2 to 8 by 8. |
| b. Of the first 4 lines by 4. | g. Of lines 2 to 9 by 9. |
| c. Of the first 5 lines by 5. | h. Of lines 2 to 10 by 10. |
| d. Of the first 6 lines by 6. | i. Of lines 2 to 11 by 11. |
| e. Of the first 7 lines by 7. | j. Of lines 2 to 12 by 12. |

For other oral exercises in division, see pages 61 and 63.

* As the teacher may indicate.

SHORT DIVISION.

Examples for the Slate.

98. ILLUSTRATIVE EXAMPLE I. At \$ 5 a day for work, how many days' work can be had for \$ 4730 ?

WRITTEN WORK. *Explanation.* — As many days' work can be had for \$ 4730 as there are 5's in 4730. For convenience, we write the dividend and divisor as in the margin, and divide the terms of the dividend separately, as far as possible, beginning with the highest.

(2) (3)
 Divisor, 5) 4 7 3 0 Dividend.
 9 4 6 Quotient.

Ans. 946 days' work.

If we divide the four thousands by 5, we shall have no thousands in the quotient, so we first divide 47 hundreds by 5.

5's in 47 (hundreds), 9 (hundred), and 2 hundreds remain. We write the 9 hundred under the line in the hundreds' place, and change the 2 hundreds remaining to 20 tens, which, with the other 3 tens of the dividend, make 23 tens.

5's in 23 (tens), 4 (tens), and 3 tens remain. We write the 4 tens under the line in the tens' place, and change the 3 tens remaining to 30 units.

5's in 30 (units), 6 (units), which we write under the line in the units' place, and have 946 for the entire quotient. *Ans.* 946 days' work.

In dividing, the pupil may simply say, "5's in 47, 9 and 2 over; in 23, 4 and 3 over; in 30, 6." Or, abbreviating still more, "5's in 47, 9; in 23, 4; in 30, 6."

1. How many cords of wood at \$ 6 a cord can be bought for \$ 522 ? for \$ 3804 ? *1st Ans.* 87 cords.

2. How many hours will it take to ride 3216 miles at 8 miles an hour ? at 12 miles ? *1st Ans.* 402 hours.

3. At 7 cents an hour for work, how many hours must I work to earn 2835 cents ?

4. How many packages of tea, 9 pounds in a package, can be made from 8847 pounds ?

99. ILLUSTRATIVE EXAMPLE II. How many barrels of flour at \$ 8 a barrel can I buy for \$ 2597 ?

WRITTEN WORK.

$$\begin{array}{r} 8) \ 2597 - 5 \text{ Rem.} \\ \underline{324} \end{array}$$

Explanation. — Here, after dividing, we have a remainder of \$ 5: hence, 324 barrels can be bought and \$ 5 remain unexpended, which may be expressed as in the margin.

Ans. 324 barrels; \$ 5 remain.

The work may be proved by finding the product of the quotient and divisor (Art. 92, Note III.) and adding the remainder. Thus, $324 \times 8 + 5 = 2597$.

5. How many weeks are there in 585 days? in 730 days?

1st Ans. 83 weeks; 4 days remain.

6. How many 8 quart cans can be filled with 1865 quarts of milk? with 2587 quarts? *1st Ans.* 233 cans; 1 quart remains.

7. How many years of 12 months each are there in 200 months?

8. There are in an orchard 1608 trees, 12 in a row. How many rows of trees are there?

9. At 11 cents a yard, how many yards of cloth can I buy for 5972 cents?

10. At 9 cents apiece, how many oranges can be bought for 29415 cents?

100. ILLUSTRATIVE EXAMPLE III. If 8 men buy 9675 acres of land which they are to divide equally among themselves, what is each man's share?

WRITTEN WORK.

$$8) \ 9675 \text{ acres.}$$

Ans. 1209 $\frac{3}{8}$ acres.

Explanation. — Each one will have 1 eighth of 9675 acres. We divide, briefly, thus:

One eighth of 9 thousand is 1 thousand, and 1 thousand (equal to 10 hundreds) remains. One eighth of 16 hundreds is 2 hundreds; of 7 tens, 0 tens and 7 tens (equal to 70 units) remain. One eighth of 75 units is 9 units, with a remainder of 3 units yet to be divided.

If 1 eighth of each of the 3 acres is taken, we shall have 3 eighths of an acre. This we express as in the margin, and have 1209 $\frac{3}{8}$ acres for the entire quotient.

11. What is the price of 1 hat if 6 hats cost 375 cents? if 12 cost 2700 cents? *1st Ans.* $62\frac{3}{4}$ cents.
12. How far must a man travel each day to go 1761 miles in 4 days? in 9 days? *1st Ans.* $440\frac{1}{4}$ miles.
13. Mr. Stewart promises to sell me 5 rods of land for \$1578. What is his price per rod?
14. At \$8 a thousand, how many thousands of bricks can be bought for \$3287?
15. A man left by his will \$45267 to be divided equally among his 6 children. What should each child receive?
16. Eight times a certain number equals 324787. What is that number?
17. How many 9's are there in 10000?
18. To what number is $\frac{36857}{12}$ equal?
19. To what number is $\frac{149367}{4}$ equal?
20. How many are $10101019 \div 7$?
21. How many are $98306572 \div 5$?
22. Divide 864024 by 7.
24. Divide 369801 by 9.
23. Divide 164408 by 8.
25. Divide 120087 by 11.

101. Division of Decimals.

ILLUSTRATIVE EXAMPLE IV. What is 1 twelfth of 109.92?

Explanation. — Briefly thus: 1 twelfth of 109 is 9, and 1 remains; of 19 tenths is 1 tenth, and 7 tenths remain; of 72 hundredths is 6 hundredths. *Ans.* 9.16.

WRITTEN WORK.

$$\begin{array}{r} 12) 109.92 \\ \underline{9.16} \end{array}$$
 In the example above it will be seen that we have hundredths in the quotient as there are hundredths in the dividend. *In dividing a decimal by a whole number, the quotient is of the same denomination as the dividend.*

In dividing a decimal by a whole number, fix the decimal point in the quotient as soon as you reach the decimal point in the dividend.

26. What is 1 fifth of 86.4055? (28.) $\$234.54 \div 9 = ?$
27. What is 1 eighth of 94076.8? (29.) $\$907.34 \div 7 = ?$

102. To Divide, carrying the Division to Decimals.

ILLUSTRATIVE EXAMPLE V. Find 1 eighth of 9675 acres.

WRITTEN WORK.

$$\begin{array}{r} 8) 9675.000 \\ \underline{1209.375} \end{array}$$

Explanation. — We divide as in Illustrative Example III., until we come to the remainder, 3 acres. This we change to 30 tenths. One eighth of 30 tenths is 3 tenths, and 6 tenths remain, which are equal to 60 hundredths. One eighth of 60 hundredths is 7 hundredths, and 4 hundredths remain, which are equal to 40 thousandths. One eighth of 40 thousandths is 5 thousandths. The entire quotient is 1209.375 acres.

Perform Examples 11 to 16 in Article 100, carrying the division to decimals.

103. Where the divisor is not greater than 12, it is customary to divide as shown above without expressing all the operations. Such a process is *short division*.

For other examples in short division, see pages 61 and 63.

LONG DIVISION.

104. ILLUSTRATIVE EXAMPLE VI. Divide 33075 by 82.

WRITTEN WORK.

$$\begin{array}{r} 82) 33075 \quad (403\frac{1}{2} \\ \underline{328} \\ 275 \\ \underline{246} \\ 29 \end{array}$$

Explanation. — We write the dividend and divisor as in the margin, and draw a curved line at the right of the expression for the dividend.

Since the divisor 82 is a larger number than 3 or than 33, we first divide 330 hundreds by 82.

Now 330 divided by 82 will give about the same quotient as 33 divided by 8,* which is 4.

The first term of the quotient is then 4 hundreds, which we express by writing a figure 4 at the right of the curved line. Multiplying 82 by 4 hundreds, and subtracting the product, we find 2 hundreds remain; uniting with these 2 hundreds the 7 tens of the dividend, we have 27 tens.

Dividing the 27 tens by 82, we have no tens in the quotient; so we write a zero to show that there are no tens in the quotient, and unite with the 27 tens the 5 units of the dividend, making 275 units.

* So we make 8 our trial divisor.

Dividing the 275 units by 82, using 8 for a trial divisor, we have 3 units in the quotient, which we write. Multiplying and subtracting as before, 29 units remain.

Dividing each of the 29 units by 82, we have $\frac{3}{82}$, which we write with the units, and have for the entire quotient $403\frac{3}{82}$.

105. When the divisor is larger than 12, it is usually convenient to express in full, as above, the work of dividing. The process is then called *long division*.

To Divide, carrying the Division to Decimals.

106. ILLUSTRATIVE EXAMPLE VII. Divide 33075 by 82.

WRITTEN WORK.

82) 33075 (403.35...
328

275

246

290 *Tenths.*

246

440 *Hundredths.*

410

30

Explanation. — We divide as in the last illustrative example until we reach the remainder, 29 units. We now put a decimal point in the expression for the quotient, and, changing the remainder to 290 tenths, divide as before; and so we keep on dividing as far as desirable, or until there is no remainder. In this example we stop dividing at hundredths, and indicate that the division is incomplete by writing a few dots.

107. Give answers to the following examples as in Art. 104, or with the quotient carried to thousandths, as the teacher may direct: *

30. Divide 4684 by 31.

34. Divide 12157 by 23.

31. Divide 9632 by 43.

35. Divide 24898 by 72.

32. Divide 5872 by 54.

36. Divide 36872 by 84.

33. Divide 6748 by 62.

37. Divide 36072 by 91.

108. ILLUSTRATIVE EXAMPLE VIII. Divide 1849 by 192.

WRITTEN WORK.

192) 1849 ($9\frac{1}{3}$
1728
121

Explanation. — As 192 is nearly 200, 1849 divided by 192 will give about the same quotient as 1800 divided by 200, or as 18 divided by 2. We then make 2 our trial divisor.

* The answers in the Key are given in both forms.

38. Divide 26832 by 96. 40. Divide 232848 by 56.
 39. Divide 97684 by 79. 41. Divide 682345 by 88.

109. From the preceding examples we derive the following

Rule for Division.

1. *Write the dividend ; at the left draw a curved line ; and at the left of this line write the divisor.*
2. *Divide the highest term or terms of the dividend by the divisor.*
3. *Express the result for the first term of the quotient at the right in long division, beneath in short division.*
4. *Multiply the divisor by this term.*
5. *Take the product thus obtained from the part of the dividend used.*
6. *Unite the next term of the dividend with the remainder for a new partial dividend ; divide, multiply, and subtract as before ; and so continue till all the terms of the dividend are used.**
7. *Express the division of the final remainder, should there be any, in the fractional form. (Or*
Change the remainder to tenths, hundredths, thousandths, etc., and continue the division as far as desirable.)

Proof.

Find the product of the quotient and divisor, and add to it the remainder, if there is one. The result ought to equal the dividend.

42. How many are $36247 \div 189$?
 43. How many are $53004 \div 398$?
 44. How many are $932480 \div 287$?
 45. How many are $750010 \div 677$?

* If at any time the divisor is not contained in a partial dividend, write a zero for the next figure of the quotient, and unite with the partial dividend the next term of the given dividend.

Contractions in Division.

110. ILLUSTRATIVE EXAMPLE IX. Divide 12367 by 10; by 100; by 1000.

If the decimal point be moved one place to the left, each figure will express a number 1 tenth as great as before (Art. 30); therefore, 1 tenth of 12367 is 1236.7.

For a similar reason, 1 hundredth of 12367 is 123.67, and 1 thousandth of 12367 is 12.367. Hence, when the divisor is 10, 100, 1000, etc., we may find the quotient by moving the decimal point of the dividend as many places to the left as there are zeros in the divisor.

46. There are 100 cents in a dollar; how many dollars are there in 2742 cents? in 12367 cents? *1st Ans.* \$ 27.42.

47. How many dollars are there in 14863 cents?

48. There are 1000 mills in a dollar; how many dollars are there in 56849 mills?

49. Divide 25000 by 10, by 100, by 1000, and add the quotients.

50. Divide 380768 by 100, by 1000, by 10, by 10000, and add the quotients.

111. ILLUSTRATIVE EXAMPLE X. Divide 20864 by 6300.

WRITTEN WORK.

63) 208.64 (3.31...

189

196

189

74

63

11

Explanation. — Since $6300 = 63 \times 100$, we may first divide by 100, obtaining 208.64 (Art. 110), and then divide this quotient by 63, as shown in the margin (Art. 106).

NOTE. In cases where the exact remainder is wanted, the common form of written work is better. It may be abbreviated, as in the written work of this note.

63|00) 208|64 ($3\frac{19}{63}$
189
1964

Explanation. — Indicate first, by a vertical line, the division

by 100; this gives 208 for a quotient, and 64 remain.

Dividing now 208 by 63, we have for a quotient 3, and 19 hundreds remain. Uniting the first remainder 64 with the last remainder 19 hundreds, we have for the entire remainder 1964.

For other contractions of division, see Appendix, page 302.

112. Examples in Division.

- a. Divide 5864 by 9. *Ans.* 651 $\frac{5}{9}$.
- b. At \$8 apiece, how many sheep can be bought for \$2595? *Ans.* 324 sheep; \$3 remain.
- c. If the dividend is 86445 and the divisor 51, what is the quotient? *Ans.* 1695.
- d. What is the quotient of $40076 \div 98$? *Ans.* 408 $\frac{2}{49}$.
- e. 48 times a certain number equals 38256. What is that number? *Ans.* 797.
- f. What number multiplied by 87 gives a product of \$22446? *Ans.* \$258.
- g. Divide 759000 by 10, by 1000, by 100, and add the quotients. *Ans.* 84249.
- h. What is the sum of 93600 divided by 20, and 93600 divided by 7200? *Ans.* 4693.
- i. How many are $493689 \div 47000$? *Ans.* 10 $\frac{23689}{47000}$.
- j. 37884 is 42 times what number? *Ans.* 902.
- k. What is 1 thirty-eighth of 856406? *Ans.* 22537.
- l. The product of two factors is 5063; one of them is 83. What is the other? *Ans.* 61.
- m. The product of three factors is 28350; two of them are 42 and 75. What is the third? *Ans.* 9.
- n. $\frac{3130548}{12} + \text{one fourth of } 2700 = \text{what number?}$ *Ans.* 261554.

Examples with Decimals.

- o. Divide 42.8116 by 13. *Ans.* 3.2932.
- p. What is 1 ninth of \$76.842? *Ans.* \$8.538.
- q. What is the cost of each chair if 25 chairs can be bought for \$247? *Ans.* \$9.88.

NOTE. The examples on this page embrace the principal varieties in form of examples in division. Examples for additional practice will be found on pages 61 and 63.

113. Applications.

51. If I travel 42 miles a day, in how many days can I travel 273 miles ?

52. How many barrels are required to hold 5488 pounds of flour, if one flour-barrel holds 196 pounds ?

53. How many days are there in 9684 hours ?

54. How many days will it take a ship to sail 13724 miles, at the rate of 133 miles a day ?

55. There are 5280 feet in a mile. How many miles high is Mount Everest, which is 29002 feet high ?

56. A. B. bought a farm for \$18785 at \$95 an acre. How many acres were there in the farm ?

57. A produce dealer packed 19152 eggs in boxes containing 144 eggs each. How many boxes did he fill ?

58. If the dealer would put 19152 eggs into 84 equal-sized boxes, how many eggs should he put in a box ?

59. In one year, Missouri produced 4164 tons of lead, worth \$353940. What was the value of a ton ?

60. There was sent to the U. S. Mint in 13 years \$4377984 worth of gold. What was the average value sent a year ? If gold was worth 16 dollars an ounce, and 12 ounces make a pound, how many pounds were sent ?

114. Examples with Decimals.

61. A man divided among his three sons 887.625 acres of land. What was each son's share ?

62. What is the price of 1 comb, when 48 combs can be bought for \$53.76 ?

63. When 234 oranges are bought for \$7.02, what is paid for 1 orange ?

In the following examples continue dividing to the third order of decimals :

64. Find 1 ninth of 1.28 acres. (67.) $8.1 \div 21 = ?$

65. Find 1 twelfth of 3.75 tons. (68.) $0.5 \div 33 = ?$

66. Find 1 fifteenth of 128.5 miles. (69.) $1.868 \div 215 = ?$

SECTION VI.

MISCELLANEOUS EXERCISES.

115. General Review, No. 1.

1. $287 + 5 \text{ million} + 36 \text{ thousand} + 59481 = ?$
2. Add 567 to the sum of the following numbers: 121, 232, 343, 454, 565, 676, 787, and 898.
3. The difference between two numbers is 95478. The larger number is 148769; what is the smaller?
4. Which of the two numbers 15672 or 10560 is nearer to 13465, and how much?
5. Take 987 from each of the following numbers, and add the remainders: 3644; 7573; 2432; 4001.
6. What number must be added to the difference between 58 and 7003 to equal 938000?
7. What number taken from the quotient of $1833000 \div 24$ leaves 25?
8. What number equals the product of the three factors 1785, 394, and $624 - 48$?
9. If 5872 be the multiplicand, and half that number the multiplier, what is the product?
10. If 4832796 is the product, and 1208199 the multiplicand, what is the multiplier?
11. If 894869 is the minuend, and the sum of the numbers in the fifth example is the subtrahend, what is the remainder?
12. If 700150 is a dividend, and 3685 the quotient, what is the divisor?
13. If 28936 is the divisor, and 86 is the quotient, what is the dividend?
14. Divide 87 million by 15 thousand.

For other questions in review, see pages 59 - 63.

116. Oral Examples for Analysis.

(See Appendix, page 302.)

a. If a car runs 69 miles in 3 hours, how far can it run in 5 hours?

b. If 18 rows of potatoes yield 36 bushels, how many bushels will 20 similar rows yield?

c. If \$5 pay for 35 quarts of berries, how many quarts will \$12 buy?

d. If, when flour is \$8 a barrel, a ten-cent loaf weighs 25 ounces, what should it weigh when flour is \$10 a barrel?

e. If 5 oxen consume 185 pounds of hay in 2 days, how much will be required for 1 yoke of oxen for the same time?

f. If 6 cows were bought for \$224 and sold for \$260, what was the gain on each cow?

g. If 150 barrels of apples were bought for \$200 and sold for \$350, what would be gained by selling 45 barrels at the same rate?

h. I bought a lot of paint for \$3.90 and sold it for \$5.10, gaining 12 cents on a pound. How many pounds did I buy?

i. If a quantity of hay lasts 22 oxen 10 days, how many days will it last 5 yoke?

j. A field of wheat was reaped by 10 men in 6 days; what length of time would be required for 15 men to reap the same amount?

k. A cistern can be emptied in 15 minutes by 7 pipes; in what time can it be emptied, if only 5 of the pipes are open?

l. If 8 operatives can do a piece of work in 12 days, in what time will 24 operatives perform the same work?

m. If a certain piece of work can be performed by 50 men in 4 weeks, how many more must be employed to perform it in a week?

n. Ten hunters have provisions to last them 6 weeks; if 2 men be killed, how long will the provisions last the remainder?

117. Miscellaneous Examples.

15. A merchant bought goods for \$1084, and sold them for \$594 more than he gave. How much did he receive for them?

16. From a farm containing 984 acres there were sold at one time 347 acres, at another time 157 acres. How many acres remained?

17. A merchant bought goods for \$2467, and sold them for \$875 less than he gave. How much did he receive for them?

18. If I take 7642 gallons from 21002 gallons twice, what will remain?

19. Of 30070 men who went into battle, 4564 were slain and 1675 were taken prisoners. How many were left?

20. Bought two horses; the first cost \$215, the second \$40 less than the first. How much did the two horses cost?

21. If \$19.74 were paid for 14 bushels of wheat, what must be paid for 25 bushels?

22. If 19 tons of coal run an engine 798 miles, how far will 14 tons run it?

23. The area of the New England States is as follows: Maine, 31766 square miles; New Hampshire, 9280; Vermont, 10212; Massachusetts, 7800; Connecticut, 4674; Rhode Island, 1306. How many more square miles are there in Maine than in the three States of Vermont, New Hampshire, and Massachusetts?

24. How many States of the size of Rhode Island might be made out of Massachusetts, and how many square miles would remain?

25. How much smaller is Connecticut than Vermont?

26. Texas contains 237504 square miles. How many States of the size of New England might be made out of it, and how many States of the size of New Hampshire out of the remainder?

27. If 5 bushels of wheat of 60 pounds each are required to make 1 barrel of flour, how many pounds of wheat are required to make 100 barrels of flour?

28. In a certain schoolhouse 9 of the rooms will seat 52 pupils each, and 4 will seat 48 pupils each. How many pupils can be seated in all ?

29. How many feet of fencing will be required to enclose a lot of land measuring on each of two sides 489 feet, on the third 548 feet, and on the fourth 596 feet ?

30. In a school there are 7 classes of 54 pupils each; 196 of these are boys. How many are girls ?

31. A horse cost \$262, a chaise \$228, and a hack 3 times as much as both. What did all cost ?

32. A farmer exchanged 4 cows, worth \$68 each, for a span of horses. What were the horses worth apiece ?

33. A merchant bought 45 bales of cotton, each bale containing 42 pieces, and each piece 38 yards, at 9 cents a yard, and sold the whole at 11 cents a yard. How much did he gain ?

34. A man raised in one year 364 bushels of corn, the next year twice as much as he did the first year, and the third year three times as much as the second year. How many bushels did he raise in all ?

35. A grocer bought 8 chests of tea, each chest containing 48 pounds, at 50 cents a pound. He sold one half of the tea at 65 cents a pound and the other half at 72 cents a pound. How much did he gain ?

36. After \$158 were taken from a box there remained \$15 more than twice that sum. How many dollars remained ?

37. Mrs. Keyes, having \$2000 to invest, bought 10 United States bonds at \$112 each, and then as many railroad shares at \$92 each as she could pay for. How much money was left ?

38. Mr. Oaks bought a piano for \$375, paid \$14 for freight and cartage, and \$2 for tuning, then let it 7 quarters at \$15 a quarter, and afterwards sold it for \$325. Did he gain or lose, and how much ?

39. A man paid \$270 for a threshing-machine, and hired help to run it at \$5 a day. He then let the machine at \$8 a day, including the help he hired. How many days must he let the machine to pay its first cost ?

40. How many posts and how many rails will be required for a fence 156 feet long if the posts are set 12 feet apart and the fence is 5 rails high?

41. A man sold three houses; for the first he received \$3525, for the second \$950 less than he received for the first, and for the third as much as for the other two. How much did he receive for the three?

42. A jeweller sold 15 clocks and 22 watches; for the clocks he received \$12 apiece, and for each watch 7 times as much as for a clock. What did he receive for all?

43. If 28 men can grade a road in 72 days, how long will it take 36 men to do half the work?

44. If a man earns \$180 a month and spends \$36 for board and \$50 for clothes and other expenses, in how many months can he save \$1410?

45. Mr. Brown bought 18 cords of wood for \$110. For how much must he sell it a cord to gain \$34 on the whole?

46. Mr. Snow bought some land for \$13825. He sold 100 acres at \$55 an acre, and then found, in order not to lose on his bargain, that he must sell the remainder for \$62 an acre. How many acres were there in the remainder?

47. A had \$45; B twice as much less \$17; and C as much as A and B together. How much money had C?

48. One half of one number is 1764, and four times another number is 5876. What is their sum?

49. A and B, 450 miles apart, travel towards each other. A travels at the rate of 30 miles a day and B of 35 miles a day. If B rests the second day, how far apart are they at the end of the fourth day?

50. A man bought 163 barrels of flour at \$9 a barrel; 15 barrels were spoiled, and the remainder he sold at \$11 a barrel. Did he gain or lose, and how much?

51. At an election the sum of the votes received by two opposing candidates was 4324; the successful candidate received 218 more votes than his opponent. How many votes did each receive?

52. On commencing business a merchant had \$ 7852 in cash, \$ 7919 in real estate, goods valued at \$ 9728, a lot of lumber valued at \$ 6930, a ship valued at \$ 16834; during the first year he was in trade he gained above all his expenses \$ 3195. What was he worth at the end of the year ?

53. The Gulf Stream carries 2787840000000 cubic feet of water past a given point every hour, which is 1200 times as much as the hourly discharge of the Mississippi. What is the hourly discharge of the Mississippi ?

54. There were 1032467 cigars made in Westfield in 1 month. If these were bought for 5 cents apiece, how many families would the money thus spent supply with bread for a year (365 days) if each family should consume two 8-cent loaves a day ?

55. The distance from Boston to Albany is 202 miles, from Albany to Buffalo, 298 miles. How long will it take a train to pass over the road at the rate of 28 miles an hour, allowing 2 hours for detentions between Boston and Albany, 1 hour at Albany, and 3 between Albany and Buffalo ?

56. If it takes 5 yards of cloth to make a pair of shirts, what will 24 pairs cost at 15 cents per yard for the cloth, 45 cents apiece for bosoms, wristbands, and buttons, and 95 cents apiece for making ?

57. In how many days, of 6 hours each, can the president of a bank sign 90000 bank-notes, if he signs 5 in a minute ?

58. If 8 presses can coin 19200 pieces of money in an hour, how many pieces can one press coin in a minute, 60 minutes making an hour ?

Papyrus is said to have been used to write upon 2000 years before Christ, and parchment to have been invented 1810 years later; from the invention of parchment to that of paper in China was 20 years; to printing by movable types was 1608 years more; stereotyping was invented 273 years still later.

59. How many years from the first use of papyrus, as given above, to stereotyping ?

60. In what year before Christ was paper invented in China ?

61. In what year after Christ was printing invented ?

62. There are in a certain school 47 pupils 14 years old; 96 pupils 12 years old; 114, 11 years; 149, 10 years; and 168, 9 years old. What is their average age?

63. If the earth is 92000000 of miles from the sun, and the moon at its full is 224000 miles farther on, and light travels at the rate of 185172 miles a second, how many seconds is it in passing from the sun to the moon and back to the earth?

118. Questions for Review.

What is ADDITION? What is the amount? Add orally 64 and 87. How do you write numbers to be added? Is this absolutely necessary? Add five numbers expressed by four figures each, and explain. Give the rule; the proof. Illustrate adding at once numbers expressed in two or more columns.

What is SUBTRACTION? What is the minuend? the subtrahend? the remainder? Take orally 28 from 91. Find the difference between 368 and 7006, and explain. Give the rule; the proof. When the minuend and difference are given, how can you find the subtrahend? When the subtrahend and difference are given, how can you find the minuend?

What is MULTIPLICATION? What is the multiplicand? the multiplier? the product? What are factors? Multiply orally 45 by 6. Perform and explain an example in which the multiplier has at least two terms. Give the rule; the proof. How do you multiply by 10, 100, 1000, etc.? How do you proceed if there are zeros at the right of the expression of the multiplicand or the multiplier, or both?

Tens \times units = what? Units \times tens? Thousands \times tens? Tens \times hundreds? Ten-thousands \times hundreds?

What is DIVISION? What is the dividend? the divisor? the quotient? the remainder? Perform and explain an example in short division; prove the work. Perform and explain an example in long division. Give the rule; the proof. How do you divide by 10, 100, 1000, etc.? How do you divide when the expression of the divisor contains zeros at the right? When the dividend and quotient are given, how can you find the divisor? When the divisor and quotient are given, how can you find the dividend? When the multiplier and product are given, how can you find the multiplicand? When the multiplicand and product are given, how can you find the multiplier?

DRILL EXERCISES.

119. EXPLANATION OF THE USE OF THE DRILL TABLES.

The object of the Drill Tables and Exercises which are found on the six following pages is to extend indefinitely practice in arithmetical operations without additional labor on the part of the teacher.

The exercises are not to be assigned in order, nor is any one pupil expected to perform them all ; they may be used, however, like other examples. (See Notes on pages 16, 25, 35, and 48.)

The following illustration shows how they may be used for class drill, and each pupil have a different example.

Addition.

1. Let the members of the class number themselves 1, 2, 3, etc., to any given number up to 25 ; and let each member find his number in the left-hand margin of the table.

2. The teacher then gives a direction in this form : "Add A, B, and C." (See Exercise 1, page 59.)

3. In obedience to this direction, each pupil will add the numbers that he finds expressed under the letters A, B, and C, and in the line of his own number. Thus, pupil No. 1 will add 65, 512, and 7901 ; No. 2 will add 34, 724, and 3053 ; and so on.

Thus a series of examples is given out at a single dictation, and the pupils are taught to work independently.

4. The key contains answers to all these examples.

Subtraction, Multiplication, and Division.

By changing slightly the form of direction described above, the same table will afford abundant practice in the other fundamental operations. (See pages 59, 61, and 63.)

120. DRILL TABLE No. 1.

Simple Numbers.

Examples.	A	B	C	D	E	F
	fg	hij	k lmn	opq	r stu	vw xyz
1.	65	512	7 901	274	2 865	40 912
2.	34	724	3 053	613	3 742	83 640
3.	79	235	5 360	769	8 604	20 357
4.	46	941	1 604	133	6 821	67 904
5.	98	858	8 029	486	4 930	13 580
6.	56	467	7 940	918	5 439	80 439
7.	32	673	4 809	675	7 108	50 672
8.	48	388	6 580	436	4 583	38 204
9.	87	747	2 096	577	8 057	29 073
10.	76	599	8 920	814	6 974	68 370
11.	54	252	2 031	239	9 107	90 521
12.	95	381	6 150	721	1 580	15 023
13.	82	817	4 706	544	7 362	56 807
14.	71	426	9 059	715	2 115	80 195
15.	23	794	4 270	645	4 276	41 730
16.	37	261	3 011	978	6 583	67 054
17.	93	638	5 490	851	2 941	57 026
18.	28	372	1 705	327	8 724	20 687
19.	62	919	9 630	163	6 239	34 610
20.	57	485	5 108	784	4 037	25 048
21.	89	591	7 502	516	5 482	90 572
22.	45	183	2 610	297	9 372	72 056
23.	92	868	3 703	466	6 048	46 190
24.	64	655	6 207	349	3 659	20 839
25.	25	942	8 054	922	4 176	81 704
	1	2	3	4	5	6 7

121. Exercises upon the Table.

Addition.

1. Add A,* B, and C.
2. Add B, C, and D.
3. C plus D plus E plus F equals what number?
4. $A + B + C + D + 16042 = ?$
5. What is the sum of B, C, D, E, F, and 61375?
6. Find the amount of A, B, C, D, E, F, and 23456.

In each column indicated by figures at the bottom of pages 58, 60, and 63,

7. Add the upper six numbers.
8. Add the upper ten numbers.
9. Add the upper fifteen numbers.
10. Add all the numbers.

Subtraction.

11. From C take B.
12. Subtract D from E.
13. Take E from F.
14. Find the difference between C and E.
15. F minus C equals what number?

Multiplication.

16. Multiply B by 6.
17. Multiply C by 7.
18. Multiply D by 8.
19. Multiply E by 9.
20. Multiply B by A.
21. Multiply C by B.
22. Multiply C by D.
23. Multiply E by D.
24. Find the product of F by D.
25. Find the product of F by C.

* See the explanation, page 57.

Review.

26. What number added to the amount of A and B will equal C?
27. Add together C, E, and the difference between B and D.
28. Subtract C from 12304, and from the remainder take B.
29. Multiply D by 1002, and from the product take F.
30. Multiply C by 6; D by 7; E by 8; and find the sum of the products.
31. Multiply B by 10; D by 11; and add the products with C plus E.
32. A man having F dollars paid E dollars to one man and D dollars to another. How much did he have left?
33. Bought a house for C dollars; paid B dollars for repairs; then sold it at a loss of D dollars. How much did I receive for the house?
34. A merchant had B barrels of flour. He sold A barrels at \$12 a barrel, and the remainder at \$9 a barrel. How much did he receive for the flour?

Oral Practice.

35. How many are $8 + f + g + h$, etc. to z?
36. How many are $27 + h + i$, etc. to z?
37. How many are $55 - f - g - h - i - j$?
38. How many are $100 - o - p$, etc. to z?
39. How many are $7 + f$ to n less o less p less q?
40. How many are h times i less j plus k to z?
41. How many are 100 less A?
42. Find the difference between 43 and A.

122. DRILL TABLE No. 2.

Simple Numbers.

Examples.	A	B	C	D	E
	ef	ghi	j klm	nop qrs	t uvw xyz
1.	45	648	6 068	469 007	8 046 352
2.	68	473	5 406	743 500	6 530 781
3.	74	835	8 049	620 085	4 654 380
4.	56	592	7 250	800 659	7 820 463
5.	48	726	3 087	389 700	9 068 318
6.	64	954	4 503	956 800	5 782 630
7.	78	367	2 790	285 004	3 905 746
8.	83	289	5 608	500 376	2 849 370
9.	54	635	9 160	675 400	6 470 856
10.	76	489	6 085	732 900	5 783 062
11.	58	375	8 507	486 300	7 560 849
12.	47	689	4 130	840 015	3 672 083
13.	65	764	7 082	570 068	4 921 608
14.	53	865	3 706	962 400	9 430 572
15.	49	796	8 154	435 600	4 187 063
16.	84	347	4 370	700 843	6 410 342
17.	79	586	5 480	659 700	1 598 706
18.	63	627	1 094	800 875	7 421 089
19.	57	395	8 406	643 007	6 450 713
20.	69	738	6 530	350 092	2 835 068
21.	75	579	7 209	800 765	5 306 739
22.	67	486	8 560	469 007	3 540 687
23.	59	942	4 308	782 500	7 390 468
24.	46	278	3 960	946 007	6 083 745
25.	73	587	6 805	465 300	7 408 653
	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u> <u>12</u>	<u>13</u> <u>14</u>

123. Exercises upon the Table.

Division.

43. Divide D by 4.
44. Divide D by 5.
45. Divide E by 6.
46. Divide E by 7.
47. Divide D by 8.
48. Divide D by 9.
49. Divide C by 12.
50. Divide C by 15.
51. Divide D by 16.
52. Divide D by 18.
53. Divide E by 27.
54. Divide C by A.
55. Divide D by A.
56. Divide E by B.
57. Divide D by C.
58. Divide E by C.
59. Divide D by 800.
60. Divide E by 4200.

Addition.

61. How many are
 $46872 + A$ to D?
62. How many are
 $65478 + A$ to E?

Subtraction.

63. From E take D.
64. Find the difference between E and $D \times 10$.

Multiplication.

65. Multiply D by C.
66. Multiply E by D.

Review.

67. How many more than C are B times B?
68. What number added to ten times the amount of B and C will equal D?
69. A man owns three tracts of land; the first is valued at C dollars, the second at B dollars, and the third is worth twice as much as the second. How much is the land worth?
70. By selling a house at C dollars I gained 12 times A dollars. What was the cost?
71. If a farmer should purchase B acres of land at A dollars per acre, and pay down C dollars, how much would he then owe for the land?
72. A man having C dollars spent B dollars and lost A dollars. How much would one third of the remainder be?
73. How many cows, at A dollars apiece, can be bought for one fifth of ten times B dollars, and how many dollars will remain?

Oral Practice.

74. How many are $6 + e + f + g$, etc. to z?
75. How many are $15 + g + h$, etc. to z?
76. How many are $29 + j + k$, etc. to z?
77. How many are $e \times f - g - h$?
78. How many are $g \times h \div i$?
79. How many are $h \times i \div j$?
80. Divide A by 2; by 3; by 4; 5; 6; 7; 8; 9.
81. Divide gh (64, 47, 88, etc.) by 3; by 4; etc.

Other dividends and divisors can be indicated, as jk by 7; no by 8; tu by 9; etc.

124. DRILL TABLE No. 3.**Simple Numbers.****Examples.****A**

- | | |
|-----|---|
| 1. | Nine hundred fourteen thousand, forty-one. |
| 2. | One million, forty thousand, fourteen. |
| 3. | Nine hundred seventy-six thousand, sixty-seven. |
| 4. | Sixteen hundred seventy-eight. |
| 5. | Sixty-three million, three hundred six thousand. |
| 6. | Nineteen million, nine hundred thousand, 19. |
| 7. | One hundred seventy million, seven. |
| 8. | Ten million, one thousand, one hundred one. |
| 9. | Three hundred five million, fifty thousand. |
| 10. | Twelve million, two hundred thousand, two. |
| 11. | One million, eighteen thousand, eight. |
| 12. | One billion, six hundred thousand, six. |
| 13. | Nine hundred four million, ninety-four. |
| 14. | Three billion, thirty million, three hundred three. |
| 15. | Two billion, four hundred twelve thousand, 14. |
| 16. | One hundred one million, one thousand, one. |
| 17. | Six billion, sixty thousand, six hundred. |
| 18. | Four hundred three billion, thirty. |
| 19. | One trillion, seventeen million. |
| 20. | 506 billion, sixty-five thousand, five. |
| 21. | Four trillion, four billion, four thousand. |
| 22. | Six hundred eighty-nine billion, six thousand, 89. |
| 23. | Forty-two trillion, forty thousand, two hundred. |
| 24. | Eighteen trillion, 108 million, eighteen. |
| 25. | Four trillion, forty-seven billion, 4700. |

DRILL TABLE No. 3

(continued).

Ex- amples.	B		C		D		E		
	q	r	s	t	u	v	w	x	y z
1.	7	6	3	8	1	9	3	2	76
2.	4	5	2	3	6	4	4	3	68
3.	9	8	1	4	8	6	9	2	34
4.	3	3	6	3	2	4	2	5	92
5.	8	6	4	5	7	6	7	4	88
6.	5	3	5	5	9	5	6	5	45
7.	2	2	8	7	2	8	4	3	68
8.	4	6	8	9	5	2	7	6	16
9.	3	5	1	6	1	2	4	8	96
10.	6	7	2	6	4	8	7	1	28
11.	8	8	8	3	5	2	8	4	48
12.	7	5	6	3	9	2	8	2	32
13.	2	4	8	9	1	2	2	7	36
14.	5	4	5	5	8	5	2	9	25
15.	9	5	4	8	6	4	6	9	12
16.	6	6	6	5	9	4	4	1	58
17.	3	5	7	6	2	7	8	7	78
18.	7	4	9	5	8	8	9	9	96
19.	5	6	5	7	1	5	6	4	35
20.	8	9	6	7	6	8	5	3	76
21.	2	3	4	8	8	4	6	1	88
22.	4	8	4	6	7	2	8	7	36
23.	9	9	9	4	9	5	7	4	25
24.	7	9	1	2	7	3	9	2	82
25.	6	7	8	6	2	4	3	7	44
		15		16				17	

125. Exercises upon the Table.

82. Express A by figures.
83. Add, in A, the 1st and 2d; 2d and 3d; etc.
84. Add, in A, from 1 to 6; 2 to 7; etc.
85. Find, in A, the difference between the 1st and 2d; the 2d and 3d; etc.
86. Multiply A by 6.
87. Multiply A by 7.
88. Multiply A by 8.
89. Multiply A by 9.
90. Divide A by 6.
91. Divide A by 7.
92. Divide A by 8.
93. Divide A by 9.
94. Multiply C by B; add D to the product; and find the difference between the amount and E.
95. Divide D by B; also divide E by B; and find the difference between the sum of the quotients and D.
96. Divide E by D; subtract the quotient from C; and multiply the remainder by B.
97. Subtract C from D; divide the remainder by B; and with the quotient divide E.

Oral Practice.

98. How many are $9 + q$ to x less y less z ?
99. How many are $34 + r$ to v , divided by q ?
100. How many are $45 + t$ to z , divided by q ?
101. How many are r times $s + t$ to z , divided by q ?
102. How many are t times $u + v$ to z , divided by q ?

SECTION VII.

UNITED STATES MONEY.



126. The picture above represents pieces of metal weighed and stamped by authority of government, and used in buying and selling. Such pieces of metal are *coins*. Each coin represents a unit of value.

127. *Dollars* and *cents* are the units of value chiefly used in business. *Eagles*, *dimes*, and *mills* are also used, but there is no coin to represent a mill.

TABLE.

10 mills	= 1 cent, marked <i>ct.</i> or <i>¢</i> .
10 cents	= 1 dime.
10 dimes or } 100 cents	= 1 dollar, marked <i>\$</i> .
10 dollars	= 1 eagle.

To read and write numbers in United States Money.

128. The dollar, being the principal unit of United States money, is expressed at the left of the decimal point; dimes, cents, and mills, being tenths, hundredths, and thousandths of a dollar, are expressed at the right of the decimal point.

Thus, 11 dollars, 2 dimes, 3 cents, and 4 mills are written
 $\$11.234$;
 and the expression is read, "Eleven dollars twenty-three
 cents four mills."

For exercises in reading and writing, turn to page 73.

129. Oral Exercises in Reduction.

a. How many mills in 1 cent? in 18 cents 5 mills?

By what do you multiply to change cents to mills? dollars to cents?
 dollars to mills?

- | | |
|---|-------------------------------|
| b. Change $\$14.08$ to cents. | e. Change $\$2.625$ to mills. |
| c. Change $\$1.62$ to cents. | f. Change $\$5.02$ to mills. |
| d. Change $\$0.48$ to cents. | g. Change $\$4$ to mills. |
| h. How many dollars are there in 500 cents? | |

By what do you divide to change cents to dollars? mills to dollars?

- i. How many dollars are there in 170 cents? in 3689 cents?
 j. How many dollars are there in 1875 mills? in 4728 mills?

130. In performing the examples above, you have
 changed numbers expressing a certain amount of money
 to numbers whose units are larger or smaller, but without
 changing the amount itself. Such a process is called
reduction.

For other examples in reduction of United States money, see page 73.

131. Examples for the Slate.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

How do you write dollars, cents, and mills, when you are to add or
 subtract them? (Art. 46.)

1. My deposits in a bank were $\$192.92$ and $\$155.37$; of
 this I have withdrawn $\$79.48$, $\$71.62$, and $\$78.21$. What
 is the balance in the bank?

2. What must I pay for 23 yards of silk at $\$2.37$ a yard,
 and 5 yards of lace at $\$1.68$ a yard?

3. What is 1 fifteenth of \$287.40? 1 seventeenth of \$722.50?

In the following examples continue the division to cents and mills. (Art. 102.) In the answers, reject mills when less than 5, and call 5 mills or more 1 cent.

4. What is 1 fifth of \$17? of \$83? *1st Ans.* \$3.40.

5. What is 1 sixteenth of \$981?

6. Eight men chartered a schooner for \$295. What was each man's share of the cost?

7. When 32 lawn-mowers were bought for \$696, what was the price of each?

8. Mr. Rice paid \$198.45 for 35 school desks. What would 168 desks cost at the same price?

To divide one sum of money by another.

132. ILLUSTRATIVE EXAMPLE. At \$2.12 per pair, how many pairs of slippers can be bought for \$100?

WRITTEN WORK.

$$\begin{array}{r}
 212) 10000 \text{ (47} \\
 \underline{848} \\
 1520 \\
 \underline{1484} \\
 36
 \end{array}$$

Explanation. — To divide one sum of money by another, both dividend and divisor must be expressed in the same denomination. Here the divisor being cents, the dividend must be changed to cents. (Art. 129.) Dividing 10000 cents by 212 cents, we have 47 for a quotient, with a remainder of 36 cents. *Ans.* 47 pairs; 36 cents remain.

9. I paid \$80 for turkeys at \$2.50 apiece. How many turkeys did I buy? Divide \$42 by \$1.75. *1st Ans.* 32.

10. A conductor took up \$1224 worth of railroad tickets from Springfield to New York at \$4.25 apiece. How many tickets did he take?

11. How many boxes at 33 cents a box can be bought for \$20, and how many cents will be left?

12. How many veils at 92¢ each can be bought for \$30?

13. How many dinners at \$0.625 each will \$22 pay for?

For additional examples, see page 73.

Coins and Paper Currency.

133. The legal coins of the United States are :

Gold.		Silver.	
Double-Eagle	= \$20.00	Dollar	= \$1.00
Eagle	= 10.00	Half-dollar	= 0.50
Half-Eagle	= 5.00	Quarter-dollar	= 0.25
Quarter-Eagle	= 2.50	Twenty-cent piece	= 0.20
Three-dollar piece	= 3.00	Dime	= 0.10
One-dollar piece	= 1.00		

Copper and nickel 3-cent and 5-cent pieces and bronze 1-cent piece.

NOTE. The gold coin is hardened by an alloy of 1 tenth copper and silver (the silver not to exceed 1 tenth of the whole alloy). The silver coin is hardened by 1 tenth copper. The bronze cent has 95 parts of copper to 5 parts of tin and zinc. The 3-cent and 5-cent pieces have 75 parts of copper to 25 parts of nickel.

The silver 5-cent and 3-cent pieces, the bronze 2-cent piece, and the old copper coins, are no longer issued.

Bank bills and United States Treasury notes (greenbacks) are largely used in place of coins. These represent the values of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, and \$1000.

134. Exercises in reckoning Money.

Perform as many as possible of the following examples without written work :

How much money in

- a.* Two 20-dollar bills, three 10's, four 5's, and seven 1's ?
- b.* Eight 5-dollar bills, seven 2's, five 10's, and three 1's ?
- c.* One 50-dollar bill, six 5's, two 1's, with 3 half-dollars, 5 quarters and 4 dimes ?

How much more money must you receive to have \$50 if you now have

- d.* Three 5-dollar bills, seven 2's, four 1's, with 2 half-dollars, 3 quarters, and four 5-cent pieces ?
- e.* Two 5-dollar bills, three 2's, and one 10, with 5 quarters, 4 dimes, two 3-cent, and three 5-cent pieces ?

f. How much money shall I have left of six 5-dollar bills, and 2 quarters, after paying for 6 yards of brilliant at 65¢ a yard, for Silesia, 28¢, and for buttons \$1.15?

g. What must you pay for 2 dozen eggs, 5 pounds of sugar, 2 gallons of vinegar, and 2 bushels of apples at the present prices where you live?

135. Accounts and Bills.

[EXTRACT FROM THE ACCOUNT-BOOK OF T. SMITH & Co.]

EDWARD WILLIAMS,				<i>Dr.</i>	
1876.				\$	¢
<i>April</i>	<i>6</i>	<i>To 5 bbls. Western Flour, @ \$8</i>		<i>40</i>	<i>00</i>
<i>May</i>	<i>2</i>	<i>" 25 lbs. Malaga Raisins, " 15¢</i>		<i>3</i>	<i>75</i>
<i>June</i>	<i>1</i>	<i>" 16 lbs. Java Coffee, " 30¢</i>		<i>4</i>	<i>80</i>
<i>"</i>	<i>28</i>	<i>" 1 day's work of hired man.....</i>		<i>1</i>	<i>75</i>

136. Above is a record kept by T. Smith & Co. of articles sold and services rendered by them to Mr. Williams.

137. It is customary for persons who buy or sell goods or services to keep a record of the articles bought or sold, the kind and amount of services rendered, their value, etc., as above. Such a record is *an account*.

138. The person to whom a debt is owed is a *creditor*.

139. The person who owes a debt is a *debtor*.

Who is debtor in the account stated above? Who is creditor?

140. A written statement of an account prepared for the debtor by the creditor is a *bill*.

141. When the bill is paid, the creditor, or some one authorized by him, signs his name to the bill, with the words "Received payment." The bill is thus *receipted*. (See bills Nos. 2 and 3.)

Examples for the Slate.

142. Find the cost of each article in the following bills, and their several amounts :

14. (1) *New York, Nov. 12, 1877.*

Mr. George Brewster

Bought of FOWLE, PRATT, & CO.

100 bbls.	St. Louis Flour, extra,	@	* \$9.50		
12 "	Minnesota Flour,	"	10.25		
14 bu.	Corn,	"	87½		
Received payment,				\$	

15. (2) *Cincinnati, Dec. 18, 1876.*

Mr. D. Searles

Bought of J. SMITH.

1876.					
Nov.	1	65 lbs.	Paris Green.	@	35¢
"	3	300 "	Whiting,	"	1¢
Dec.	7	450 "	White Lead,	"	11¢
"	"	35 "	Glue,	"	50¢
Received payment,				J. SMITH.	

16. (3) *Worcester, Oct. 9, 1877.*

Mr. J. E. Drew

To OTIS LERNED, Dr.†

To	2 bbls.	Pork, prime,	@	\$ 15.50		
"	250 lbs.	Hams,	"	9¢		
"	475 "	Butter,	"	24¢		
Received payment,				OTIS LERNED.		
				By JOHN WATTS.		

* This sign means "at," and is commonly used in stating prices.

† This means that Mr. Drew is debtor to Mr. Larned. Dr. is read "debtor."

17.

(4)

New York, Mar. 7, 1877.

Mr. A. M. Brown

To CHARLES DAY, Dr.

1877.					
Jan.	8	To 12 lbs. Tartaric Acid, @	85¢		
"	"	" 7 " Blue Vitriol, "	15¢		
"	"	" 3 oz. Morphine, "	\$7.00		
"	"	" 5 " Quinine, "	4.00		
Feb.	2	" 2 lbs. Cardamoms, "	2.50		
"	"	" 8 " Camphor, "	43¢		
"	"	" 8 gals. Cod-Liver Oil, "	1.75		

Received payment,

CHARLES DAY.

18.

(5)

Bristol, Jan. 1, 1877.

Mr. Thomas Butler

To A. R. PEASE, Dr.

1876.					
Apr.	3	To 96 lbs. Rice, @	7¢		
"	"	" 3 " Salsaparilla, "	8¢		
"	"	" 20 " Starch, "	9¢		
May	9	" 210 " Crushed Sugar, "	13¢		
June	3	" 46 " Breakfast Tea, "	80¢		
July	10	" 37 " Japan Tea, "	90¢		
Cr.*					
May	10	By 1 Wagon.....	\$23.00		
"	16	" 2 Cows, @ \$34			
"	"	" Cash.....	10.00		
			\$		
Balance due A. R. Pease.....					

Received payment,

* This means that Mr. Butler is credited for goods or cash delivered. Cr. is read "creditor."

Examples for Bills.

143. Find the amounts due in the following examples, and make out the bills, supplying dates, etc., when wanting.

19. Charles Miller bought of James Gibbs, Jan. 4, 1877, 1 horse for \$95.00, 2 cows at \$50 apiece, 1 wagon for \$62.00, 2 shovels at \$1.12 apiece, 30 bushels of corn at 65 ¢ per bushel, and 17 bushels of wheat at \$1.62 per bushel.

20. Samuel Briggs sold to Alfred Loomis 2 pieces flannel, of 62 yards each, at 49 ¢ per yard; 38 yards ticking, at 29 ¢; 86 yards brown sheeting, at 27 ¢; and 42 yards broadcloth at \$3.65.

21. Dr. Holland bought of John Avery 9 pounds oil of peppermint at \$2.50; 4 pounds oil of cassia at \$1.62; 4 pounds oil of orange at \$3; 6 pounds oil of lemon at \$3.25; 5 pounds oxalic acid at 13 ¢; and 5 pounds Seneca root at 95 ¢.

22. Banks & Searles, of Cleveland, bought of Snow & Rising, Albany, 24 sack coats at \$15.75; 36 vests at \$3.50; 9 dozen felt hats at \$36 per dozen; 4 dozen pairs suspenders at 42 ¢ per pair; and 23 dozen pairs gloves at 68 ¢ per pair.

23. J. D. Furber bought of C. O. Clement, Nov. 8, 1876, 2 Dictionaries, at 87 ¢ apiece; 9 Vocal Cultures, at 90 ¢, and 24 Spellers, at 20 ¢. Dec. 2, he bought 2 reams of paper at \$2.12, 3 dozen pencils at 50 ¢, and 12 slates at 17 ¢. Dec. 10, he paid Mr. Clement \$20.00, and Jan. 1, 1877, Mr. Clement made out his bill. Required the balance due.

24. Sell to your neighbor 4 pear-trees at \$1.75 each, 9 tomato-plants at 7 ¢ each, 5 geraniums at 30 ¢ each, and make out the bill.

25. Sell three different articles from a dry-goods store, and make out the bill.

26. Make out a bill for 3 days' work at 75 ¢ a day, 4 days' work at \$1.50 a day, and 2 bushels of cranberries at \$4 a bushel, crediting the person against whom you make the bill with 5 hours' work at 35 ¢ an hour.

144. DRILL TABLE No. 4.

United States Money.

Examples.	A	B
1.	\$ 18.40	Twelve dollars, twenty-five cents.
2.	\$ 83.22	Seventy-one dollars, ninety cents.
3.	\$ 36.41	Twenty-five dollars, sixty-two cents.
4.	\$ 30.05	Eighteen dollars, nine cents.
5.	\$ 204.75	One hundred thirty dollars, six cents.
6.	\$ 9.208	Five dollars, seven cents, five mills.
7.	\$ 5.632	One dollar, ninety cents, eight mills.
8.	\$ 876.	One hundred dollars, twenty cents.
9.	\$ 100.35	Forty-nine dollars, seventy-two cents.
10.	\$ 15.207	Six dollars, seven cents, three mills.
11.	\$ 1.36	Twenty-seven cents, five mills.
12.	\$ 20.95	Twelve dollars, nineteen cents.
13.	\$ 0.402	Twenty-five cents, five mills.
14.	\$ 19.005	Sixteen dollars, six mills.
15.	\$ 63.072	Forty-nine dollars, twenty-four cents.
16.	\$ 7.645	Five dollars, sixty-seven cents, five mills.
17.	\$ 419.28	Ninety-nine dollars, fifty-six cents.
18.	\$ 0.625	Seventeen cents, eight mills.
19.	\$ 500.57	Thirty-eight dollars, five mills.
20.	\$ 268.06	89 dollars, fifty cents, three mills.
21.	\$ 29.70	Ninety-two cents, five mills.
22.	\$ 11.005	Seventy-five cents, five mills.
23.	\$ 100.02	Fifty-four dollars, nine cents.
24.	\$ 444.44	Four dollars, forty-four cents, four mills.
25.	\$ 100.10	Nine dollars, nine cents, nine mills.

145. Exercises on Table No. 4.

103. Read as dollars, cents, and mills, the numbers expressed in A.
 104. Read decimally the numbers expressed in A.
 105. Write in figures the numbers expressed in B.
 106. Disregarding the mills, change the numbers expressed in A to cents.
 107. Change the numbers expressed in B to mills.
 108. Add the numbers from 1 to 8* in A to each number expressed in B.
 109. Add the numbers expressed in A and B, (1st) from 1 to 4*; (2d) from 2 to 5; (3d) from 3 to 6, etc.
 110. $\$900 - A = ?$ 114. $A \times 9 = ?$ 118. $A \div 7 = ?$
 111. $A - B = ?$ 115.† $A \div 10 = ?$ 119.† $A \div \$0.25 = ?$
 112. $A \times 6 = ?$ 116.† $B \div 11 = ?$ 120.† $A \div \$0.16 = ?$
 113. $B \times 7 = ?$ 117.† $B \div 12 = ?$ 121. $B \div \$0.005 = ?$
 122. If a person saves a sum equal to A in one month, how much will he save in 18 months?
 123. How many pounds of sugar, at 8 cents a pound, can be bought for each sum of money expressed in A?

146. Questions for Review.

What are the units of United States money? Give the table. How are dollars, cents, and mills expressed by figures? What is considered the principal unit? Give the sign for dollars. How do you change dollars to cents? dollars to mills? cents to mills? mills to dollars? cents to dollars?

How do you add numbers in United States money? How do you subtract? When you multiply, where do you put the decimal point in the product? Divide \$185 by 7, continue the division to mills, and explain. What is necessary in order to divide one sum of money by another? Divide \$900 by 36 cents.

What are coins? Why is paper money sometimes used in place of coins? Name the gold coins; the silver coins.

What is a creditor? a debtor? an account? a bill? How is a bill receipted?

* Inclusive.

† See page 66, note.

‡ Reject mills.

147. Miscellaneous Examples.

27. A girl bought a pair of boots for \$2.37, another pair for \$1.65, slippers for \$1.25, and shoes for 82¢. What was the whole cost?

28. I bought a horse for \$95.00, a wagon for \$63.00, and a harness for \$15.00; kept them a week, paying \$2.50 for board of the horse, then sold them for \$175.00. Did I gain or lose, and how much?

29. What should I pay for 2 dozen pigeons at 85¢ per dozen, 2 dozen at \$1.10 per dozen, and 1 dozen for 90¢.

30. There were sold in one week 8874 sheep at \$4.13 per head. What did they bring?

31. There were sold 4778 beeves, averaging 874 pounds apiece, at 7¢ per pound. What was received for them?

32. What did I gain by buying 2 pieces of cambric, each containing 62 yards, for \$39.68, and selling them for 40 cents per yard?

33. A man paid \$16.25 for 13 days' work. What was that a day?

34. Among how many boys must \$12 be distributed, that each may receive 75 cents?

35. I sold 35 barrels Pippins at \$1.75 per barrel, 17 barrels Pome Royals at \$1.80 per barrel, 13 barrels Golden Sweets at \$1.25 per barrel, and 25 of Russets at \$2.25 per barrel. Paid 17 cents a barrel for picking, and \$12.00 for freight. What remained after my expenses were paid?

36. Paid \$3.00 for 1 dozen apple-trees, \$3.36 for 1 dozen peach-trees, \$3.30 for half a dozen pear-trees. What did I pay for the whole, and how much apiece for each kind?

37. A carpenter paid for stock and work for a barn, \$450.75; for mason's work, \$38.25; for digging and stoning cellar, \$47.18; for painting, \$40.00; to the plumber, \$8.12. He then sold the barn, and lost, in so doing, \$14.30; how much did he sell it for?

SECTION VIII.

FACTORS.

148. What numbers multiplied together will produce 10? *Answer*, 2 and 5; also 1 and 10; thus, $2 \times 5 = 10$ and $1 \times 10 = 10$.

A number that may be used as multiplicand or as multiplier to make another number is a *factor* of that number.

Name two factors of 15; of 16; of 18; of 24; of 36; of 45.

NOTE I. The word *factor* will be used in this Arithmetic to denote only such factors as are not fractional.

NOTE II. If a number be divided by any of its factors there will be no remainder. Hence a factor of a number is also called a *divisor* or a *measure* of that number.

149. Name some factors of 12 besides the number itself and 1. Has the number 13 factors besides itself and 1? Has the number 14? 15? 17? 18? 19?

A number that has other factors besides itself and one is a *composite number*.

Which of the numbers 12, 13, 14, 15, 17, 18, 19 are composite numbers?

150. A number that has no other factors besides itself and one is a *prime number*.

Which of the numbers 12, 13, 14, 15, 17, 18, 19 are prime numbers?

Name the composite numbers from 1 to 40.

Name the prime numbers from 1 to 40.

NOTE. In speaking of the factors of a number, we do not usually include the number itself and one. Thus, we frequently say that a prime number has no factors.

151. Name the factors of 12 that are prime numbers. Name those that are not prime numbers.

A factor that is a prime number is a *prime factor*.

152. Oral Exercises.

- a. What are the prime factors of 6? 8? 14? 24? 27?
- b. What are the prime factors of 22? 36? 28? 20? 35?
- c. What are the prime factors of 16? 21? 15? 33? 26?

153. In seeking for the factors of a number we may use certain tests, the more convenient of which are the following:

1. A number whose units' figure is 0, 2, 4, 6, or 8, is divisible by 2.

NOTE. A number that is divisible by 2 is an *even number*; a number that is not divisible by 2 is an *odd number*.

2. A number is divisible by 3 if the sum of its digits* is divisible by 3. Thus, 285 is divisible by 3, for $2+8+5=15$ is divisible by 3.

3. A number is divisible by 4 if its tens and units together are divisible by 4. Thus, 6724 is divisible by 4, while 6731 is not.

4. A number is divisible by 5 if the units' figure is either 0 or 5.

5. A number is divisible by 6 if it is an even number and divisible by 3.

6. A number is divisible by 8 if its hundreds, tens, and units are divisible by 8. Thus, 6728 is divisible by 8, while 6724 is not.

7. A number is divisible by 9 if the sum of its digits is divisible by 9.

8. A number is divisible by 11 if the sums of its alternate digits are equal, or if their difference is divisible by 11. Thus, 1782 and 1859 are divisible by 11, while 4987 is not.

9. A number is divisible by a *composite number*, if it is divisible by all the factors of the composite number. Thus 3555 is divisible by 15, for it is divisible by 3 and by 5.

NOTE. For the reasons of these tests, see Appendix, page 303.

* A digit here means the number denoted by a figure without regard to its place.

154. Oral Exercises.

Using the tests described above,

- a. Name the numbers expressed in B, page 58, that contain the factor 2; 4; 5.
- b. Name the numbers in C, page 58, that contain the factor 3; 6; 9.
- c. Name the numbers in D, page 58, that contain the factor 8; 9; 10; 100.

To find the Prime Factors of a Number.

155. ILLUSTRATIVE EXAMPLE I. What are the prime factors of 2205?

WRITTEN WORK.

$$\begin{array}{r|l} 3 & 2205 \\ .3 & 735 \\ 5 & 245 \\ 7 & 49 \\ & 7 \end{array}$$

Ans. 3, 3, 5, 7, 7.

Explanation. — Applying the tests (Art. 153) to the given number, we find that 2 is not, but that 3 is, a factor of 2205; and, by dividing, see that $2205 = 3 \times 735$.

Seeking, in the same way, a prime factor of 735, we find that $735 = 3 \times 245$. Continuing this process, we find that $245 = 5 \times 49$, and that $49 = 7 \times 7$. Therefore, $2205 = 3 \times 3 \times 5 \times 7 \times 7$, and the prime factors are 3, 3, 5, 7, and 7.

156. ILLUSTRATIVE EXAMPLE II. What are the prime factors of 409?

WRITTEN WORK.

$$\begin{array}{r|l} 19) 409 (21 & 23) 409 (17 \\ 38 & 23 \\ \hline 29 & 179 \\ 19 & 161 \\ \hline 10 & 18 \end{array}$$

Explanation. — Applying the tests (Art. 153), we find that 409 is not divisible by 2, 3, or 5. We then try to divide by the other prime numbers in order until we reach 23, when we see that the quotient is less than the divisor. There can then be no prime factor

in 409 greater than 23, for if there were, there would be another factor (the quotient) less than 23, which we should have found before reaching 23. The number 409 is therefore prime.

157. As we have found in Art. 155 that 2205 equals the product of all its prime factors, so we shall always find that *A composite number equals the product of all its prime factors.*

158. When a composite number is expressed as a product of prime factors, it is said to be *separated into its prime factors*.

159. From the above examples may be derived the following

Rule.

To separate a number into its prime factors :

1. *Divide the given number by one of its prime factors.*
2. *Divide the quotient thus obtained by one of its prime factors ; and so continue dividing until a quotient is obtained that is a prime number.*
3. *This quotient and the several divisors are the prime factors sought.*

Proof.

Multiply together the prime factors thus found. The product ought to equal the given number.

NOTE. If no prime factor is readily found by which to divide, we try to divide by the several prime numbers in order. If no prime factor is found before the quotient becomes less than the trial divisor, the given number is prime. See Illustrative Example II.

160. Examples for the Slate.

Separate into prime factors the following numbers :

- | | | | |
|-----------|-----------|-----------|-------------|
| (1.) 180. | (4.) 208. | (7.) 329. | (10.) 644. |
| (2.) 192. | (5.) 260. | (8.) 338. | (11.) 684. |
| (3.) 176. | (6.) 169. | (9.) 357. | (12.) 2500. |

Select the prime numbers and find the prime factors of the composite numbers among the following :

- | | | | |
|------------|------------|------------|-------------|
| (13.) 341. | (18.) 450. | (23.) 704. | (28.) 945. |
| (14.) 344. | (19.) 590. | (24.) 711. | (29.) 972. |
| (15.) 362. | (20.) 560. | (25.) 762. | (30.) 2688. |
| (16.) 367. | (21.) 596. | (26.) 808. | (31.) 1164. |
| (17.) 408. | (22.) 689. | (27.) 836. | (32.) 3248. |

SYMBOLS OF OPERATION.

161. The signs $+$, $-$, \times , and \div , since they indicate that certain operations (adding, subtracting, multiplying, and dividing) are to be performed, are called *symbols of operation*.

162. In expressing a series of operations by aid of these signs, it is often necessary to indicate that an operation is to be performed on two or more numbers combined. This is done by writing the numbers to be operated upon, with the proper signs, and enclosing the whole expression in marks of parenthesis or brackets. *The expression so enclosed is then treated as if it denoted a single number.*

Thus,

$(7 + 2) \times 5$ means that the sum of 7 and 2 is to be multiplied by 5; but $7 + 2 \times 5$ means that 7 is to be increased by 5 times 2.

$(7 - 2) \times 3$ means that the difference between 7 and 2 is to be multiplied by 3; but $7 - 2 \times 3$ means 7 diminished by 3 times 2.

$(7 + 2) \div 5$, or $\frac{7+2}{5}$,* means that the sum of 7 and 2 is to be divided by 5.

$[(2 + 3) \times 5 - 11] \times 2$ means that the sum of 2 and 3 is to be multiplied by 5, the product diminished by 11, and the remainder multiplied by 2.

163. In performing a series of operations indicated by signs,

First, operate on the numbers that are written within parentheses as indicated by the signs. Next, multiply and divide as indicated by the signs \times and \div . Finally, add and subtract as indicated by the signs $+$ and $-$.

* The horizontal line here drawn between $7 + 2$ and 5 is equivalent to marks of parenthesis.

164. Oral Exercises.

[The Key contains answers to the following examples.]

a. $(6 + 8) \times 5 = ?$

h. $3 \times 8 \div 4 \times 3 = ?$

b. $6 + 8 \times 5 = ?$

i. $3 \times 8 \div (4 \times 3) = ?$

c. $(8 - 3) \times 2 = ?$

j. $14 - \frac{3 \times 4 - 2 \times 3}{2} = ?$

d. $8 - 3 \times 2 = ?$

e. $8 + 12 \div 4 = ?$

k. $\frac{8 + 3}{2} + \frac{8 - 3}{2} = ?$

f. $(8 + 12) \div 4 = ?$

g. $(2 + 1) \times (7 - 2) = ?$

l. $[(4 + 6) \times 4 - 5 \times 3] \times 3 = ?$

CANCELLATION.

165. ILLUSTRATIVE EXAMPLE I. If 4 be multiplied by 3 and the product divided by 3, what is the result?

WRITTEN WORK.

From this example we see that

$$\frac{4 \times \cancel{3}}{\cancel{3}} = 4$$

Ans. 4.

If a given number be multiplied by a number, and the product be divided by the same number, the result will be the given number.

In such cases, both the multiplication and the division may be omitted.

NOTE. This omission is indicated in the written work above by drawing a mark through the 3 thus, $\cancel{3}$.

166. ILLUSTRATIVE EXAMPLE II. What is the result of dividing the product of 4 and 6 by 3?

WRITTEN WORK.

$$\frac{4 \times \cancel{6}}{\cancel{3}} = 8$$

Explanation. — As $6 = 2 \times 3$, the dividend in this example is $4 \times 2 \times 3$, and the divisor is 3, so that we may strike out the factor 3 in both dividend and divisor, and multiply by 2 only, thus shortening the work.

The process of shortening work by striking out equal factors in dividend and divisor is *cancellation*.

167. Examples for the Slate.

All operations upon numbers should first be indicated, as far as possible, by signs, that the work to be done may be shortened, if possible, by cancellation.

33. Divide 81×42 by 99×7 .
34. Multiply 75×10 by 3×6 , and divide that product by $15 \times 25 \times 12$.
35. Divide $7 \times 8 \times 48$ by $63 \times 4 \times 5 \times 17$, and multiply the quotient by 51.
36. If 5 sets of chairs, 6 in a set, cost \$75, what did 1 chair cost?
37. If it requires 13 bushels of wheat to make 3 barrels of flour, how many bushels will be required to make 78 barrels of flour?
38. If a tree 54 feet high casts a shadow of 90 feet, what length of shadow will be cast by a flag-staff 105 feet high?
39. A grocer exchanged 561 pounds of sugar, at 12 cents per pound, for eggs at 22 cents per dozen. How many dozen were received?
40. If 12 pieces of cloth, each piece containing 62 yards, cost \$372, what do 24 yards cost?
41. If the work of 7 men is equal to the work of 9 boys, how many men's work will equal the work of 90 boys?
42. If 15 men consume a barrel of flour in 6 weeks, how long would it last 9 men?
43. If 12 men can build a wall in 42 days, how many days will be required for 21 men to build it?
44. If \$15 purchase 12 yards of cloth, how many yards will \$48 purchase?
45. A ship has provision for 15 men 12 months. How long will it last 45 men?
46. How many overcoats, each containing 4 yards, can be made from 10 bales of cloth, 12 pieces each, 42 yards in each piece?

COMMON FACTORS.

168. ILLUSTRATIVE EXAMPLE I. What numbers are factors of both 18 and 24 ?

WRITTEN WORK.

$$18 = 2 \times 3 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

Ans. 2, 3, and 6.

Explanation. — Separating 18 and 24 into their prime factors, we find 2 and 3, and consequently 6 (which is the product of 2 and 3), to be factors of both 18 and 24.

Name any common factor of 12 and 15; of 12 and 18; of 30 and 40.

169. Numbers that have no common factors are said to be *prime to each other*.

Thus, 14 and 15 are prime to each other, though they are not prime numbers.

170. The greatest factor which is common to two or more numbers is their *greatest common factor*.

What is the greatest factor which is common to 18 and 24 ? to 40 and 50 ? to 45 and 54 ?

171. We have seen that 6, the greatest common factor of 18 and 24, is the product of 2 and 3, the only prime factors common to 18 and 24. *The greatest common factor of any two or more numbers is the product of all the prime factors which are common to those numbers.*

NOTE. The letters g. c. f. are used for *greatest common factor*.

To find the Greatest Common Factor.

172. ILLUSTRATIVE EXAMPLE II. Find the greatest common factor of 12, 30, and 48.

WRITTEN WORK.

$$12 = 2 \times 2 \times 3$$

$$\text{g. c. f.} = 2 \times 3 = 6$$

Explanation. — The prime factors of 12 are 2, 2, and 3. The product of such of these as are common to 30 and 48 must be the g. c. f. required.

We find that 2 is a factor of both 30 and 48 ; therefore 2 is a factor of the g. c. f. We find that but one 2 is a factor of 30; therefore only

one 2 is used as a factor of the g. c. f. We find that 3 is a factor of both 30 and 48 ; therefore 3 is a factor of the g. c. f. Thus the g. c. f. sought is 2×3 , equal to 6. Hence the following

Rule.

173. To find the greatest common factor of two or more numbers : *Separate one of the numbers into its prime factors, and find the product of such of them as are common to the other numbers.*

174. Examples for the Slate.

Find the greatest common factor

47. Of 48, 56, and 60.

48. Of 24, 42, and 54.

49. Of 108, 45, 18, and 63.

50. Of 18, 36, 12, 48, and 42.

NOTE. In Example 50, 18 is a factor of 36, and 12 of 48. The g. c. f. of 18 and 12 must be the g. c. f. of 18, 12, and their *multiples* 36 and 48 ; hence we need only find the g. c. f. of 18, 12, and 42.

Find the greatest common factor

51. Of 42, 28, and 84.

53. Of 32, 18, 108, and 25.

52. Of 26, 52, and 65.

54. Of 114, 102, 78, and 66.

55. What is the width of the widest carpeting that will exactly fit either of two halls, 45 feet and 33 feet wide, respectively ?

56. A has a piece of ground 90 feet long and 42 feet wide. What is the length of the longest rails that will exactly suit both its length and its width ?

57. What is the length of the longest stepping-stones that will exactly fit across each of three streets, 72, 51, and 87 feet wide, respectively ?

58. What is the length of the longest curb-stones that will exactly fit each of four strips of sidewalk, the first being 273 feet long, the second 294, the third 567, the fourth 651 ?

175. When numbers cannot readily be separated into their factors, the following method for finding the greatest common factor may be adopted.

ILLUSTRATIVE EXAMPLE. Find the greatest common factor of 52 and 91.

WRITTEN WORK.

$$\begin{array}{r}
 52) 91 \text{ (1)} \\
 \underline{52} \\
 39) 52 \text{ (1)} \\
 \underline{39} \\
 13) 39 \text{ (3)} \\
 \underline{39} \\
 \hline
 \end{array}$$

Divide the greater number by the less, and then divide the less number by the remainder, if there be any. Continue dividing the last divisor by the last remainder until nothing remains. The last divisor will be the g. c. f. sought.

NOTE. As the explanation of this method is somewhat difficult for younger pupils, it is not given here, but will be found in the Appendix, page 304.

To find the g. c. f. of more than two numbers, *find the g. c. f. of any two of them and then of that common factor and a third number, and so on till all the numbers are taken.*

176. Find the greatest common factor

59. Of 323 and 663.

61. Of 6581 and 1127.

60. Of 147 and 966.

62. Of 187, 442, and 969.

For other examples in factoring, see page 123.

MULTIPLES.

177. Name some numbers which are made by using 3 as a factor. *Ans.* 3, 6, 9, 12, etc.

Any number made by using another number as a factor is a *multiple* of the number thus used.

178. Name the multiples of 4 and of 6 to 36.

Ans. { Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36.
 { Multiples of 6 are 6, 12, 18, 24, 30, 36.

Which of these numbers are multiples of both 4 and 6?

Numbers which are multiples of two or more numbers are *common multiples* of these numbers.

Thus 12, 24, and 36 are common multiples of 4 and 6.
Name a common multiple of 3 and 5; name two more.

179. Oral Exercises.

Name any six multiples of 5. Name three multiples of 12.
Name all the multiples of 11 up to 140. Name any common multiple of 10 and 6. Of 3, 6, and 5.

Least Common Multiple.

180. Name the least number which is a multiple of both 4 and 6. *Ans.* 12.

The least number which is a multiple of two or more numbers, is the *least common multiple* of those numbers.

Name the least common multiple of 2 and 5; of 6 and 9.

NOTE. The letters l. c. m. are used for *least common multiple*.

181. As any number contains all its prime factors, a multiple of any number must contain all the prime factors of that number.

A common multiple of two or more numbers must contain all the prime factors of those numbers, and

The least common multiple of two or more numbers is the least number which contains all the prime factors of those numbers.

182. ILLUSTRATIVE EXAMPLE I. What is the least common multiple of 6, 9, and 15?

WRITTEN WORK.

$$6 = 2 \times 3$$

$$9 = 3 \times 3$$

$$15 = 3 \times 5$$

$$l. c. m. = 2 \times 3 \times 3 \times 5 = 90$$

Explanation. — The least multiple of 6 is 6, which may be expressed in the form 2×3 .

The least multiple of 9 is 9, which may be expressed in the form 3×3 . But in 6 we have already one of the factors (3) of 9; hence if we put with the prime

factors of 6 the remaining factor (3) of 9, we shall have $2 \times 3 \times 3$, which are all the factors necessary to produce the l. c. m. of 6 and 9.

The least multiple of 15 is 15, which may be expressed in the form 3×5 . In the l. c. m. of 6 and 9 we have one of the prime factors (3) of 15; hence if we put with the prime factors of 6 and 9 the remaining factor (5) of 15, we shall have $2 \times 3 \times 3 \times 5$, which are all the prime factors necessary to produce the l. c. m. of 6, 9, and 15.

The product of these factors is 90, which is the l. c. m. sought.

NOTE. In finding the least common multiple, the factors of the given numbers seldom need to be expressed, and the written work may be greatly reduced. Thus, in this example the written work may be simply $\text{l. c. m.} = 2 \times 3 \times 3 \times 5 = 90$.

183. From the explanation above may be derived

Rule I

To find the least common multiple of two or more numbers: *Take the prime factors of one of the numbers; with these take such prime factors of each of the other numbers in succession as are not contained in any preceding number, and find the product of all these prime factors.*

184. Oral Exercises.

What is the least common multiple

- | | |
|---------------------|----------------------|
| a. Of 4, 5, and 8? | c. Of 6, 14, and 21? |
| b. Of 6, 8, and 12? | d. Of 3, 4, and 5? |

When several numbers are prime to each other, what must their least common multiple equal?

185. Examples for the Slate.

Find the least common multiple

63. Of 8, 18, 20, and 21. 64. Of 3, 5, 12, 36, and 45.

NOTE. When one of the given numbers is contained in another, the smaller may be disregarded in the operation; thus, in the preceding example, 3, 5, and 12 may be rejected. Why?

Find the least common multiple

- | | |
|----------------------------|----------------------------|
| 65. Of 18, 36, 60 and 72. | 68. Of 18, 32, 48, and 52. |
| 66. Of 12, 42, 56, and 70. | 69. Of 16, 28, 35, and 63. |
| 67. Of 13, 28, 39, and 49. | 70. Of the nine digits. |

186. The above is a good method for finding the least common multiple when the numbers are easily separated into their prime factors. For larger numbers observe the following method:

ILLUSTRATIVE EXAMPLE II. Find the l. c. m. of 18, 56, 38, and 30.

WRITTEN WORK.

2) 18, 56, 38, 30

3) 9, 28, 19, 15

3, 28, 19, 5

l. c. m. = $2 \times 3 \times 3 \times 28 \times 19 \times 5 = 47880$

and those that are not common must be the l. c. m. sought.

Explanation. — Here, by repeated divisions, we take out all the factors that are common to two or more of the given numbers. The product of these factors (2 and 3)

Rule II.

187. To find the least common multiple of two or more numbers:

1. *Write the given numbers in a line as dividends. Make any prime number which is a factor of two or more of the given numbers a divisor of those numbers.*

2. *Write the quotients and undivided numbers beneath as new dividends, and so continue dividing till the last quotients and undivided numbers are prime to each other.*

3. *The product of all the divisors, last quotients, and undivided numbers is the least common multiple required.*

188. Examples for the Slate.

Find the least common multiple

71. Of 338, 364, and 448.

75. Of 165, 9500, and 855.

72. Of 184, 390, and 552.

76. Of 1146, 484, and 24.

73. Of 308, 616, and 77.

77. Of 880, 9680, and 176.

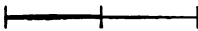
74. Of 84, 336, and 472.


78. Of 187, 539, and 8470.

For other examples in multiples, see page 123.

SECTION IX.

COMMON FRACTIONS.

189. If a unit, as 1 inch, is divided into two equal parts,
 one of the parts is called one half.

If the unit is divided into three equal parts, one of the
 parts is called one third; two of the parts are called two thirds.

One of the equal parts of a unit is a *fraction*, or *fractional unit*. A collection of fractional units is a *fractional number*.

NOTE I. For the sake of brevity, fractional units and fractional numbers are both called *fractions*.

NOTE II. A number whose units are entire things is an *integral* number, or an *integer*.

Name a fractional unit; a fractional number; an integer.

190. The unit of which the fraction is a part is the *unit of the fraction*.

191. The number of equal parts into which the unit of the fraction is divided is the *denominator* of the fraction.

Thus, in the fraction two thirds the denominator is *three*.

192. The number of equal parts taken is the *numerator* of the fraction.

Thus, in the fraction two thirds the numerator is *two*.

193. The numerator and denominator are called the *terms* of the fraction.

NOTE. Decimal fractions have been treated of in previous articles. All fractions except decimal fractions are called *common fractions*.

Writing Common Fractions.

194. The terms of a fraction are written, the numerator above and the denominator below a line. Thus, *two thirds of an inch* is written as in the margin.

Numerator, $\frac{2}{3}$ inch.
Denominator, $\frac{2}{3}$

195. Exercises.

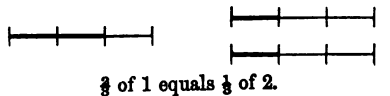
Write in figures the following:

- a. One half of a mile.
- b. One third of a day.
- c. Seven tenths of a dollar.
- d. Twenty twenty-fifths.
- e. Twelve thirds.
- f. Seven sevenths.
- g. Write any fraction you please, having for a denominator five; seven; ten; seventeen; one hundred.
- h. Write any fraction you please, having for a numerator six; eight; sixty; one hundred.
- i. Where is the denominator of a fraction written? Where is the numerator written?
- j. Which is the greater part of a thing, $\frac{1}{3}$ or $\frac{1}{4}$? $\frac{1}{2}$ or $\frac{1}{3}$?

196. The form of writing fractions as shown above is the same as the fractional form used to indicate division. (Art. 94.) Thus the expression $\frac{2}{3}$ may mean *two thirds of one* or *one third of two*.

ILLUSTRATION.

The fact that $\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of 2 may be illustrated as in the margin.



197. Exercises.

- a. What is meant by the expression $\frac{5}{9}$?
Ans. It means 5 of the 9 equal parts into which a unit is divided, or it means 1 ninth of 5 units.
- b. What is meant by the expression $\frac{7}{8}$? $\frac{11}{16}$? $\frac{4}{7}$? $\frac{13}{18}$? $\frac{15}{16}$?
- c. Illustrate the fact that $\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of 3; that $\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of 2.

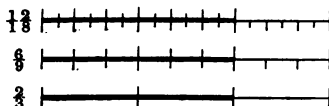
REDUCTION.

To change a Fraction to smaller or larger terms.

198. ILLUSTRATIVE EXAMPLE I. Change $\frac{1}{2}$ to equivalent fractions of smaller terms.

WRITTEN WORK. *Explanation.* — By dividing both terms of $\frac{1}{2}$ by 2, we make the terms half as large, and have the fraction $\frac{1}{4}$. Now dividing both terms of the fraction $\frac{1}{4}$ by 3, we make its terms one third as large, and have the fraction $\frac{1}{12}$. If we had divided both terms of $\frac{1}{2}$ by 6, we should have made the terms one sixth as large, and obtained at once the fraction $\frac{1}{6}$.

ILLUSTRATION.



The illustration shows that the same part of the unit is expressed by $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{12}$. In obtaining $\frac{1}{4}$ and $\frac{1}{12}$ from $\frac{1}{2}$, the number of parts taken has been diminished as the size of the parts has been increased.

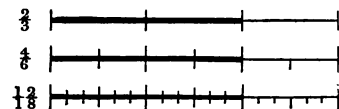
If both terms of a fraction are divided by the same number, the value of the fraction will not be changed.

199. ILLUSTRATIVE EXAMPLE II. Change $\frac{2}{3}$ to equivalent fractions of larger terms.

WRITTEN WORK.

$$\frac{2}{3} = \frac{4}{6}; \quad \frac{2}{3} = \frac{1}{1.5}$$

ILLUSTRATION.



Explanation. — By multiplying both terms of $\frac{2}{3}$ by 2, we make the terms twice as large, and have the fraction $\frac{4}{6}$. By multiplying both terms of $\frac{2}{3}$ by 6, we make the terms six times as large, and have the fraction $\frac{12}{18}$. Here the number of parts in each case is

increased as the size of the parts is diminished.

If both terms of a fraction are multiplied by the same number, the value of the fraction will not be changed.

200. When the terms of a fraction have no common factor, the fraction is said to be expressed in its *smallest terms*.

201. Oral Exercises.

Perform mentally the examples given below, naming results merely; thus, " $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$," and so on.

a. Change to their smallest terms: $\frac{2}{4}$; $\frac{3}{6}$; $\frac{4}{8}$; $\frac{5}{10}$; $\frac{6}{12}$; $\frac{7}{14}$; $\frac{8}{16}$; $\frac{9}{18}$; $\frac{10}{20}$; $\frac{11}{22}$; $\frac{12}{24}$.

b. Change to their smallest terms: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{10}$; $\frac{1}{11}$; $\frac{1}{12}$; $\frac{1}{13}$; $\frac{1}{14}$; $\frac{1}{15}$; $\frac{1}{16}$; $\frac{1}{17}$; $\frac{1}{18}$; $\frac{1}{19}$; $\frac{1}{20}$.

c. Change to their smallest terms: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{10}$; $\frac{1}{11}$; $\frac{1}{12}$; $\frac{1}{13}$; $\frac{1}{14}$; $\frac{1}{15}$; $\frac{1}{16}$; $\frac{1}{17}$; $\frac{1}{18}$; $\frac{1}{19}$; $\frac{1}{20}$.

d. Change $\frac{2}{3}$ to equivalent fractions, having 12, 16, 28, 44, 100, and 120 for denominators.

e. Change $\frac{1}{2}$ to equivalent fractions, having 27, 54, 99, and 900 for denominators.

f. Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, each to an equivalent fraction having 120 for a denominator.

g. How many thirtieths in $\frac{1}{2}$? in $\frac{1}{3}$? in $\frac{1}{4}$? in $\frac{1}{5}$? in $\frac{1}{6}$? in $\frac{1}{7}$? in $\frac{1}{8}$?

h. How many 24ths in $\frac{1}{2}$? in $\frac{1}{3}$? in $\frac{1}{4}$? in $\frac{1}{5}$? in $\frac{1}{6}$? in $\frac{1}{7}$? in $\frac{1}{8}$?

To change a Fraction to its smallest terms.

202. From previous illustrations we may derive the following

Rule.

To change a fraction to an equivalent fraction of the smallest terms: *Strike out all the factors which are common to the numerator and denominator; or divide both terms by their greatest common factor.*

203. Examples for the Slate.

Change to equivalent fractions of smallest terms:

- | | | | |
|------------------------|------------------------|-------------------------|-------------------------|
| (1.) $\frac{36}{50}$. | (4.) $\frac{56}{80}$. | (7.) $\frac{34}{100}$. | (10.) $\frac{12}{36}$. |
| (2.) $\frac{48}{72}$. | (5.) $\frac{64}{96}$. | (8.) $\frac{56}{88}$. | (11.) $\frac{34}{48}$. |
| (3.) $\frac{32}{72}$. | (6.) $\frac{72}{96}$. | (9.) $\frac{37}{84}$. | (12.) $\frac{36}{84}$. |

For other examples, see page 123.

To change Improper Fractions to Integers or to Mixed Numbers.

204. A fractional number, the numerator of which equals or exceeds the denominator, is called an *improper fraction*.

205. ILLUSTRATIVE EXAMPLE III. Change $\frac{60}{12}$ and $\frac{47}{12}$ as far as possible to integers.

WRITTEN WORK.

$$(1.) \quad \begin{array}{r} 12 \overline{) 60} \\ \underline{12} \\ 48 \\ \underline{12} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

Ans. 5

$$(2.) \quad \begin{array}{r} 12 \overline{) 47} \\ \underline{36} \\ 11 \end{array}$$

Ans. $3\frac{11}{12}$

Explanation.—(1.) Since 12 twelfths make a unit, in 60 twelfths there are as many units as there are 12's in 60, which is 5. Ans. 5.

(2.) In $\frac{47}{12}$ there are as many units as there are 12's in 47, which is 3 and $\frac{11}{12}$. Ans. $3\frac{11}{12}$.

206. The number $3\frac{11}{12}$ consists of an integer and a fraction. A number consisting of an integer and a fraction is a *mixed number*.

207. Oral Exercises.

a. Change to integral numbers: $\frac{8}{4}$; $\frac{10}{5}$; $\frac{15}{3}$; $\frac{25}{5}$; $\frac{40}{8}$; $\frac{6}{3}$; $\frac{14}{7}$; $\frac{24}{3}$; $\frac{28}{4}$; $\frac{36}{9}$; $\frac{42}{6}$; $\frac{56}{8}$; $\frac{72}{9}$; $\frac{84}{12}$; $\frac{96}{12}$; $\frac{45}{5}$; $\frac{60}{6}$.

b. Change to mixed numbers: $\frac{13}{3}$; $\frac{8}{3}$; $\frac{13}{5}$; $\frac{15}{8}$; $\frac{12}{5}$; $\frac{22}{5}$; $\frac{28}{3}$; $\frac{35}{5}$; $\frac{42}{3}$; $\frac{18}{5}$; $\frac{51}{8}$; $\frac{44}{11}$; $\frac{37}{16}$; $\frac{56}{8}$; $\frac{63}{9}$; $\frac{68}{8}$; $\frac{71}{7}$.

c. Change to integers or to mixed numbers: $\frac{7}{7}$; $\frac{21}{3}$; $\frac{8}{2}$; $\frac{38}{2}$; $\frac{55}{5}$; $\frac{48}{6}$; $\frac{54}{9}$; $\frac{31}{12}$; $\frac{110}{11}$; $\frac{108}{9}$; $\frac{22}{2}$; $\frac{37}{7}$.

208. From previous illustrations we may derive the following

Rule.

To change an improper fraction to an integer or a mixed number: *Divide the numerator by the denominator.*

209. Examples for the Slate.

Change to integers or to mixed numbers:

$$(13.) \quad \frac{25}{5}. \quad (15.) \quad \frac{175}{10}. \quad (17.) \quad \frac{1254}{10}. \quad (19.) \quad \frac{527}{12} \text{ days.}$$

$$(14.) \quad \frac{108}{13}. \quad (16.) \quad \frac{575}{12}. \quad (18.) \quad \frac{1789}{36}. \quad (20.) \quad \frac{281}{4} \text{ years.}$$

To change an Integer or a Mixed Number to an Improper Fraction.

210. ILLUSTRATIVE EXAMPLE IV. Change $23\frac{1}{4}$ to fourths.

WRITTEN WORK.

Explanation.— Since in 1 there are 4 fourths, in $23\frac{1}{4} = 2\frac{3}{4}$ Ans. 23 there are 23 times 4 fourths, or 92 fourths, which, with 1 fourth added, are 93 fourths.

$$\begin{array}{r} 4 \\ 93 \end{array}$$

Ans. $2\frac{3}{4}$.

211. Oral Exercises.

a. Change to improper fractions: $2\frac{1}{2}$; $3\frac{2}{3}$; $2\frac{3}{4}$; $5\frac{1}{8}$; $2\frac{3}{4}$; $3\frac{1}{2}$; $6\frac{1}{2}$; $5\frac{2}{3}$; $5\frac{1}{2}$; $7\frac{2}{3}$; $7\frac{1}{2}$; $8\frac{2}{3}$; $8\frac{2}{3}$; $9\frac{1}{2}$; $9\frac{2}{3}$; $10\frac{1}{2}$.

b. Change to improper fractions: $2\frac{1}{8}$; $2\frac{3}{8}$; $3\frac{1}{11}$; $3\frac{4}{7}$; $4\frac{1}{3}$; $4\frac{2}{3}$; $5\frac{2}{3}$; $9\frac{2}{3}$; $6\frac{2}{3}$; $7\frac{1}{8}$; $8\frac{2}{3}$; $9\frac{4}{7}$; $4\frac{2}{7}$; $4\frac{2}{3}$; $8\frac{1}{3}$; $7\frac{1}{2}$.

c. Change 5 to ninths; 11 to fifths; 14 to thirds; 8 to twelfths; 15 to fourths; 1 to sevenths.

d. Among how many persons must 7 melons be divided that each may receive $\frac{1}{4}$ of a melon? $\frac{1}{8}$? $\frac{1}{10}$?

e. How many persons will $5\frac{1}{2}$ cords of wood supply if each person receives $\frac{1}{2}$ of a cord? $\frac{1}{4}$ of a cord? $\frac{1}{8}$ of a cord?

212. From previous illustrations may be derived the following

Rule.

To change an integer or a mixed number to an improper fraction: *Multiply the integer by the denominator of the fraction, and to the product add the numerator; the result will be the numerator of the required fraction.*

213. Examples for the Slate.

Change the following to improper fractions :

- (21.) $69\frac{2}{3}$. (24.) $76\frac{4}{5}$. (27.) Change 48 to ninths.
 (22.) $272\frac{1}{4}$. (25.) $10\frac{1}{2}$. (28.) Change 567 to tenths.
 (23.) $109\frac{3}{8}$. (26.) $66\frac{2}{3}$. (29.) Change 93 to forty-thirds.

For other examples in reduction of fractions, see page 123.

ADDITION OF FRACTIONS.

To add Fractions having a Common Denominator.

214. ILLUSTRATIVE EXAMPLE I. Add $\frac{3}{8}$ of an apple, $\frac{2}{8}$ of an apple, and $\frac{1}{8}$ of an apple. *Ans.* $\frac{6}{8}$ of an apple.

These fractions are like parts (eighths) of the same or similar units (apples). Such fractions are *like fractions*.

215. Like fractions have the same denominator, which, because it belongs to several fractions, is called a *common denominator*.

216. Oral Exercises.

- a. Add $\frac{7}{15}$, $\frac{4}{15}$, and $\frac{3}{15}$. e. Add $\frac{3}{40}$, $\frac{9}{40}$, and $\frac{2}{40}$.
 b. Add $\frac{15}{180}$, $\frac{28}{180}$, and $\frac{7}{180}$. f. Add $\frac{1}{12}$, $\frac{2}{12}$, and $\frac{4}{12}$.
 c. Add $\frac{3}{8}$, $\frac{5}{8}$, $\frac{8}{8}$, and $\frac{2}{8}$. g. Add $\frac{20}{2000}$, $\frac{20}{2000}$, and $\frac{70}{2000}$.
 d. Add $\frac{7}{18}$, $\frac{2}{18}$, $\frac{1}{18}$, and $\frac{4}{18}$. h. Add $\frac{44}{360}$, $\frac{2}{360}$, and $\frac{32}{360}$.

How do you add fractions which have a common denominator?

To add Fractions not having a Common Denominator.

217. ILLUSTRATIVE EXAMPLE. Add $\frac{5}{6}$, $\frac{4}{9}$, and $\frac{7}{15}$.

WRITTEN WORK.

$$2 \times 3 \times 3 \times 5 = 90 \text{ l. c. denom.}$$

$$\frac{5}{6} = \frac{5 \times 15}{6 \times 15} = \frac{75}{90}$$

$$\frac{4}{9} = \frac{4 \times 10}{9 \times 10} = \frac{40}{90}$$

$$\frac{7}{15} = \frac{7 \times 6}{15 \times 6} = \frac{42}{90}$$

$$\text{Ans. } \frac{157}{90} = 1\frac{67}{90}.$$

Explanation. — To be added, these fractions must be changed to like fractions, or to fractions having a common denominator. (Art. 215.) The new denominator must be *some multiple of the given denominators*. A convenient multiple is their *least common multiple*, which is 90. (Art. 182.)

To change $\frac{5}{6}$ to 90ths, the denominator 6 must be multiplied by 3×3 , or 15; hence the numerator 5 must be multiplied by 15. (Art. 199.) Thus, $\frac{5}{6}$ is found to equal $\frac{75}{90}$.

In a similar way $\frac{4}{9}$ will be found to equal $\frac{40}{90}$, and $\frac{7}{15}$ to equal $\frac{42}{90}$. Adding these fractions, we have $\frac{157}{90}$, or $1\frac{67}{90}$, for the sum.

218. Oral Exercises.

a. Add $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{3}$.

Ans. $\frac{3}{12} = \frac{3}{4}$.

b. Add $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$.

Ans. $\frac{13}{8} = 2\frac{1}{8}$.

c. Add $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{2}{3}$.

d. Add $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{1}{12}$.

e. Add $\frac{7}{8}$, $\frac{1}{2}$, and $\frac{3}{4}$.

f. Add $\frac{1}{10}$, $\frac{2}{15}$, and $\frac{1}{6}$.

NOTE. When the denominators are prime to each other, the new denominator will be the product of all the denominators, and the new numerators will be found by multiplying each numerator by the product of all the denominators except its own.

g. Add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{2}$ and $\frac{1}{6}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{4}$ and $\frac{1}{3}$; $\frac{1}{11}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{7}$; $\frac{1}{3}$ and $\frac{1}{10}$; $\frac{1}{3}$ and $\frac{1}{5}$.

h. Add $\frac{3}{4}$ and $\frac{2}{4}$; $\frac{3}{8}$ and $\frac{5}{8}$; $\frac{3}{8}$ and $\frac{2}{8}$; $\frac{3}{4}$ and $\frac{3}{4}$; $\frac{1}{2}$ and $\frac{1}{4}$.

i. Add $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{4}$; $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{2}{8}$; $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{1}{3}$; $\frac{1}{10}$, $\frac{3}{4}$, and $\frac{1}{2}$.

j. If you should spend $\frac{1}{4}$ of your time in school, $\frac{1}{24}$ in practising music, and $\frac{1}{8}$ in sewing and studying, what time would you spend in all?

k. Owning $\frac{3}{8}$ of a paper-mill, I bought the shares of two other persons who owned $\frac{1}{10}$ and $\frac{2}{8}$ respectively. What part of the mill did I then own?

219. From the above examples may be derived the following

Rule.

To change fractions to equivalent fractions having the least common denominator:

1. For the common denominator, find the least common multiple of the given denominators.

2. For the new numerators, multiply the numerator of each fraction by the number by which you multiply its denominator to produce the common denominator.

NOTE. If the number to multiply the numerator by is not readily seen, it may be found by dividing the common denominator by the denominator of the given fraction.

220. From what we have now learned of the addition of fractions, we may derive the following

Rule.

To add fractions :

1. If they have a common denominator, *add their numerators.*

2. If they have not a common denominator, *change them to equivalent fractions that have a common denominator, and then add their numerators.*

221. Examples for the Slate.

$$(30.) \frac{4}{7} + \frac{2}{10} + \frac{1}{2} + \frac{3}{8} = ?$$

$$(35.) \frac{4}{21} + \frac{7}{15} + \frac{1}{3} = ?$$

$$(31.) \frac{4}{8} + \frac{1}{3} * + \frac{1}{2} + \frac{5}{8} = ?$$

$$(36.) \frac{1}{2} + \frac{7}{8} + \frac{5}{2} = ?$$

$$(32.) \frac{5}{8} + \frac{3}{10} + \frac{1}{2} + \frac{3}{8} = ?$$

$$(37.) \frac{3}{4} + \frac{5}{8} + \frac{1}{3} = ?$$

$$(33.) \frac{7}{4} + \frac{4}{8} + \frac{3}{8} = ?$$

$$(38.) \frac{1}{3} + \frac{4}{9} + \frac{2}{3} = ?$$

$$(34.) \frac{2}{5} + \frac{5}{10} + \frac{1}{2} = ?$$

$$(39.) \frac{1}{2} + \frac{8}{11} + \frac{5}{18} + \frac{1}{3} = ?$$

Add the integers and fractions of the following, and similar examples, separately :

40. In my furnace there were burned $2\frac{3}{4}$ tons of coal in December, $2\frac{1}{2}$ tons in January, and $3\frac{1}{2}$ in February. How many tons were burned in all ?

$$(41.) 72\frac{1}{2} + 16\frac{1}{4} + 18\frac{3}{8} + 23\frac{1}{2} + 37\frac{7}{8} = ?$$

42. A horse travelled $43\frac{1}{10}$ miles in one day, $52\frac{1}{2}$ the next, $36\frac{1}{4}$ the third, and $40\frac{3}{4}$ the fourth. How far did he travel in all ?

43. A merchant had three barrels of sugar, the first containing $247\frac{1}{2}$ pounds ; the second, $229\frac{5}{8}$ pounds ; and the third, $260\frac{1}{4}$ pounds. What was the weight of the whole ?

For other examples in addition of fractions, see page 123.

* What operation should first be performed on this fraction ?

SUBTRACTION.

222. Oral Exercises.

- a. $\frac{4}{5}$ less $\frac{3}{8}$ are how many ninths? *Ans.* $\frac{3}{8}$.
 b. $\frac{1}{2} - \frac{2}{3} =$ what? d. $\frac{3}{5} - \frac{1}{3} =$ what?
 c. $\frac{3}{8} - \frac{7}{10} =$ what? e. $\frac{1}{2} - \frac{5}{11} - \frac{7}{11} =$ what?
 f. Find the difference between $\frac{1}{4}$ of a day and $\frac{3}{4}$ of a day.
 g. What must be added to $\frac{7}{10}$ to make $\frac{1}{2}$? $\frac{1}{10}$?

When the minuend and subtrahend are like fractions, how do you subtract?

223. ILLUSTRATIVE EXAMPLE. If $\frac{3}{4}$ of a yard of velvet is cut from a piece containing $\frac{5}{4}$ of a yard, what part of a yard will be left?

WRITTEN WORK.

$$\frac{5}{4} - \frac{3}{4} = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2} \text{ Ans.}$$

Explanation. — That the subtraction may be performed, these fractions must be changed to equivalent fractions having a common denominator. The least common denominator is 12. $\frac{5}{4} = \frac{15}{12}$ and $\frac{3}{4} = \frac{9}{12}$.

$$\frac{15}{12} - \frac{9}{12} = \frac{6}{12} = \frac{1}{2} \text{ Ans.}$$

- h. $\frac{1}{3} - \frac{1}{6}$? $\frac{1}{8} - \frac{1}{4}$? $\frac{1}{8} - \frac{1}{8}$? $\frac{1}{4} - \frac{1}{11}$? $\frac{1}{2} - \frac{1}{8}$? $\frac{1}{3} - \frac{1}{10}$?
 i. $\frac{3}{4} - \frac{2}{4}$? $\frac{3}{8} - \frac{2}{4}$? $\frac{5}{8} - \frac{1}{8}$? $\frac{3}{4} - \frac{1}{8}$? $\frac{1}{11} - \frac{1}{8}$? $\frac{7}{8} - \frac{3}{4}$?
 j. $2 - \frac{5}{8}$? $8 - \frac{7}{8}$? $11 - \frac{3}{4}$? $9 - 3\frac{1}{2}$? $7 - 2\frac{3}{4}$? $8 - 3\frac{3}{4}$?
 k. How many yards will be left if from a piece containing $6\frac{3}{10}$ yards there be taken $1\frac{1}{2}$ yards?
 l. What is the difference in the height of two boys, one being $3\frac{1}{2}$ feet, the other $2\frac{3}{4}$ feet high?
 m. A pole is standing so that $\frac{3}{4}$ of it is in the water, $\frac{1}{4}$ in the mud, and the rest in the air. What part is in the air?
 n. How much will be left of a piece of cloth containing 7 yards, after cutting from it 2 vests and a coat, allowing $\frac{3}{4}$ of a yard for a vest and $4\frac{1}{2}$ yards for a coat?
 o. From a bin containing $23\frac{1}{2}$ bushels of wheat there were taken out $3\frac{3}{4}$ bushels at one time and $4\frac{1}{2}$ bushels at another. How much remained?

224. From the previous illustrations we may derive the following

Rule.

To subtract one fraction from another :

1. If they have a common denominator, *find the difference of their numerators.*

2. If they have not a common denominator, *change them to equivalent fractions which have a common denominator, and then find the difference of their numerators.*

225. Examples for the Slate.

$$(44.) \quad \frac{5}{12} - \frac{2}{21} = ?$$

$$(50.) \quad 12\frac{1}{2} - \frac{3}{4} = ?$$

$$(45.) \quad \frac{7}{13} - \frac{4}{38} = ?$$

$$(51.) \quad 17\frac{1}{2} - 12\frac{9}{16} = ?$$

$$(46.) \quad \frac{9}{36} - \frac{7}{37} = ?$$

$$(52.) \quad 26\frac{3}{4} - 1\frac{1}{8} = ?$$

$$(47.) \quad 36 - \frac{3}{8} = ?$$

$$(53.) \quad 10\frac{3}{4} - 5\frac{1}{8} = ?$$

$$(48.) \quad 19 - 2\frac{1}{8} = ?$$

$$(54.) \quad 17\frac{1}{2} - 2\frac{3}{8} = ?$$

$$(49.) \quad 75 - 15\frac{1}{2} = ?$$

$$(55.) \quad 18\frac{1}{2} - 15\frac{1}{3} = ?$$

For other examples in subtraction of fractions, see page 123.

226. Examples in Addition and Subtraction.

$$(56.) \quad \frac{3}{4} - \frac{2}{8} + \frac{5}{8} = ?$$

$$(60.) \quad 20 - 5\frac{1}{2} + \frac{8}{11} = ?$$

$$(57.) \quad \frac{5}{8} + \frac{4}{11} - \frac{1}{12} = ?$$

$$(61.) \quad 8\frac{2}{7} - 2\frac{3}{4} + 7\frac{2}{3} = ?$$

$$(58.) \quad \frac{3}{4} - \frac{1}{8} + \frac{5}{8} - \frac{1}{12} = ?$$

$$(62.) \quad 7 - (\frac{7}{12} - \frac{2}{36}) = ?$$

$$(59.) \quad \frac{8}{9} - \frac{1}{3} - \frac{4}{9} - \frac{7}{18} = ?$$

$$(63.) \quad 5 - (\frac{3}{8} + \frac{7}{4}) = ?$$

64. Two men start at the same place and travel in opposite directions, one at the rate of $3\frac{1}{2}$ miles per hour, the other at the rate of $4\frac{1}{3}$ miles per hour. How far apart were they at the end of an hour ?

65. Two boats are 5280 feet apart and rowing towards each other, one at the rate of $320\frac{7}{10}$ feet per minute, the other at the rate $309\frac{1}{6}$ feet per minute. How far apart are they at the end of one minute ?

66. From 8 trees I gathered apples as follows: $2\frac{1}{2}$ barrels, $3\frac{1}{2}$ barrels, $5\frac{1}{2}$ barrels, $4\frac{1}{2}$ barrels, $3\frac{1}{2}$ barrels, $1\frac{1}{2}$ barrels, $3\frac{1}{2}$ barrels, and $2\frac{1}{2}$ barrels. If I sold $15\frac{1}{2}$ barrels of the apples to one man and $2\frac{1}{2}$ to another, how many had I left?

67. A lady who had \$50 received $\$8\frac{1}{2}$ more, spent $\$17\frac{3}{4}$, lost $\$4\frac{3}{10}$, and collected $\$15\frac{1}{2}$ of a debt. How much money in dollars and cents had she?

68. A man having a sum of money spent $\frac{1}{5}$ of it for a house, $\frac{1}{10}$ of it for furniture, $\frac{3}{10}$ of it for horses and carriages, and $\frac{3}{10}$ of it to build a church. What part of his money had he left?

69. How much more is the sum of $12\frac{3}{5}$ and $6\frac{7}{10}$ than their difference?

For other examples in addition and subtraction of fractions, see page 123.

MULTIPLICATION.

To multiply a Fraction by an Integer.

227. ILLUSTRATIVE EXAMPLE I. If it takes $\frac{2}{3}$ of a yard of cloth to make 1 apron, how much will it take to make 2 aprons?

Solution. — If it takes $\frac{2}{3}$ of a yard to make 1 apron, to make 2 aprons it will take 2 times $\frac{2}{3}$, or $\frac{4}{3}$ of a yard, equal to $1\frac{1}{3}$ yards.

In multiplying the fraction $\frac{2}{3}$ by 2, which term of the fraction was multiplied? How will you multiply any fraction by an integer?

228. Oral Exercises.

a. Multiply $\frac{2}{3}$ by 2; $\frac{4}{5}$ by 3; $\frac{3}{8}$ by 7; $\frac{1}{10}$ by 4; $\frac{5}{6}$ by 5; $\frac{7}{8}$ by 8; $\frac{9}{10}$ by 6.

b. How many are 3 times $\frac{2}{3}$? 4 times $\frac{3}{4}$; 5 times $\frac{1}{11}$? 8 times $\frac{1}{10}$? 9 times $\frac{2}{10}$?

c. How many are $\frac{1}{2} \times 3$? $\frac{7}{8} \times 2$? $\frac{3}{8} \times 4$? $\frac{1}{10} \times 2$? $\frac{7}{10} \times 6$? $\frac{7}{15} \times 5$? $\frac{7}{10} \times 9$? $\frac{3}{8} \times 20$?

d. If $2\frac{1}{2}$ pounds of cane are required to seat 1 chair, how many pounds will be required to seat 12 chairs?

NOTE. Multiply the integer and the fraction separately.

- e. At \$18 $\frac{1}{2}$ a dozen, what is the cost of 5 dozen lamps?
- f. If 7 men can build a dam in 4 $\frac{1}{2}$ days, in what time can 1 man build it?
- g. At \$10 $\frac{1}{2}$ each in currency, what is the value of 5 gold eagles?
- h. In a piece of land 1 foot long and 1 foot wide there is 1 square foot. How many square feet are there in a piece 8 $\frac{1}{2}$ feet long and 1 foot wide? in a piece 18 $\frac{1}{2}$ feet long and 5 feet wide?
- i. If a man receives \$ $\frac{3}{4}$ for shoeing a horse and \$ $\frac{1}{2}$ for shoeing an ox, how much will he receive for shoeing 4 horses and a yoke of oxen?

229. Examples for the Slate.

ILLUSTRATIVE EXAMPLE II. Multiply $\frac{3}{16}$ by 56.

WRITTEN WORK.

$$\begin{array}{r} 7 \\ 3 \times \cancel{56} = \frac{21}{2} = 10\frac{1}{2} \text{ Ans.} \\ \cancel{16} \end{array}$$

70. If a man can mow $\frac{1}{3}$ of an acre of meadow in 1 hour, how much can he mow in 38 hours?
71. How many yards of cloth are required for 6 suits, each suit requiring 7 $\frac{5}{8}$ yards?
72. What is the width of 18 house lots, each 5 $\frac{1}{2}$ rods wide?
73. What distance can a vessel sail in 33 hours, going at the rate of 5 $\frac{1}{2}$ miles an hour?
74. There are 16 $\frac{1}{2}$ feet in a rod. How many feet are there in 40 rods? in 320 rods, or 1 mile?
75. How much ivory worth \$1 a pound can be bought for the same sum that will pay for 15 $\frac{1}{2}$ pounds worth \$12 a pound?
76. One quart dry measure contains 67 $\frac{1}{2}$ cubic inches. How many cubic inches are there in a bushel, or 32 quarts?
77. If by working 11 hours a day a piece of work can be done in 45 $\frac{1}{2}$ days, in what time can it be done by working 1 hour a day?

78. If 17 men can shear a lot of sheep in $9\frac{3}{5}$ days, in what time can 1 man shear the lot?

79. Multiply $14\frac{1}{2}$ by 9.

82. Multiply $365\frac{1}{2}$ by 39.

80. Multiply $16\frac{3}{5}$ by 7.

83. Multiply $256\frac{1}{4}$ by 18.

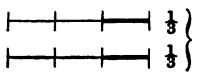
81. Multiply $23\frac{1}{3}$ by 11.

84. Multiply $376\frac{3}{4}$ by 21.

To multiply an Integer by a Fraction.

230. ILLUSTRATIVE EXAMPLE III. What is $\frac{1}{3}$ of 2 inches?

ILLUSTRATION.

 If $\frac{1}{3}$ of each of the 2 inches is taken, we shall have $\frac{2}{3}$ of an inch. *Ans.* $\frac{2}{3}$ of an inch. (See illustration; also Art. 196.)

$\frac{1}{3}$ of 2 = $\frac{2}{3}$.

231. Oral Exercises.

a. What is $\frac{1}{2}$ of 2? $\frac{1}{3}$ of 8? $\frac{1}{4}$ of 4? $\frac{1}{5}$ of 7? $\frac{1}{6}$ of 6?

b. What is $\frac{1}{10}$ of 3? $\frac{1}{4}$ of 2? $\frac{1}{11}$ of 9? $\frac{1}{6}$ of 5? $\frac{1}{12}$ of 5?

232. ILLUSTRATIVE EXAMPLE IV. What is $\frac{2}{3}$ of 2 inches?

Solution. — $\frac{1}{3}$ of 2 inches is $\frac{2}{3}$ of an inch, and $\frac{2}{3}$ of 2 inches must be 3 times $\frac{2}{3}$, or $\frac{4}{3}$ of an inch, equal to $1\frac{1}{3}$ inches. *Ans.* $1\frac{1}{3}$ inches.

c. What is $\frac{2}{3}$ of 7? $\frac{2}{3}$ of 6? $\frac{2}{3}$ of 4? $\frac{2}{3}$ of 5? $\frac{2}{3}$ of 9?

In finding the fractional part of a number, as in the example above, what was the first operation? *Ans.* Dividing the number by the denominator of the fraction. By what was the result multiplied? How then will you find the fractional part of a number?

233. The process by which the fractional part of a number is found is called *multiplying by a fraction*.

ILLUSTRATIVE EXAMPLE V. Multiply 11 by $\frac{4}{5}$.

Solution. — To multiply 11 by $\frac{4}{5}$ is to take $\frac{4}{5}$ of 11. $\frac{1}{5}$ of 11 is $\frac{11}{5}$, and $\frac{4}{5}$ of 11 must be 4 times $\frac{11}{5}$, or $\frac{44}{5} = 8\frac{4}{5}$. *Ans.* $8\frac{4}{5}$.

d. Multiply 8 by $\frac{3}{4}$.

f. Multiply 10 by $\frac{4}{5}$.

e. Multiply 6 by $\frac{7}{8}$.

g. Multiply 12 by $\frac{3}{4}$.

h. In 1 yard there are 3 feet. How many feet are there in $\frac{5}{8}$ of a yard?

Solution.—If in 1 yard there are 3 feet, in $\frac{5}{8}$ of a yard there are $\frac{5}{8}$ of 3 feet; etc.

i. What is the cost of $\frac{7}{10}$ of a ream of paper at \$6 a ream?

j. What is the cost of $\frac{3}{8}$ of an acre of land at \$100 an acre?

Examples for the Slate.

234. ILLUSTRATIVE EXAMPLE VI. What must I pay for $\frac{5}{12}$ of an acre of land at \$165 an acre?

WRITTEN WORK.

$$\begin{array}{r} 55 \\ 165 \times 5 = 825 \\ \underline{12} \\ 4 \end{array} = \frac{275}{4} = 68\frac{3}{4}. \text{ Ans. } \$68\frac{3}{4}, \text{ or } \$68.75.$$

85. A man bought a carriage for \$645, and sold it for $\frac{4}{5}$ of what it cost him. How much did he receive for it?

86. If it requires 9 bushels of apples to make a barrel of cider, how many bushels will be required to make $13\frac{1}{2}$ barrels of cider?

NOTE. Multiply by the integral and the fractional number, separately; then add the results.

87. In 1 pound there are 16 ounces. How many ounces in $13\frac{7}{8}$ pounds?

88. 4 quarts equal 1 gallon. How many quarts are there in $65\frac{3}{4}$ gallons?

89. At \$3 a bundle, what will $26\frac{1}{2}$ bundles of shingles cost?

90. If \$1 will buy 9 pounds of sugar, how much will \$ $19\frac{3}{4}$ buy?

91. What must I pay for $63\frac{1}{2}$ yards of flannel at \$0.54 a yard?

(92.) $36 \times 5\frac{3}{4} = ?$

(95.) $3681 \times 4\frac{5}{8} = ?$

(93.) $568 \times \frac{5}{8} = ?$

(96.) $5432 \times 3\frac{1}{2} = ?$

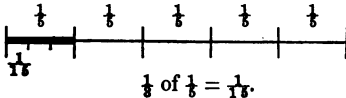
(94.) $385 \times \frac{7}{25} = ?$

(97.) $87036 \times \frac{2}{27} = ?$

To multiply a Fraction by a Fraction.

235. ILLUSTRATIVE EXAMPLE VII. Multiply $\frac{1}{3}$ by $\frac{1}{3}$.

ILLUSTRATION.



Explanation.—To multiply $\frac{1}{3}$ by $\frac{1}{3}$ is to take 1 third of $\frac{1}{3}$. To find 1 third of $\frac{1}{3}$, the $\frac{1}{3}$ must be divided into 3 equal parts. Since the entire unit $\frac{1}{3}$ will contain 5 times 3 or 15 such parts,

one of the parts will be $\frac{1}{15}$ of the unit; hence $\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{15}$. (See illustration.) *Ans.* $\frac{1}{15}$.

In multiplying $\frac{1}{3}$ by $\frac{1}{3}$, how was the new denominator obtained?

236. Oral Exercises

- a. Multiply $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{1}{10}$; $\frac{1}{3}$ by $\frac{1}{7}$; $\frac{1}{3}$ by $\frac{1}{3}$.
 b. Multiply $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{12}$ by $\frac{1}{2}$; $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{11}$ by $\frac{1}{3}$.

237. ILLUSTRATIVE EXAMPLE VIII. Multiply $\frac{4}{5}$ by $\frac{2}{3}$.

Explanation.—To multiply $\frac{4}{5}$ by $\frac{2}{3}$ is to take $\frac{2}{3}$ of $\frac{4}{5}$. $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$ ($\frac{1}{3} \times \frac{4}{5}$); then $\frac{2}{3}$ of $\frac{4}{5}$ must be $\frac{8}{15}$ ($\frac{2}{3} \times \frac{4}{5}$), and $\frac{2}{3}$ of $\frac{4}{5}$ must be 2 times as much ($\frac{2}{3} \times \frac{4}{5}$), or $\frac{8}{15}$. *Ans.* $\frac{8}{15}$.

In multiplying $\frac{4}{5}$ by $\frac{2}{3}$, how was the new numerator obtained? the new denominator? Then how do you multiply one fraction by another?

- c. What is $\frac{2}{3}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{2}{3}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{4}$ of $\frac{2}{3}$? $\frac{2}{3}$ of $\frac{5}{6}$?
 d. Multiply $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{1}{3}$ by $\frac{2}{11}$; $\frac{1}{10}$ by $\frac{2}{3}$; $\frac{1}{11}$ by $\frac{2}{3}$; $\frac{2}{3}$ by $\frac{2}{3}$.

238. From the previous illustrations may be derived the following

Rules.

1. To multiply an integer by a fraction, or a fraction by an integer: *Find the product of the integer and the numerator of the fraction for the numerator of the answer, and take the denominator of the fraction for the denominator of the answer.*

2. To multiply a fraction by a fraction: *Find the product of the numerators for the numerator of the answer, and the product of the denominators for the denominator of the answer.*

239. Examples for the Slate.

98. What is $\frac{2}{10}$ of $\frac{5}{21}$? (101.) $\frac{6}{8} \times \frac{4}{5} = ?$
 99. What is $\frac{8}{15}$ of $\frac{2}{20}$? (102.) $\frac{1}{2} \times \frac{2}{3} = ?$
 100. What is $\frac{3}{8}$ of $\frac{4}{5}$? (103.) $4\frac{6}{11} \times \frac{2}{3} = ?$

NOTE. Change mixed numbers to fractional numbers before multiplying.

104. Multiply $8\frac{4}{10}$ by $24\frac{1}{2}$. 105. Multiply $4\frac{2}{3}$ by $15\frac{1}{3}$.
 106. If a man can walk $4\frac{3}{8}$ miles in an hour, how far can he walk in $\frac{1}{5}$ of an hour?
 107. What is the cost of $22\frac{3}{4}$ thousand bricks at \$11 $\frac{1}{4}$ a thousand?

108. What is the cost of $37\frac{1}{2}$ pounds of lead at \$12 $\frac{1}{2}$ per pound?

109. A woman who inherited $\frac{2}{3}$ of a ship divided $\frac{2}{3}$ of her part equally between her two daughters. What part of the ship did each daughter receive?

NOTE. Each daughter received $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{2}{3}$ of the ship. The operation may be expressed thus: $\frac{2 \times 2 \times 1}{3 \times 3 \times 3}$.

110. If a dress-pattern cost \$11 $\frac{1}{2}$, the trimmings $\frac{1}{2}$ as much, and the making $\frac{1}{3}$ as much as the trimmings, what was the cost of making?

111. In a school of 840 pupils, $\frac{1}{4}$ cipher, $\frac{1}{4}$ as many write as cipher, $\frac{1}{4}$ as many study grammar as write, $\frac{1}{4}$ as many study geography as study grammar. How many study geography?

240. The following, which are usually called "compound fractions," are simply expressions for multiplication of fractions, *the finding of a part of a part*:

112. What is $\frac{2}{3}$ of $\frac{7}{12}$ of $\frac{4}{5}$?
 113. What is $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{1}{3}$?

114. What is $\frac{7}{18}$ of $\frac{8}{21}$ of $\frac{5}{8}$?
 115. What is $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{3}{4}$ of $5\frac{3}{4}$?
 116. What is $\frac{7}{8}$ of $\frac{6}{14}$ of $\frac{5}{15}$ of $6\frac{3}{4}$?
 117. What is $\frac{1}{9}$ of $\frac{2}{18}$ of $\frac{8}{9}$ of $3\frac{3}{4}$?
 118. What is $\frac{4}{13} \times \frac{1}{12}$ of $\frac{1}{10} \times 6\frac{2}{3}$?
 119. What is $\frac{9}{10}$ of $\frac{2}{11} \times \frac{7}{8}$ of $\frac{3}{4} \times 8\frac{2}{3} \times 15$?

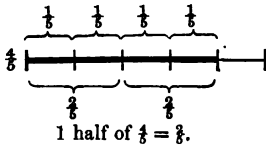
For other examples in multiplication of fractions, see page 123.

DIVISION.

To divide a Fraction by an Integral Number.

241. ILLUSTRATIVE EXAMPLE I. If $\frac{4}{5}$ of a melon be divided equally between 2 boys, how many fifths of a melon will each boy have ?

ILLUSTRATION.



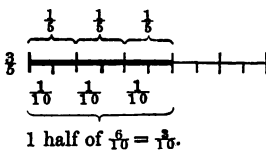
Explanation. — If $\frac{4}{5}$ of a melon be equally divided between two boys, each boy will have 1 half of $\frac{4}{5}$, or $\frac{2}{5}$ of a melon. (See illustration.)

Ans. $\frac{2}{5}$ melon.

In dividing the fraction $\frac{4}{5}$ by 2, what was done to the numerator ?

242. ILLUSTRATIVE EXAMPLE II. If $\frac{3}{5}$ of a melon be divided equally between 2 boys, what part of a melon will each boy have ?

ILLUSTRATION.



Explanation. — As the number of parts 3 cannot be divided by 2 without a remainder, each one of the parts, fifths, may be divided into two equal parts; these parts will be tenths of the whole melon. We shall then have 6

tenths, of which each boy's share is 3 tenths. (See illustration.)

In dividing $\frac{3}{5}$ by 2, what was done to the denominator ?

243. From the preceding illustrations we learn that to divide a fraction by an integer, *we may divide the numerator or multiply the denominator by the integer.*

NOTE. In dividing by 2, as above, we find one half of a number; this is equivalent to multiplying by $\frac{1}{2}$.

Dividing by 3 is equivalent to multiplying by what? Dividing by 6 is equivalent to multiplying by what?

244. Oral Exercises.

- a. Divide $\frac{3}{5}$ by 5; $\frac{2}{3}$ by 11; $\frac{3}{8}$ by 7; $\frac{2}{9}$ by 13; $\frac{7}{8}$ by 25.
- b. Divide $\frac{2}{3}$ by 5; $\frac{7}{8}$ by 3; $\frac{7}{11}$ by 9; $\frac{1}{12}$ by 7; $\frac{5}{8}$ by 8.
- c. Divide $\frac{3}{8}$ by 3; $\frac{8}{9}$ by 4; $\frac{9}{10}$ by 7; $\frac{1}{11}$ by 3; $\frac{5}{8}$ by 2.
- d. If \$3 will buy $\frac{9}{10}$ of a yard of broadcloth, what part of a yard will \$1 buy?
- e. If 1 man can do a piece of work in $\frac{3}{4}$ of a month, in what time can 12 men do it?
- f. If \$2 $\frac{3}{4}$ be paid for 7 pounds of butter, what is the price of 1 pound?

NOTE. Change \$2 $\frac{3}{4}$ to fifths before dividing.

g. At \$2 $\frac{1}{2}$ per day, what are the wages of a man for 1 third of a day?

h. If by 1 pipe a cistern can be emptied in 1 $\frac{3}{4}$ hours, in what time can the cistern be emptied by 4 like pipes?

Examples for the Slate.

245. ILLUSTRATIVE EXAMPLE III. If 5 $\frac{5}{8}$ yards of ribbon are required for 18 knots of trimming, how much is required for 1 knot?

WRITTEN WORK.

$$5\frac{5}{8} = \frac{45}{8}. \quad \frac{\frac{45}{8}}{18} = \frac{5}{16}. \quad \text{Ans. } \frac{5}{16} \text{ yd.}$$

120. If 39 $\frac{1}{2}$ inches of rain fell in a year, what was the average fall per week?

121. If a steamer goes $77\frac{1}{2}$ miles in 5 hours, what is her rate per hour?

NOTE. First divide 77 by 5, then change the remainder to thirds, and divide.

122. If $44\frac{1}{2}$ yards of cloth be required to make 8 suits, how many yards are required for 1 suit?

123. If land that extends along the street $103\frac{1}{2}$ rods is made into 18 house-lots of equal width, what is the width of each lot?

124. What is the length of one side of a square that can be enclosed by a string $89\frac{1}{2}$ feet long?

125. Divide $\frac{1}{2}\frac{2}{3}$ by 32. (130.) $47\frac{3}{4} \div 10 = ?$

126. Divide $\frac{2}{3}$ by 16. (131.) $121\frac{1}{4} \div 16 = ?$

127. Divide $\frac{1}{2}\frac{3}{4}$ by 72. (132.) $272\frac{1}{4} \div 20 = ?$

128. Divide $\frac{1}{2}\frac{3}{4}$ by 65. (133.) $5\frac{1}{4} \div 11 = ?$

129. Divide $93\frac{1}{2}$ by 9. (134.) $1240\frac{1}{2} \div 90 = ?$

246. To divide an Integer or a Fraction by a Fraction.

In 1 there are how many fourths? sixths? eighths? ninths?

In 2 there are how many times $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$?

ILLUSTRATIVE EXAMPLE IV. How many baskets of peaches at $\frac{2}{3}$ of a dollar a basket can be bought for \$5?

WRITTEN WORK.

$$5 = 1\frac{1}{2}$$

Explanation.—As many baskets can be bought as there are times $\frac{2}{3}$ in 5.

$$1\frac{1}{2} \div \frac{2}{3} = 15 \div 2 = 7\frac{1}{2}$$

We first change 5 to thirds, making $1\frac{1}{2}$.

There are as many times $\frac{2}{3}$ in $1\frac{1}{2}$ as there are 2's in 15, or $7\frac{1}{2}$. Ans. $7\frac{1}{2}$ baskets.

NOTE. For different analysis of this example, see Appendix, page 304.

247. Oral Exercises.

a. Eight are how many times $\frac{2}{3}$? $\frac{4}{5}$? $\frac{1}{2}$? $\frac{3}{4}$? $\frac{5}{6}$? $1\frac{1}{2}$ or $\frac{3}{2}$?

b. Twenty are how many times $1\frac{1}{2}$? $1\frac{2}{3}$? $2\frac{1}{4}$? $2\frac{3}{5}$? $1\frac{3}{4}$? $1\frac{1}{2}$?

c. Divide 4 by $\frac{2}{3}$; 7 by $\frac{2}{3}$; 9 by $\frac{2}{3}$; 6 by $\frac{2}{3}$; 8 by $1\frac{1}{2}$.

How do you change an integer to divide it by a fraction? How do you then divide? How do you divide by a mixed number?

d. If a person walks a mile in $\frac{3}{8}$ of an hour, how many miles can he walk in 8 hours?

e. At $\$ \frac{3}{4}$ a pound for coffee, how many pounds can be bought for $\$ 4$?

f. How many chairs at $\$ 1\frac{1}{4}$ each can be bought for $\$ 10$?

NOTE. Change $1\frac{1}{4}$ to fourths.

g. How many tons of coal at $\$ 6\frac{3}{4}$ can be bought for $\$ 25$?

h. The old shilling of New England was worth $16\frac{3}{4}$ cents. How many shillings made a dollar?

i. If a boy can write a page in $\frac{1}{15}$ of an hour, how many pages can he write in $\frac{1}{2}$ of an hour? in 1 hour?

j. If a hat can be made from $\frac{3}{4}$ of a yard of velvet, how many hats can be made from $3\frac{3}{4}$ yards?

k. Divide $\frac{1}{10}$ by $\frac{1}{10}$; $\frac{1}{8}$ by $\frac{1}{8}$; $\frac{3}{4}$ by $\frac{3}{4}$; $\frac{1}{8}$ by $\frac{5}{16}$.

When fractions have a common denominator, how do you divide?

248. ILLUSTRATIVE EXAMPLE V. Divide $\frac{4}{5}$ by $\frac{2}{3}$.*

WRITTEN WORK.

$$\frac{4}{5} = \frac{12}{15}; \quad \frac{2}{3} = \frac{10}{15}$$

$$\frac{12}{15} \div \frac{10}{15} = 12 \div 10 = 1\frac{1}{5}$$

Explanation.— $\frac{4}{5}$ and $\frac{2}{3}$ changed to fractions having a common denominator are $\frac{12}{15}$ and $\frac{10}{15}$. $\frac{12}{15}$ divided by $\frac{10}{15}$ equals 12 divided by 10, or $1\frac{1}{5}$. Ans. $1\frac{1}{5}$.

1. Divide $\frac{8}{9}$ by $\frac{2}{3}$; $\frac{2}{3}$ by $\frac{1}{3}$; $\frac{7}{8}$ by $\frac{3}{8}$; $\frac{5}{8}$ by $\frac{1}{8}$; $\frac{1}{2}$ by $\frac{3}{8}$.

When fractions have different denominators, how do you prepare them to divide?

In the written work of Illustrative Example V., after obtaining a common denominator we have $12 \div 10$, or the new numerator of the dividend divided by the new numerator of the divisor. If, in the place of these numbers, we put the factors which formed them, we shall have $(4 \times 3) \div (5 \times 2)$ or $\frac{4 \times 3}{5 \times 2}$ or $\frac{4}{5} \times \frac{3}{2}$, in which the expression for the divisor, $\frac{2}{3}$, is inverted, becoming $\frac{3}{2}$, and the answer, found by multiplying $\frac{4}{5}$ by $\frac{3}{2}$, is $\frac{12}{10}$, or $1\frac{1}{5}$, as before.

249. To divide one fraction by another, we may then invert the divisor and proceed as in the multiplication of

* For other explanations of division of fractions, see Appendix, page 304.

fractions. The written work of Illustrative Example V. will then be merely $\frac{2 \times 3}{5 \times 2} = \frac{3}{5} = 1\frac{1}{5}$.

Perform the following examples by either of the methods illustrated above:

m. How many are $\frac{4}{5} \div \frac{1}{8}$? $\frac{2}{3} \div \frac{3}{4}$? $\frac{5}{8} \div \frac{7}{11}$? $\frac{4}{5} \div \frac{7}{9}$?

n. How many are $\frac{4}{5} \div \frac{2}{3}$? $\frac{2}{3} \div \frac{4}{5}$? $\frac{5}{8} \div \frac{7}{11}$? $\frac{7}{11} \div \frac{4}{5}$?

250. From the previous illustrations may be derived the following

Rules.

1. To divide a fraction by an integer, *Divide the numerator or multiply the denominator by the integer.*

2. To divide an integer or a fraction by a fraction, *Change the dividend and divisor to fractions having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.* Or,

2. *Invert the divisor, and proceed as in the multiplication of fractions.*

251. Examples for the Slate.

135. Divide $1\frac{2}{3}$ by 6.

138. Divide 181 by $\frac{1}{81}$.

136. Divide $\frac{1}{15}$ by 7.

139. Divide 96 by $\frac{1}{12}$.

137. Divide $1\frac{2}{3}$ by 18.

140. Divide 108 by $\frac{1}{90}$.

141. At $\$ \frac{1}{12}$ per pound, how many pounds of rice can be bought for \$11?

142. At $\$ \frac{1}{2}$ per foot for rubber hose, how many feet can be bought for \$41?

(143.) $18 \div \frac{4}{5} = ?$

(147.) $\frac{4}{5} \div \frac{1}{15} = ?$

(144.) $21 \div \frac{3}{10} = ?$

(148.) $\frac{3}{10} \div \frac{1}{15} = ?$

(145.) $98 \div \frac{7}{10} = ?$

(149.) $\frac{3}{10} \div \frac{7}{15} = ?$

(146.) $54 \div \frac{9}{8} = ?$

(150.) $\frac{1}{8} \div \frac{1}{15} = ?$

151. How many bushels of peas at $\$ \frac{2}{3}$ a bushel can be bought for \$18? for \$12?

152. At $\$ \frac{2}{10}$ per thousand ems for setting type, how many thousand ems can be set for \$75?

153. If 1 yard of cloth can be made from $\frac{1}{3}$ of a pound of wool, how many yards can be made from 5 tons of 2000 pounds each?

154. One rod equals $16\frac{1}{2}$ feet. How many rods in 100 feet?

155. How many breadths of paper, each $\frac{3}{4}$ of a yard wide, will reach around a room, the distance being $27\frac{1}{2}$ yards?

156. At \$ $2\frac{1}{2}$ per yard, how many yards of cloth can be bought for \$ $45\frac{1}{4}$?

157. How many lengths of $7\frac{1}{2}$ feet are there in a fence $1706\frac{1}{2}$ feet long?

158. How many square rods, each containing $30\frac{1}{2}$ square yards, are there in $75\frac{1}{2}$ square yards?

159. A man had \$1.50, which he exchanged for francs at $18\frac{1}{2}$ cents each. How many francs did he receive?

$$(160.) \quad 3\frac{3}{4} \div 4\frac{2}{5} = ? \qquad (162.) \quad 26\frac{1}{2} \div 3\frac{7}{11} = ?$$

$$(161.) \quad 5\frac{1}{2} \div 6\frac{1}{2} = ? \qquad (163.) \quad 1 \div 541\frac{1}{2} = ?$$

252. ILLUSTRATIVE EXAMPLE VI. Change the expression $\frac{9\frac{3}{4}}{2\frac{7}{8}}$ to its simplest form.

WRITTEN WORK.

$$9\frac{3}{4} \div 2\frac{7}{8} = \frac{39}{4} \div \frac{23}{8} = \frac{39 \times 8}{4 \times 23} = \frac{312}{92} = 2\frac{4}{23} = 3\frac{1}{23}. \quad \text{Ans. } 3\frac{1}{23}.$$

Expressions like that above are sometimes called *complex fractions*. But they merely indicate division.

Change the form of the following expressions, and perform the division indicated:

$$\begin{array}{llll} (164.) \quad \frac{\frac{1}{2}}{\frac{3}{5}} & (167.) \quad \frac{\frac{7}{8}}{\frac{1}{12}} & (170.) \quad \frac{4}{1\frac{1}{2}} & (173.) \quad \frac{2\frac{1}{2}}{4\frac{1}{4}} \\ (165.) \quad \frac{\frac{3}{4}}{\frac{5}{8}} & (168.) \quad \frac{\frac{5}{6}}{\frac{4}{6}} & (171.) \quad \frac{18}{1\frac{1}{3}} & (174.) \quad \frac{19\frac{1}{2}}{32\frac{1}{4}} \\ (166.) \quad \frac{1\frac{1}{2}}{9} & (169.) \quad \frac{1\frac{1}{2}}{160} & (172.) \quad \frac{1\frac{1}{2}}{1\frac{1}{2}} & (175.) \quad \frac{1062}{47\frac{3}{8}} \end{array}$$

For other examples in division of fractions, see page 123.

TO FIND THE WHOLE WHEN A PART IS GIVEN.

Oral Exercises.

253. ILLUSTRATIVE EXAMPLE I. If $\frac{3}{4}$ of a ton of hay costs \$16, what will $\frac{1}{2}$ of a ton cost? what will 1 ton cost?

a. If $\frac{2}{3}$ of a certain number is 28, what is the entire number?

b. 81 is $\frac{3}{4}$ of what number?

c. A man bought a harness for \$75, which was $\frac{3}{4}$ of what he paid for his carriage. What did he pay for his carriage?

d. I paid \$6 a week for board in Albany, which was $\frac{2}{3}$ of what I paid in Buffalo; this was $\frac{3}{4}$ of what I paid in Chicago; and this was $\frac{1}{2}$ of what I paid in San Francisco. What did I pay in San Francisco?

e. An exploring party having lost $\frac{1}{3}$ of their bread, are obliged to subsist on 14 ounces a day. What were they allowed at first?

NOTE. If $\frac{1}{3}$ is lost, $\frac{2}{3}$ remain.

f. If $\frac{3}{4}$ of a piece of work be performed in 24 days, how many days will it take to do the remainder?

g. A vessel, having lost $\frac{1}{3}$ of her cable, has 200 feet remaining. How many feet had she at first?

h. Mary is 24 years old, and her age is equal to once and $\frac{1}{2}$ the age of her brother. How old is her brother?

NOTE. Mary's age is $\frac{3}{2}$ of that of her brother.

i. A mother and her son have \$45 in a purse; the son's part is $\frac{2}{3}$ as great as the mother's. What is each one's part?

Solution. — The mother's part must be $\frac{1}{3}$ of itself, and her son's part added to her part must be $\frac{2}{3}$ of her part. But the two together have \$45; then \$45 is $\frac{3}{2}$ of the mother's part.

j. If I sell an article for \$80, and thereby gain a sum equal to $\frac{1}{3}$ of the cost, what is the cost?

k. If I sell an article for \$80, and thereby lose a sum equal to $\frac{1}{3}$ of the cost, what is the cost?

254. Examples for the Slate.

- (176.) $16\frac{1}{2}$ is $\frac{2}{5}$ of what number ?
- (177.) $25\frac{3}{4}$ is $\frac{3}{8}$ of what number ?
- (178.) $\frac{4}{5}$ of $1\frac{1}{2}$ is $\frac{3}{4}$ of what number ?
- (179.) $\frac{3}{8}$ of $6\frac{3}{4}$ is $\frac{4}{5}$ of what number ?
- (180.) $\frac{4}{11}$ of $1\frac{2}{3}$ is $\frac{5}{8}$ of what number ?
- (181.) $2\frac{1}{2} \times 7\frac{7}{8}$ is $3\frac{1}{2}$ times what number (or $\frac{1}{2}$ of what number) ?
- (182.) $182 \div (12 \times 2\frac{1}{2})$ is $3\frac{1}{4}$ times what number ?
183. An author's copyright on a book was \$54.57. If this was $\frac{3}{5}$ of the whole profit, what was the whole profit ?
184. Mr. Smith owns $\frac{3}{4}$ of an acre of land; his neighbor Mr. French owns $\frac{3}{4}$ as much, which is $\frac{4}{5}$ of what Mr. Brown owns. What does Mr. Brown own ?
185. If $\frac{1}{2}$ of my property is in real estate, $\frac{3}{4}$ in trade, and the balance, which is \$33000, is in stocks: what is the value of my property ?
186. A man sold a lot of land for \$1440, which was $2\frac{1}{2}$ times what it cost him. What did it cost him ?
187. Having lost $\frac{1}{11}$ of my money in trade, I now have \$2476.50. What had I at first ?
188. A person against whom I had an account has failed, and I have lost $\frac{4}{5}$ of what he owed me. If I receive \$1584.72, how much did he owe me ?
189. A body of 4800 troops had $\frac{1}{4}$ as many cavalry as infantry. What was the number of each ?
190. A lot of land yielded 4140 bushels of grain in two years, yielding $\frac{4}{5}$ as much the second year as the first. What was the yield each year ?
191. What number is that to which if $\frac{3}{4}$ of itself be added the sum will equal 275 ?
192. In counting his fowls, a farmer finds that he has 396 in all, which is $\frac{1}{2}$ more than he had the previous year. How many had he then ?

TO FIND WHAT FRACTION ONE NUMBER IS OF ANOTHER.

Oral Exercises.

255. ILLUSTRATIVE EXAMPLE I. 1 is what part of 5?

Answer. 1 is $\frac{1}{5}$ of 5 because it is one of the five equal parts into which 5 may be divided.

a. 1 is what part of 7? of 9? of 10? Why?

In comparing 1 with any number to see what fraction it is of that number, what do you take as the numerator? as the denominator?

b. 1 is what part of 7? 2 is what part of 7? Why?

c. What part of 9 is 2? What part of 10 is 7?

d. What part of 200 is 20? 50? 25? 40?

e. 1 peach is what part of 7 peaches? 3 pears of 13 pears?

f. $\frac{1}{8}$ is what part of $\frac{7}{8}$? $\frac{3}{20}$ is what part of $\frac{1}{2}$?

g. $\frac{1}{3}$ is what part of $\frac{1}{2}$. NOTE. Change $\frac{1}{2}$ and $\frac{1}{3}$ to sixths.

h. What part of 10 is $3\frac{1}{2}$? is $2\frac{1}{4}$?

256. From the foregoing illustrations we may derive the following

Rule.

To find what fraction one number is of another, *Make the number which is the part the numerator of a fraction, and the number with which it is compared the denominator.*

i. If a piece of work can be performed in 9 days, what part of the work can be performed in 7 days?

j. If Mr. Chase has \$54 and spends \$18 for a coat, what part of his money does he spend?

k. Stock originally worth \$50 a share now sells for \$40. What part of the original value does it bring?

l. When goods which cost 75 cents sell for \$1, what is the gain? What part of the cost is the gain?

m. A and B hired a pasture together; A pastured 12 cows in it and B 13 cows. What part of the price should each pay?

257. Examples for the Slate.

193. A man owing \$316, paid \$84 of the debt. What part of the debt did he pay?

194. What part of $272\frac{1}{2}$ square feet is 9 square feet?

195. I bought a house for \$3000 and sold it for \$4500. What part of the original cost was the gain?

196. Four men were hired to work on a farm. A worked 7 days, B worked 5 days, C 8 days, and D 4 days. They received \$72. What was each man's share?

What part (201.) $12\frac{3}{4}$ is what part of 19?

(197.) Of 75 is 30? (202.) 1 is what part of $2\frac{7}{8}$?

(198.) Of 267 is 89? (203.) 1 is what part of $1\frac{1}{2}$?

(199.) Of 8 is $\frac{1}{3}$? (204.) $\frac{1}{11}$ is what part of $\frac{1}{11}$?

(200.) Of 11 is $3\frac{2}{3}$? (205.) $2\frac{3}{8}$ is what part of $3\frac{3}{8}$?

206. What part of 100 is $33\frac{1}{3}$? $66\frac{2}{3}$? $87\frac{1}{2}$? $37\frac{1}{2}$? $12\frac{1}{2}$? $62\frac{1}{2}$? $6\frac{1}{2}$? $56\frac{1}{2}$?

To solve Examples by using Aliquot Parts of Numbers.

258. What is one of the three equal parts of 9? of 10?

One of the equal parts of a number is an *aliquot* part of the number. Thus, $3\frac{1}{3}$ is an aliquot part of 10.

259. Oral Exercises.

Find such aliquot parts of the following numbers as are indicated below:

a. Of the number 30 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{10}$; $\frac{2}{3}$; $\frac{3}{4}$.

b. Of the number 60 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{20}$.

c. Of the number 100 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$.

d. Of the number 100 find $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{6}{7}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{9}{10}$.

e. Of the number 144 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{12}$; $\frac{3}{4}$; $\frac{4}{5}$.

f. Of the number 200 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{3}{4}$.

g. Of the number 1000 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{3}{4}$; $\frac{4}{5}$.

260. By using the aliquot parts of numbers, the work of multiplying and dividing may often be shortened, thus :

ILLUSTRATIVE EXAMPLE. What is the cost of 25350 ft. of gas at $3\frac{1}{2}$ mills per foot ?

Operation. — $3\frac{1}{2}$ m. = $\frac{1}{2}$ of 10 mills, or $\frac{1}{2}$ of a cent. 25350 ft. at 1 cent a foot costs \$253.50, and at $\frac{1}{2}$ of a cent a foot it must cost $\frac{1}{2}$ of \$253.50, or \$84.50. *Ans.* \$84.50.

Oral Exercises.

Find the cost

- a. Of 1872 lbs. of butter at \$0.33 $\frac{1}{2}$ per lb. ?
- b. Of 64 bu. potatoes at \$0.87 $\frac{1}{2}$ per bu. ?
- c. Of 44 yds. of silk at \$1.12 $\frac{1}{2}$ per yd. ?
- d. Of fencing 50 rods of road, both sides, at \$3.75 per rod ?
- e. Of insuring a house 5 years at \$6.66 $\frac{2}{3}$ per year ?
- f. Of 750 feet of boards at \$12 per thousand ?
- g. Of 80 pounds of butter at 37 $\frac{1}{2}$ ¢ per pound ?
- h. How many pounds of cheese at 16 $\frac{2}{3}$ ¢ a pound can be bought for \$10 ?
- i. For \$20 how many yards of cloth can be bought at \$1 a yard ? at 12 $\frac{1}{2}$ ¢ ? at 16 $\frac{2}{3}$ ¢ ? at 25¢ ? at 50¢ ? at 37 $\frac{1}{2}$ ¢ ?

261. Examples for the Slate.

- | | |
|---------------------------------------|---------------------------------------|
| (207.) $987 \div 16\frac{2}{3} = ?$ | (210.) $496 \div 12\frac{1}{2} = ?$ |
| (208.) $864 \div 33\frac{1}{3} = ?$ | (211.) $684 \times 66\frac{2}{3} = ?$ |
| (209.) $572 \times 62\frac{1}{2} = ?$ | (212.) $487 \times 37\frac{1}{2} = ?$ |

262. Questions for Review.

What is a **FACTOR** of a number ? What is a composite number ? a prime number ? a prime factor ?

What is an even number ? an odd number ? What numbers are divisible by 2 ? 3 ? 4 ? 5 ? 6 ? 8 ? 9 ? 11 ?

How can you find the **PRIME FACTORS** of a number ? A composite number equals what product ? Find the prime factors of 180 and explain the process. How can you make sure that a number is prime ?

What is **CANCELLATION** ? Why should arithmetical processes first be indicated by signs ? Explain the use of the parenthesis.

When are numbers prime to each other? What is a common factor of two or more numbers? the **GREATEST COMMON FACTOR**? Find the g. c. f. of three numbers by the first method given; explain and give the rule. Find the g. c. f. of two numbers by the second method given; give the rule. In what cases would you find the g. c. f. by the second method? When do we make use of the g. c. f. of numbers?

What is a multiple? a common multiple of two or more numbers? the **LEAST COMMON MULTIPLE**? When do we make use of the l. c. m.? Explain the first method of finding it; the second. What does the l. c. m. of numbers prime to each other equal?

What is a **FRACTIONAL UNIT**? a fractional number? What name is applied to both? Name and define the terms of a fraction. Explain the expression $\frac{3}{4}$. How do you change fractions to smaller terms? to larger terms? When is a fraction expressed in its smallest terms? How do you change improper fractions to integers or mixed numbers? How do you change integers or mixed numbers to fractions?

When are fractions said to have a **COMMON DENOMINATOR**? For what operations upon fractions do we first change them to others having a common denominator? Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to fractions having a common denominator, and explain.

How do you **ADD FRACTIONS**? Take three fractions of different denominators, add and explain. How do you add mixed numbers? How do you **SUBTRACT FRACTIONS**? Give a general rule for the addition of fractions. Give a general rule for the subtraction of fractions. Let $4\frac{1}{2}$ be the minuend and $1\frac{2}{3}$ the subtrahend; subtract and explain.

How do you **MULTIPLY A FRACTION** by an integer? a mixed number by an integer? Explain, by an example, the method of multiplying an integer by a fraction. Multiply a fraction by a fraction; explain and give the rule. How do you multiply a mixed number by a mixed number or a fraction? How can you simplify the expressions called compound fractions?

How do you **DIVIDE A FRACTION** by an integer? a mixed number by an integer? an integer by a fraction? Explain, by an example, the method of dividing a fraction by a fraction, and give the rule. How can you simplify the expressions called complex fractions? How do you find what fraction one number is of another? What is an aliquot part of a number?

What effect does multiplying both terms of a fraction by the same number have upon it? Why? What effect does dividing both terms

of a fraction have upon it? Why? What effect does multiplying the numerator of a fraction have upon the fraction? Why? In what other way could you produce the same effect, and why? What effect does dividing the numerator have upon a fraction? Why? In what other way could you produce the same effect, and why?

263. General Review, No. 2.

213. What are the prime factors of 420?
214. Divide $18 \times 7 \times 15 \times 6$ by $28 \times 10 \times 3 \times 4$.
215. Find the greatest common factor of 35, 84, and 56.
216. Find the least common multiple of 63, 18, 14, and 28.
217. Change $\frac{1}{3}\frac{2}{3}$ and $\frac{1}{2}\frac{2}{3}$ to their smallest terms.
218. Change $465\frac{1}{2}$ and 84 to improper fractions, having 8 for their denominator?
219. Change $2\frac{2}{3}$ and $3\frac{2}{3}$ to mixed or integral numbers.
220. Change $\frac{4}{8}$, $\frac{1}{12}$, and $\frac{2}{3}$ to fractions having a common denominator.
221. Change $\frac{1}{12}$, $\frac{1}{8}$, and $9\frac{1}{2}$ to fractions having the least common denominator.
222. Add $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{3}{4}$, and $8\frac{1}{2}$. Add $25\frac{1}{2}$, $6\frac{1}{2}$, and $46\frac{1}{2}$.
223. From 24 take $12\frac{1}{2}$. Subtract $\frac{3}{8}$ from $\frac{1}{2}$ of $\frac{1}{10}$.
224. Multiply $7\frac{1}{2}$ by 4; $7\frac{1}{2}$ by 5; $\frac{1}{5}$ by $8\frac{3}{4}$.
225. Simplify the expression $\frac{3}{5}$ of $\frac{2}{4}$ of $\frac{1}{10}$ of $2\frac{3}{4}$.
226. Divide $\frac{2}{10}$ by $\frac{3}{4}$; $\frac{4}{8}$ of $\frac{7}{10}$ by $1\frac{1}{12}$.
227. Simplify the expressions $\frac{3}{6}$, $\frac{1}{5}\frac{4}{10}$, and $\frac{4}{3\frac{1}{2}}$.
228. What part of $4\frac{3}{4}$ is $3\frac{1}{4}$?
229. Two trains which are 75 miles apart are running towards each other, one $30\frac{1}{2}$ miles an hour, the other $40\frac{1}{2}$ miles an hour. How far apart will they be in half an hour?
230. A man paid \$18 $\frac{3}{4}$ for a load of hay weighing $1\frac{1}{2}$ tons. At the same rate what should he pay for $\frac{2}{3}$ of a ton?
231. Having spent $\frac{2}{3}$ of his money, Fred has \$13 $\frac{1}{2}$. How much had he at first?
232. Make out a bill of sale for three barrels of sugar, weighing respectively 235 pounds, 241 pounds, and 254 pounds, at 11 $\frac{3}{4}$ ¢ a pound.

264. Miscellaneous Oral Examples.

a. If $\frac{7}{8}$ of a pound of candles cost 35 cents, what is the price of 1 pound? of $\frac{1}{8}$ of a pound?

b. In $\frac{3}{4}$ of an acre of land there are 120 square rods. How many square rods are there in $\frac{7}{8}$ of an acre?

c. When $1\frac{1}{2}$ bushels of oats will feed 10 horses for a certain time, how many horses will $2\frac{1}{2}$ bushels feed for the same time?

d. At $\$ \frac{3}{8}$ each, how many cedar posts can be bought for $\$12$? for $\$7\frac{1}{2}$?

e. At $\$2\frac{1}{2}$ per day, how many days' work can be paid for with $\$20$? with $\$37\frac{1}{2}$?

f. At $\$2$ per day, how many days' work can be paid for with $\$7\frac{3}{4}$? with $\$9\frac{1}{2}$?

g. If it requires 12 yards of carpeting $\frac{3}{4}$ of a yard wide to carpet a hall, how much will be required of that which is $1\frac{1}{2}$ yards wide?

h. What number is that, $\frac{1}{4}$ of which exceeds $\frac{1}{5}$ of it by 2?

i. If $\frac{2}{3}$ of the distance from Springfield to Albany is 80 miles, what is $\frac{3}{4}$ of the distance?

j. If $\$5\frac{1}{2}$ pays for the lodging and breakfast of 7 persons, for how many persons will $\$11\frac{1}{2}$ pay?

k. What is that number to which if $\frac{3}{8}$ of itself be added the sum will equal 64?

l. I sold my watch for $\$72$, which was $\frac{1}{4}$ more than I gave for it. What did it cost me?

m. Bought a horse and saddle for $\$75$, giving $\frac{2}{3}$ as much for the saddle as for the horse. What was the cost of each?

n. A can build a wall in 3 days, and B can do the same work in 4 days. What part of the work can each do in one day? What part can both do in one day? In how many days can both do it working together?

o. C can do a piece of work in 5 days, and D in 8 days. What time will be required for both to do it?

265. Miscellaneous Examples for the Slate.

233. What will $16\frac{1}{2}$ yards of cloth cost at $53\frac{1}{2}$ a yard?
234. What will $9\frac{1}{2}$ bushels of corn cost at $87\frac{1}{2}$ a bushel?
235. What will $271\frac{1}{2}$ acres of land cost at $\$31\frac{1}{2}$ per acre?
236. I paid $65\frac{1}{2}$ for 2 boxes of strawberries. What will be the cost of 45 boxes at the same rate?
237. What is my bill for 7 pear-trees at $87\frac{1}{2}$ cents apiece for the trees, and $\$2$ a dozen for setting?
238. What do I receive per pound by selling 15 pounds of coffee for $\$3.75$?
239. If $\frac{1}{3}$ of a man's property is in land, valued at $\$2324\frac{3}{4}$, what is the value of his whole property?
240. Bought $\frac{2}{3}$ of a ship for $\$4075$. What would the whole ship cost at the same rate?
241. What is the cost of 3 pieces of calico, $37\frac{1}{2}$ yards in a piece, at $19\frac{1}{2}$ cents per yard?
242. Sold my house and farm of $47\frac{1}{2}$ acres for $\$6150$. Allowing $\$3500$ for the house, what did I receive per acre for the land?
243. How long will a quantity of flour last a family of 8 persons if it lasts 3 persons $14\frac{1}{2}$ months?
244. If in $32\frac{1}{2}$ years a man saved $\$1694$, what was his average saving per year?
245. What number is that which diminished by $1\frac{1}{10}$ will leave a remainder of $1\frac{1}{10}$?
246. What number is that to which if you add $9\frac{1}{2}$ the sum will be $124\frac{3}{4}$?
247. What is that number to which if you add $\frac{2}{3}$ of $26\frac{1}{2}$ the sum will be $147\frac{1}{2}$?
248. If you buy $7\frac{1}{2}$ yards of silk at $\$5$ a yard, $14\frac{1}{2}$ yards of cashmere at $\$1.25$ per yard, $4\frac{1}{2}$ yards of silk at 75 cents per yard, and $\frac{2}{3}$ of a yard of velvet at $\$4.50$ per yard, giving in payment a $\$100$ bill, what balance will be your due?
249. What will 50 oranges cost at $62\frac{1}{2}$ a dozen?

250. How long will 200 pounds of meat last 9 persons at the rate of $\frac{1}{4}$ of a pound a day for each person?

251. A farmer has sold his eggs at an average of $23\frac{1}{2}$ cents per dozen, which is $\frac{1}{3}$ higher than they averaged the previous year. What did they average then?

252. He is paid for grain \$1.80 per bag, which is $\frac{1}{3}$ less than he was paid last year. What was he paid last year?

253. Mr. Stevens, dying, left \$75000 to his wife and two sons. To his wife he left \$30000; to his oldest son just as large a part of the remainder as his wife's portion was of the entire property; and to his youngest son the rest. What was each son's share?

254. A man sold $54\frac{3}{4}$ yards of cloth at the rate of 3 yards for 2 dollars. What did he receive for it?

255. Mr. Day bought a house and barn for \$4050, giving $\frac{1}{3}$ as much for the barn as for the house. What did he pay for each?

256. If a body falls $16\frac{1}{2}$ feet in the first second of time, 3 times $16\frac{1}{2}$ feet in the next second, and 5 times $16\frac{1}{2}$ feet in the third second, how far will it fall in the three seconds?

257. What length of time would a man require to travel around the earth if the distance is 25000 miles and he travels at the rate of $31\frac{1}{2}$ miles per day?

258. If a man can build $2\frac{3}{4}$ rods of wall in a day, how much can he build in $6\frac{1}{2}$ days?

259. What number is that $\frac{2}{3}$ of which exceeds $\frac{1}{4}$ of it by $11\frac{3}{4}$?

260. If I buy 1250 bushels of corn at 41 cents per bushel, and sell it at $52\frac{1}{2}$ cents per bushel, how much do I gain?

261. What number divided by $\frac{1}{2}$ equals $125\frac{3}{4}$?

262. What are the contents of 3 floors measuring as follows: $13\frac{3}{4}$ square yards, $32\frac{7}{8}$ square yards, and $49\frac{3}{4}$ square yards?

263. The product of three numbers is $63\frac{1}{2}$; two of them are $8\frac{1}{2}$ and $6\frac{1}{2}$. What is the third?

264. I exchanged 42 tubs of butter, averaging $48\frac{1}{2}$ pounds, at $21\frac{1}{2}$ cents per pound, for 42 barrels of flour, at \$9 $\frac{3}{4}$ per barrel, and received the balance in cash. What was the balance?

265. Owing a man in Paris 1325 francs, I have shipped to him \$375 worth of rice. If the franc is worth $19\frac{1}{4}$ cents, how much have I overpaid him in United States money? in francs?

266. I have three boxes, each containing 12 pieces of cloth, each piece $4\frac{1}{2}$ yards in length, and weighing $3\frac{1}{2}$ pounds to the yard. What is the weight of the whole?

267. What will $42\frac{1}{2}$ quires of paper weigh at $\frac{3}{4}$ pound per quire?

268. Owning $\frac{3}{4}$ of a flour-mill, I sold $\frac{1}{4}$ of my share for \$1750. What is the value of the whole mill at the same rate?

269. When hay was \$15 per ton, I gave $\frac{3}{4}$ of a ton for $1\frac{1}{2}$ tons of coal. What was the coal worth per ton?

270. If a man walks $9\frac{1}{2}$ miles in $2\frac{1}{2}$ hours, how far will he walk in $4\frac{1}{2}$ hours?

271. At the rate of $4\frac{1}{2}$ miles an hour, what time will be required to walk 122 miles?

272. In 1860 I purchased cotton at $8\frac{1}{2}$ cents a pound, which I sold in 1862 at $90\frac{1}{4}$ cents. What did I gain on 1000 lbs.?

273. If a man can earn \$2.30 per day, how many days' work will he have to give for a suit of clothes, of which the coat costs \$25 $\frac{1}{2}$, the trousers \$8, and the vest \$5 $\frac{1}{4}$?

274. If $\frac{3}{4}$ of $\frac{1}{4}$ of a ship cost \$42000, what is $\frac{3}{4}$ of it worth?

275. In a certain manufactory $\frac{1}{4}$ of the operatives are Germans, $\frac{1}{4}$ French, $\frac{1}{10}$ Scotch, $\frac{1}{8}$ English, $\frac{3}{20}$ Swedes, and the remainder, 140, native Americans. What is the whole number, and the number of each nationality?

276. If $\frac{1}{4}$ of my money is in gold, $\frac{1}{4}$ of the remainder in silver, and the balance, \$360, in bank-notes, how much money have I in all?

277. A certain piece of work can be performed by A in 8 days, by B in 10 days, and by C in 16 days. In what time can all do it working together?

278. In what time can A and B do it together?

279. In what time can A and C do it together?

280. In what time can B and C do it together?

266. DRILL TABLE No. 5.

Examples.	A	B	C	D	E	F	G	H
1.	$\frac{8}{10}$	$\frac{7}{12}$	$\frac{8}{9}$	$\frac{11}{12}$	$7\frac{1}{2}$	9	$12\frac{1}{2}$	72
2.	$\frac{10}{10}$	$\frac{5}{11}$	$\frac{7}{8}$	$\frac{31}{32}$	$8\frac{1}{2}$	15	$24\frac{1}{2}$	27
3.	$\frac{11}{12}$	$\frac{1}{11}$	$\frac{11}{12}$	$\frac{31}{32}$	$4\frac{1}{2}$	6	$18\frac{1}{2}$	64
4.	$\frac{11}{12}$	$\frac{10}{10}$	$\frac{8}{9}$	$\frac{11}{12}$	$9\frac{1}{2}$	12	$28\frac{1}{2}$	76
5.	$\frac{7}{8}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{10}{12}$	$6\frac{1}{2}$	14	$42\frac{1}{2}$	84
6.	$\frac{11}{12}$	$\frac{11}{12}$	$\frac{10}{10}$	$\frac{11}{12}$	$5\frac{1}{2}$	8	$32\frac{1}{2}$	96
7.	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{11}{12}$	$4\frac{1}{2}$	10	$15\frac{1}{2}$	80
8.	$\frac{11}{12}$	$\frac{10}{11}$	$\frac{8}{9}$	$\frac{11}{12}$	$1\frac{1}{2}$	7	$21\frac{1}{2}$	63
9.	$\frac{8}{9}$	$\frac{7}{8}$	$\frac{11}{12}$	$\frac{11}{12}$	$2\frac{1}{2}$	23	$54\frac{1}{2}$	75
10.	$\frac{11}{12}$	$\frac{7}{8}$	$\frac{10}{11}$	$\frac{11}{12}$	$5\frac{1}{2}$	20	$48\frac{1}{2}$	56
11.	$\frac{8}{9}$	$\frac{11}{12}$	$\frac{7}{8}$	$\frac{11}{12}$	$9\frac{1}{2}$	21	$35\frac{1}{2}$	91
12.	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{10}{11}$	$\frac{11}{12}$	$3\frac{1}{2}$	24	$36\frac{1}{2}$	60
13.	$\frac{8}{9}$	$\frac{11}{12}$	$\frac{11}{12}$	$\frac{11}{12}$	$2\frac{1}{2}$	5	$45\frac{1}{2}$	57
14.	$\frac{7}{8}$	$\frac{10}{12}$	$\frac{10}{11}$	$\frac{11}{12}$	$6\frac{1}{2}$	16	$56\frac{1}{2}$	88
15.	$\frac{11}{12}$	$\frac{1}{2}$	$\frac{10}{11}$	$\frac{11}{12}$	$5\frac{1}{2}$	18	$54\frac{1}{2}$	90
16.	$\frac{8}{9}$	$\frac{10}{10}$	$\frac{11}{12}$	$\frac{11}{12}$	$2\frac{1}{2}$	22	$55\frac{1}{2}$	77
17.	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{11}{12}$	$3\frac{1}{2}$	17	$51\frac{1}{2}$	85
18.	$\frac{10}{10}$	$\frac{7}{8}$	$\frac{10}{11}$	$\frac{11}{12}$	$7\frac{1}{2}$	13	$40\frac{1}{2}$	61
19.	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{10}{11}$	$\frac{11}{12}$	$8\frac{1}{2}$	19	$38\frac{1}{2}$	95
20.	$\frac{11}{12}$	$\frac{1}{2}$	$\frac{8}{9}$	$\frac{11}{12}$	$4\frac{1}{2}$	11	$44\frac{1}{2}$	66
21.	$\frac{11}{12}$	$\frac{10}{10}$	$\frac{11}{12}$	$\frac{11}{12}$	$9\frac{1}{2}$	24	$36\frac{1}{2}$	78
22.	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{8}{9}$	$\frac{11}{12}$	$3\frac{1}{2}$	12	$18\frac{1}{2}$	57
23.	$\frac{11}{12}$	$\frac{10}{11}$	$\frac{10}{11}$	$\frac{11}{12}$	$8\frac{1}{2}$	28	$63\frac{1}{2}$	84
24.	$\frac{11}{12}$	$\frac{10}{12}$	$\frac{10}{11}$	$\frac{11}{12}$	$6\frac{1}{2}$	15	$33\frac{1}{2}$	93
25.	$\frac{11}{12}$	$\frac{10}{11}$	$\frac{11}{12}$	$\frac{11}{12}$	$7\frac{1}{2}$	26	$39\frac{1}{2}$	65

267. Exercises upon the Table.

- | | |
|---|--|
| 124. Find the prime factors of each numerator in D. | 164. What number is that from which if you take A the remainder will be B? |
| 125. Find the prime factors of each denominator in D. | 165. What number is that to which if you add C of F the sum will be G? |
| 126. Find the g. c. f. of the terms of each fraction in D. | 166. What number multiplied by F will give G for a product? |
| 127. Find the l. c. m. of F, G,* and H. | 167. What number divided by E will give D for a quotient? |
| 128. Change D to lowest terms. | 168. What divisor will give E for a quotient, H being the dividend? |
| 129. Change the mixed numbers in G to improper fractions. | 169. What number is that to which if A of itself be added the sum will equal H? |
| 130. Find the sum of A and C. | 170. What number is that from which if B of itself be subtracted, the remainder will be F? |
| 131. $A + B + C$. | 171. Divide H into three such parts that the 2d shall be twice the 1st, and the 3d F more than the 2d. What is the 3d part? |
| 132. $C + D + E$. | 172. At E dollars a yard, what will F yards of cloth cost? |
| 133. $E + F + G + H$. | 173. At E dollars a yard, how many yards of cloth can be bought for F dollars? |
| 134. $A - B$. | 174. If B pounds of tea cost H cents, what will E pounds cost? |
| 135. $H - G$. | 175. John can do a piece of work in E days, and James can do the same work in F days. In what time can both together do it? |
| 136. $G - E$. | 176. If George and Albert can do a piece of work in E days, and Albert can do it alone in F days, in what time can George do it alone? |
| 137. $H + A - G$. | |
| 138. $A - B$ of C. | |
| 139. Difference of C and D. | |
| 140. C of E | |
| - A of B. | |
| 141. Simplify A of B of C. | |
| 142. Simplify A of B of C of E. | |
| 143. $A \times B$. | |
| 144. $C \times F$. | |
| 145. $C \times E$. | |
| 161. If G is B of some number, what is C of the same number? | |
| 162. H is C of how many times E? | |
| 163. C of E is B of how many times A? | |
| 146. $G \times F$. | |
| 147. $G \times E$. | |
| 148. $A \div B$. | |
| 149. $C \div E$. | |
| 150. $C \div F$. | |
| 151. $H \div A$. | |
| 152. $E \div F$. | |
| 153. $G \div E$. | |
| 154. A of $F \div E$. | |
| 155. A of C | |
| $\div B$ of E. | |
| 156. $(A + B)$ | |
| $\div (B \times C)$. | |
| 157. $(A - B)$ | |
| $\div (B \div C)$. | |
| 158. $A + C - B$. | |
| 159. $E \times F + G$. | |
| 160. | |
| $\frac{A \text{ of } E}{B + C} \times \frac{A - B}{G \div F}$. | |

* Omitting fractions.

SECTION X.

DECIMAL FRACTIONS.

268. Articles **30** to **36** treat of a series of fractions,—tenths, hundredths, thousandths, etc.,—each of which has for a denominator 10, or a number made by using 10's only as factors. Such fractions are *decimal fractions*.

NOTE. Decimal fractions are usually called *decimals*.

To read and write Decimals.

269. The method of reading and of writing decimals has been explained in Articles **34** to **36**. These the pupil may review.

270. Exercises.

- a.** Read 5.368; 0.406; 2.007; 0.039; 105.105.
b. Read 0.4721; 7.0497; 10.010; 15.0015.

Read the following:

- | | | |
|--------------------|----------------------|-------------------|
| c. 30.0094 | e. 120.250049 | g. 200.005 |
| d. 17.01845 | f. 1.001025 | h. 0.205 |

NOTE. To distinguish 200.005 (Example **g**) from 0.205 (Example **h**), use the word *decimal* before reading the decimal part. Thus, 200.005 may be read "two hundred and the decimal five thousandths"; while 0.205 may be read "decimal two hundred five thousandths."

Read the following:

- | | | |
|-----------------------|---------------------|----------------------|
| i. 0.315 | m. 500.0074 | q. 1000.00001 |
| j. 300.015 | n. 4700.0065 | r. 14.00375 |
| k. 36000.00018 | o. 430.06 | s. 0.0000027 |
| l. 0.36018 | p. 43000.06 | t. 0.1000012 |

271. To write a decimal: *Write the number as an integer, and place the decimal point so that the right-hand figure shall stand in the place required by the denomination of the decimal.*

NOTE. When the given number does not fill all the decimal places, supply the deficiency with zeros.

For other exercises in reading and for exercises in writing decimals, see page 135.

The pupil may now review addition and subtraction of decimals (Articles 45, 50, 61, and 65).

REDUCTION OF DECIMALS.

To change the Denomination of a Decimal Fraction.

272. Exercises.

a. What is the denominator of the fraction 0.5? 0.25? 0.075? 7.3? 4.86?

b. What is the numerator of the fraction 0.4? 0.04? 0.075? 0.0101? 0.000007? 0.25? 0.1125?

c. Write as a common fraction 0.3; 0.08; 0.375; 0.0204.

273. ILLUSTRATIVE EXAMPLE. Change 0.5 to thousandths.

WRITTEN WORK. *Explanation.*— Multiplying both numerator and denominator of $\frac{5}{10}$ by 100, we have $\frac{500}{1000}$, which is expressed decimally by writing 0.500.

274. From the written work above we derive the following

Rule.

To express a decimal fraction in any lower denomination: *Annex zeros to the given expression until the place of the required denomination is filled.*

275. Examples for the Slate.

1. Change 0.07 to thousandths.
2. Change 0.4, 0.75, 2.5, and 1.06 to thousandths.
3. Express 0.003, 1.75, and 0.006 as ten-thousandths.
4. Express 3 as tenths; as hundredths; as thousandths; as ten-thousandths; etc. *Answers.* 3.0; 3.00; etc.

NOTE. Read the above answers: "Thirty tenths; three hundred hundredths"; etc.

5. Express 7, 40, and 37 as tenths; as hundredths; as ten-thousandths.

To change a Decimal Fraction to a Common Fraction.

276. ILLUSTRATIVE EXAMPLES. Change 0.25 and $0.33\frac{1}{3}$ to common fractions in their simplest forms.

WRITTEN WORK.

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

$$0.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3}$$

Explanation. — After writing these fractions with their denominators, we find that the first can be changed to smaller terms (Art. 198), and that the second may be changed to a simple fraction (Art. 252) and then to its smallest terms.

277. From the examples above we derive the following

Rule.

To change a decimal fraction to a common fraction: *Write the decimal in the form of a common fraction, and then change the result, if necessary, to its simplest form.*

278. Examples for the Slate.

Change the following to common fractions in their simplest forms:

(6.) 0.4	(11.) $0.3\frac{1}{3}$	(16.) 0.750	(21.) 0.0625
(7.) 0.80	(12.) $0.37\frac{1}{2}$	(17.) 0.368	(22.) 0.0333
(8.) 0.35	(13.) $0.62\frac{1}{2}$	(18.) $0.66\frac{2}{3}$	(23.) $0.14\frac{2}{3}$
(9.) 0.75	(14.) $0.87\frac{1}{2}$	(19.) $0.666\frac{2}{3}$	(24.) 7.5
(10.) $0.7\frac{1}{2}$	(15.) 0.875	(20.) 0.072	(25.) $1.16\frac{2}{3}$

To change Common Fractions to Decimal Fractions.

279. ILLUSTRATIVE EXAMPLE. Change $\frac{3}{8}$ to a decimal fraction.

WRITTEN WORK. *Explanation.* — The fraction $\frac{3}{8}$ is the same as $\frac{1}{8}$ of 3, or $\frac{1}{8}$ of 3.000 (3000 thousandths), which is found by dividing 3.000 by 8 in the usual way (Art. 102).

280. From the example above we derive the following

Rule.

To change a common fraction to a decimal fraction :
Express the numerator as tenths, hundredths, thousandths, etc., by annexing as many zeros as may be required, and then divide it by the denominator.

281. Examples for the Slate.

Change to decimals :

- | | | | |
|-------------------------|------------------------|-------------------------|---------------------------|
| (26.) $\frac{4}{5}$. | (29.) $\frac{11}{8}$. | (32.) $1\frac{5}{8}$. | (35.) $1.06\frac{2}{5}$. |
| (27.) $\frac{7}{20}$. | (30.) $5\frac{1}{2}$. | (33.) $8\frac{1}{4}$. | (36.) $0.04\frac{1}{2}$. |
| (28.) $1\frac{3}{25}$. | (31.) $2\frac{1}{2}$. | (34.) $17\frac{1}{2}$. | (37.) $0.03\frac{1}{2}$. |

Change to decimals and add (Art. 45) the following :

- | | |
|---|--|
| (38.) $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{10}$. | (40.) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{10}$, and $\frac{2}{5}$. |
| (39.) $\frac{1}{2}$, $\frac{7}{8}$, $1\frac{1}{2}$, and $\frac{7}{20}$. | (41.) $\frac{1}{2}$, $15\frac{3}{4}$, and $1\frac{1}{10}$. |

42. A carpenter paid for a mantel-piece \$27 $\frac{3}{4}$, for a grate \$22 $\frac{1}{2}$, and for a hearth \$4 $\frac{9}{10}$. How much did he pay in all ?

(43.) $2\frac{1}{2} + 3\frac{1}{2} + 87\frac{1}{2} + 18\frac{1}{2} =$ what ?

44. A drover bought a cow and a calf for \$38.85, and sold the cow for \$32 $\frac{1}{2}$, and the calf for \$10 $\frac{1}{2}$. How much did he gain ?

45. A man owning 17.635 acres of land, sold $1\frac{1}{2}$ acres to one person, and $\frac{1}{10}$ of an acre to another. How much had he left ?

46. Change to seven decimal places, and add $1.82\frac{3}{4}$, $0.009\frac{7}{10}$, and $0.10\frac{1}{2}$.

282. ILLUSTRATIVE EXAMPLE. What is the sum of $5\frac{1}{8}$ yards, $2\frac{2}{3}$ yards, and $7\frac{1}{3}\frac{4}{3}$ yards?

WRITTEN WORK.

$$5\frac{1}{8} = 5.125$$

$$2\frac{2}{3} = 2.666\frac{2}{3}$$

$$7\frac{1}{3}\frac{4}{3} = 7.424\frac{8}{3}$$

$$\text{Exact sum, } 15.215\frac{1}{3}$$

$$5\frac{1}{8} = 5.125$$

$$2\frac{2}{3} = 2.667$$

$$7\frac{1}{3}\frac{4}{3} = 7.424$$

$$\text{Approximate sum, } 15.216$$

Explanation.—In this example there are fractions which cannot be *completely* expressed as decimals; for, however far the division be carried, there will still be a remainder.

If we choose to stop dividing at thousandths, the quotients are expressed accurately by writing $\frac{2}{3}$ of a thousandth and $\frac{8}{3}$ of a thousandth, as in the margin. But these results are no more convenient to add than the original numbers; hence nothing has been gained by changing the latter to

the decimal form if our object was to find the exact sum.

There are, however, many cases in which the error arising from the neglect of such small fractions as parts of a thousandth is of no importance. For such cases the second form of written work given in the margin is to be adopted. Here the decimal values are expressed to the nearest thousandth. This is done by increasing the last term of the decimal by 1 whenever the neglected fraction is $\frac{1}{2}$ or more.

Greater accuracy would be attained by carrying out the decimal to the nearest ten-thousandth, or to a still lower denomination.

283. Examples for the Slate.

NOTE. Unless some other direction is given, the pupil will hereafter understand that decimal values are to be expressed to the nearest ten-thousandth.

47. Find the decimal values of $\frac{3}{8}$, $\frac{1}{2}$, $\frac{4}{5}$, and add the results.
48. Change to thousandths, and add $9\frac{2}{3}$, $16\frac{1}{3}$, and $33\frac{1}{11}$.
49. Change $\frac{1}{3}$ and 0.68 to ten thousandths, and find their difference.
50. Mr. Carpenter has worked for Mr. Bates $2\frac{3}{4}$ hours, $3\frac{1}{2}$ hours, and 5.5 hours. How many hours has he worked for him in all?
51. How many rods are there in $25\frac{3}{4}$ rods, $0.48\frac{1}{11}$ rods, $105\frac{1}{2}$ rods, and $8.62\frac{1}{2}$ rods?

Other examples in addition and subtraction may be found on page 135.

Circulating Decimals.

284. We have seen (Art. 282) that in expressing $\frac{2}{3}$ decimally (0.666...) the figure 6 is repeated again and again. So in expressing $\frac{14}{33}$ decimally (0.4242...) the figures 4 and 2 are repeated again and again.

Decimal fractions that are expressed by the same figures repeated again and again are called *repeating* or *circulating decimals*.

NOTE. Circulating decimals arise from the reduction of common fractions whose denominators contain prime factors other than 2 and 5.

285. The repeating figures of a circulating decimal are called a *repetend*.

A repetend is marked by placing dots over the first and last of the figures that repeat.

Thus, $\frac{11}{33} = 0.297297 \dots = 0.29\dot{7}$;

$\frac{14}{33} = 0.4242 \dots = 0.4\dot{2}$; $3\frac{1}{3} = 3.166 \dots = 3.1\dot{6}$.

286. Change the following fractions to decimals till the figures repeat, and mark the repetends:

- | | | | |
|-----------------------|-------------------------|------------------------|------------------------|
| (52.) $\frac{1}{3}$. | (55.) $\frac{4}{9}$. | (58.) $\frac{7}{8}$. | (61.) $\frac{1}{24}$. |
| (53.) $\frac{4}{9}$. | (56.) $\frac{11}{12}$. | (59.) $\frac{1}{24}$. | (62.) $1\frac{2}{3}$. |
| (54.) $\frac{3}{7}$. | (57.) $\frac{4}{7}$. | (60.) $\frac{7}{11}$. | (63.) $3\frac{5}{7}$. |

To change a Circulating Decimal to a Common Fraction.

287. ILLUSTRATIVE EXAMPLE I. Change $0.\dot{6}\dot{3}$ to a common fraction.

WRITTEN WORK. To change a circulating decimal to a common fraction: *Take the repetend for the figures of the numerator, and for the figures of the denominator as many 9's as there are figures in the repetend. Change the fraction thus expressed to its smallest terms.*

For an explanation of this rule, see Appendix, page 305.

Change to common fractions in their smallest terms :

(64.) 0.3	(67.) 0.39	(70.) 0.016	(73.) 0.1881
(65.) 0.6	(68.) 0.27	(71.) 0.621	(74.) 0.428571
(66.) 0.42	(69.) 0.648	(72.) 0.108	(75.) 0.571428

To change a **Mixed Circulate** to a **Common Fraction**.

288. ILLUSTRATIVE EXAMPLE II. Change 0.263 to a common fraction.

WRITTEN WORK.

$$\begin{array}{r} 263 \\ 2 \overline{) 526} \\ \underline{526} \\ 0 \end{array} = 131\frac{1}{2}$$

261

To change a mixed circulate to a common fraction: *Take for the numerator the difference between the mixed circulate and the part which does not repeat, both regarded as integers, and take for the figures of the denominator as many 9's as there are figures in the repetend, with as many zeros annexed as there are figures in that part of the circulate which does not repeat.* (See Appendix, p. 305.)

Change to common fractions in their smallest terms :

(76.) 1.86	(78.) 0.033	(80.) 0.016	(82.) 2.07671
(77.) 2.73	(79.) 0.027	(81.) 0.042	(83.) 7.161881

MULTIPLICATION.

In Articles 82 and 86 the multiplication of decimals by integers has been taught. These the pupil may now review.

289. ILLUSTRATIVE EXAMPLE I. Multiply 175 by 0.01. Multiply 175 by 0.5.

WRITTEN WORK.

(1.) $175 \times 0.01 = 1.75$

$$\begin{array}{r} (2.) \quad 175 \\ \quad 0.05 \\ \hline 8.75 \end{array}$$

Explanation. — (1.) To multiply 175 by 0.01 is to take 1 hundredth of it, which we express by placing the decimal point so that the figures 175 may express hundredths; thus, 1.75.

(2.) To multiply 175 by 0.05 is to take 5 hundredths of it. One hundredth of 175 is 1.75, and 5 hundredths is 5 times 1.75, which equals 8.75. *Ans.* 8.75.

290. ILLUSTRATIVE EXAMPLE II. Multiply 0.4 by 0.9.

WRITTEN WORK.

0.4	Explanation. — To multiply 0.4 by 0.9 is to take 9 tenths of 4 tenths. One tenth of 0.4 is 4 hundredths, and 9 tenths of 4 tenths is 9 times 0.04, which equals 0.36. <i>Ans.</i> 0.36.
0.9	
0.36	
0.36	

291. From the written work above may be derived the following

Rule.

To multiply by decimals: *Multiply as in integers, and point off as many places for decimals in the product as there are decimal places in the multiplicand and the multiplier counted together.*

NOTE. If there are not figures enough in the product, prefix zeros.

292. Examples for the Slate.

Multiply

- | | |
|---------------------|--------------------------------|
| (84.) 0.048 by 9. | (93.) 40.5 by 0.016 |
| (85.) 0.027 by 34. | (94.) 1842 by 0.07 |
| (86.) 0.075 by 20. | (95.) 0.0758 by 20. |
| (87.) 84 by 0.056 | (96.) 6.6 by $33\frac{1}{3}$ |
| (88.) 600 by 0.07 | (97.) 10.75 by $8\frac{2}{3}$ |
| (89.) 8.4 by 0.56 | (98.) $18\frac{2}{3}$ by 0.054 |
| (90.) 4.65 by 2.2 | (99.) $56\frac{1}{2}$ by 2.73 |
| (91.) 0.8 by 0.0206 | (100.) 1.7 by $272\frac{1}{2}$ |
| (92.) 7.06 by 0.053 | (101.) 66.63 by 5.7 |
102. What is the sum of 75×100 and 0.001×1000 ?
103. What is the sum of 7.5×1000 and 0.0001×0.001 ?
104. How many are $56.8 \times 0.01 + 5.29 \times 1000 + 0.7 \times 0.001$?
105. How many are $48.125 \times 8.33\frac{1}{3} + 8169.5 \times 0.09$?
106. What is the cost of paving 146.74 squares at \$ 16.84 per square ?

For other examples in multiplication of decimals, see page 135.

DIVISION.

In Articles 101, 102, and 114 the division of decimals by *integers* has been taught. These the pupil may review.

To divide an Integer or a Decimal by a Decimal.**293. ILLUSTRATIVE EXAMPLE I. Divide 72 by 1.2**

WRITTEN WORK. *Explanation.* — Before dividing by a fraction, the dividend must be expressed in the same denomination as the divisor. The divisor is a number of tenths; the dividend expressed in tenths is 72.0 (720 tenths).

720 tenths divided by 12 tenths gives the same quotient as 720 divided by 12, which is 60. *Ans.* 60.

294. ILLUSTRATIVE EXAMPLE II. Divide 1.935 by 0.45

WRITTEN WORK. *Explanation.* — Here the divisor is a number of hundredths; the dividend expressed as hundredths is 193.5 hundredths (the denomination may be indicated in the written work by a caret).

193.5 hundredths divided by 45 hundredths gives the same quotient as 193.5 divided by 45, which is 4.3. *Ans.* 4.3.

295. From the preceding examples may be derived the following

Rule.

1. To divide by decimals: *Express the dividend in the same denomination as the divisor by putting a mark as many places to the right of the decimal point as there are decimal places in the divisor.*

2. *Divide as if the divisor were an integer, making a decimal point in the quotient, when the terms of the dividend have been used as far as the mark.*

NOTE. When there is a remainder after all the terms of the dividend have been used, the division may be continued, as in Articles 102 and 103.

296. Examples for the Slate.

107. How many books at \$ 0.08 each can be bought for \$ 3.84 ?

108. At \$ 2.80 per yard, how many yards of muslin can be bought for \$ 55.30 ?

109. How many rods, each 16.5 feet, are there in 99 feet ?

110. One quart dry measure equals 67.2 cubic inches ; one quart liquid measure equals 57.75 cubic inches : in a keg whose capacity is 5040 cubic inches, how many quarts dry measure ? How many quarts liquid measure ?

111. If coal is \$ 6.67 per ton, how much coal can be bought for \$ 3.335 ?

112. At \$ 0.125 per yard for cotton cloth, how many yards can be bought for \$ 25 ?

Divide

(113.) 14.91 by 7.

(114.) 8.25 by 1.5

(115.) 3.24 by 0.81

(116.) 0.00468 by 0.013

(117.) 180.375 by 1.625

(118.) 579 by 0.075

(119.) 6.9705 by 0.45

(120.) 0.0033 by 0.011

(121.) 0.705 by 7.5

(122.) 3 by 29.9

(123.) 20 by 0.013

(124.) 4066.2 by 0.648

(125.) 68077 by 71.66

Divide

(126.) 56.28 by 0.0056

(127.) 0.417196 by 58.76

(128.) 0.08 by 1.611

(129.) 1.3 by 197.59

(130.) 1203.488 by 28.6

(131.) 49.2654756 by 0.0759

(132.) 2464.176 by 57.2

(133.) 164.6156 by 1334.

(134.) 0.789 by 0.03½

NOTE. First multiply both dividend and divisor by 7.

(135.) 1.36 by 5.807½

(136.) 43.2½ by 0.58½

137. Multiply 0.648 by 100 ; divide the product by 100 ; divide this quotient by 0.001 ; and multiply the result by ½ of 0.0362.

138. Find the sum of the following : $756.02 \times \frac{1}{10}$; 18.3×100 ; $0.7 \div 0.001$; $8.16 \div \frac{1}{100}$; and $0.24 \div 16$.

For other examples in division of decimals, see page 135.

297. DRILL TABLE No. 6.

Decimals.

Ex- amples	E	F	G
1.	0.025	Twenty-five, and two <i>thousandths</i> .	5000.
2.	25.75	Two hundred six <i>ten-thousandths</i> .	1.648
3.	0.3504	Seven hundred, and eight <i>tenths</i> .	0.657
4.	1.01	404 <i>hundred-thousandths</i> .	0.002 $\frac{1}{2}$
5.	250.	505050 <i>ten-thousandths</i> .	0.005
6.	0.0008	Eight, and ninety-six <i>hundredths</i> .	448.
7.	16.005	Five thousand two, and 5 <i>hundredths</i> .	2.5
8.	8000.	Sixteen <i>hundred-thousandths</i> .	0.008
9.	5.6	One hundred twelve <i>millionths</i> .	1.68
10.	0.708	Two, and twenty-five <i>hundredths</i> .	607.
11.	19.364	Seven hundred one, and six <i>tenths</i> .	18.08
12.	0.0516	Two, and 206 <i>thousandths</i> .	9.16
13.	1.732	936 <i>ten-thousandths</i> .	800.1
14.	8016.	54, and 54 <i>thousandths</i> .	0.0052
15.	4.95	806, and 1047 <i>millionths</i> .	2.0763
16.	0.012	5 hundred, and 26 <i>hundredths</i> .	18.26 $\frac{1}{2}$
17.	12.007	One thousand <i>millionths</i> .	0.0101
18.	45.9	Twenty-nine <i>millionths</i> .	3.0712
19.	8.621	846291 <i>hundred-thousandths</i> .	80.06
20.	0.00562	Five hundred eleven <i>thousandths</i> .	5.4
21.	1002.	4271, and 4271 <i>ten-millionths</i> .	0.6805
22.	1.87 $\frac{1}{2}$	68 thousand, and 4 $\frac{1}{2}$ <i>tenths</i> .	4.071
23.	0.12 $\frac{1}{2}$	One hundred twenty-two <i>thousandths</i> .	24.40
24.	1.015	Eight, and 4 $\frac{1}{2}$ <i>hundredths</i> .	0.0002
25.	8.33 $\frac{1}{3}$	Five hundred, and $\frac{2}{3}$ <i>tenths</i> .	1.071

298. Exercises upon the Table.

- | | | |
|----------------------------|---|---------------------|
| H | 177. Read the numbers expressed in E. | |
| $\frac{1}{15}$ | 178. Read the numbers expressed in G. | |
| $\frac{1}{3}$ | 179. Write in figures the numbers expressed in F. | |
| $\frac{1}{30}$ | 180. Change E to equiv. common fractions in lowest terms. | |
| $\frac{1}{300}$ | 181. Change G to equiv. common fractions in lowest terms. | |
| $\frac{1}{3000}$ | 182. Change H to equiv. decimals (4 places). | |
| $\frac{1}{30000}$ | 183. Add E and F. | |
| $\frac{1}{300000}$ | 184. Add F and G. | |
| $\frac{1}{3000000}$ | 185. Add E F and G. | |
| $\frac{1}{30000000}$ | 186. Find the difference of E and F. | |
| $\frac{1}{300000000}$ | 187. Find the difference of E and G. | |
| $\frac{1}{3000000000}$ | 188. Find the difference of F and G. | |
| $\frac{1}{30000000000}$ | 189. Multiply E by F. | 192. Divide E by F. |
| $\frac{1}{300000000000}$ | 190. Multiply E by G. | 193. Divide G by E. |
| $\frac{1}{3000000000000}$ | 191. Multiply F by G. | 194. Divide G by F. |
| $\frac{1}{30000000000000}$ | 195. Multiply E by 10 ; divide F by 100 ; add the results to G. | |

299. Questions for Review.

- What are DECIMAL FRACTIONS ? How are their written expressions distinguished from those of integral numbers ? What indicates the denomination of the decimal ? How do you read a decimal expression ? Read 7.05 as a mixed number ; as a fraction. Read 0.504 and 500.004 so that they may be distinguished. How do you write decimals ? What is the effect of annexing ciphers to a decimal expression ?
- How do you change decimals to common fractions ? Common fractions to decimals ? What fractions cannot be changed wholly to a decimal form ? What are they called when expressed decimally ? How do you change a circulate to a common fraction ? How do you add and subtract decimals ? Perform an example in multiplication by a decimal, explain and give the rule. Perform an example in division by a decimal, explain and give the rule.
- How do you express the multiplication of a decimal by 10 ; 100 ; 1000 ; 0.1 ; 0.01 ; 0.001 ? How do you express the division of a decimal by 10 ; 100 ; 1000 ; 0.1 ; 0.01 ; 0.001 ?

SECTION XI.

WEIGHTS AND MEASURES.

MEASURES OF LENGTH.

300. We measure the length of anything by applying to it a line of known length, as 1 foot, 1 yard, and finding how many such lengths it contains. The line of known length so used is a *linear unit*.

301. The length of a line is reckoned in linear units.

302. In measuring length we employ the *mile* (m.), *rod* (rd.), *yard* (yd.), *foot* (ft.), and *inch* (in.). These are the units of

Long Measure.

12 inches = 1 foot.

3 feet = 1 yard.

5½ yards or 16½ feet = 1 rod.

320 rods or 5280 feet = 1 mile.

NOTE I. The standard unit of length is the YARD. From this the other units of length are derived.

NOTE II. For surveyors' and mariners' measures, see Appendix, page 306.

303. Oral Exercises.

- a. How many inches are there in 1 yard? in 2 yards? in half a yard? in a quarter? in an eighth? in a sixteenth?
- b. What will it cost to grade a mile of road at \$1 a rod?
- c. What is the length in feet of a hall that is 15 yards 2 feet long?
- d. How many feet in the length of a fence 5 rods long?
- e. One eighth of a mile is sometimes called a furlong. How many rods in a furlong?

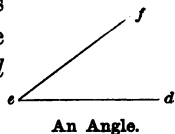
MEASURES OF SURFACE

304. Two lines meeting at a point form an *angle*. Thus the lines ab and bc form the angle abc .

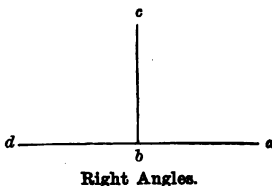
305. The lines are the *sides* of the angle, and the point where they meet is the *vertex*.



306. The size of an angle is the amount by which one side is turned away from the other. Thus the angle def is greater than the angle abc , for the side ef is turned away from ed more than bc is turned away from ba .

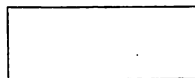


307. When one line meets another so as to form two equal angles, each of these angles is a *right angle*, and the lines are *perpendicular* to each other. Thus, the line bc is turned away equally from ba and bd , making the angles abc and cbd equal to each other.



308. A flat surface, as the surface of a slate or the top of a table, is a *plane surface*.

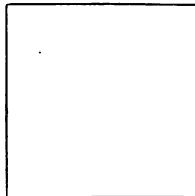
309. A *rectangle* is a plane surface bounded by four straight lines, and having all its angles right angles.



A Rectangle.

310. A *square* is a rectangle all of whose sides are equal.

311. A square each of whose sides is 1 inch long is a *square inch*; a square each of whose sides is 1 foot long is a *square foot*, etc.



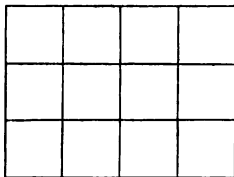
A Square.

312. The *area* of a surface is its contents reckoned in square units.

To find the Area of a Rectangle.

313. ILLUSTRATIVE EXAMPLE. If the length of a rectangle is 4 inches and its breadth is 3 inches, how many square inches does it contain?

ILLUSTRATION.



Explanation. — A rectangle which is 4 in. long and 1 in. wide will contain 4 square inches, and a rectangle of the same length and 3 in. wide must contain 3 times 4, or 12 square inches. (See illustration.)

In the same way it can be shown that the area of any rectangle is found by *multiplying the number of units in the length by the number of like units in the breadth.* This is expressed, for brevity, as *multiplying the length by the breadth.*

314. Oral Exercises.

a. How many square inches are there in a rectangle 8 in. long and 5 in. wide? 11 in. long and 10 in. wide? 12 in. (1 foot) long and 12 in. (1 foot) wide? Then how many square inches are there in a square foot?

b. How many square feet are there in a rectangle 8 ft. long and 7 ft. wide? How many square feet are there in a square whose sides are each 3 ft. (1 yard) long?

c. How would you find the number of square yards in a square whose sides are each $5\frac{1}{2}$ yards or 1 rod long?

315. In measuring surface we employ the *square mile* (sq. m.), *acre* (A.), *square rod* (sq. rd.), *square yard* (sq. yd.), *square foot* (sq. ft.), and *square inch* (sq. in.). These are the units of

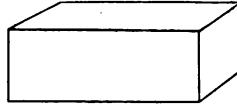
Square Measure.

144 square inches	= 1 square foot.
9 square feet	= 1 square yard.
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	= 1 square rod.
160 square rods	= 1 acre.
640 acres	= 1 square mile.

MEASURES OF VOLUME.

316. A *rectangular solid* is a solid bounded by six rectangles.

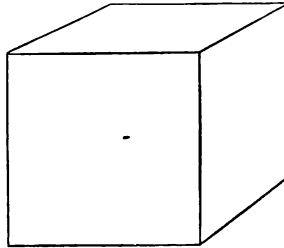
317. The rectangles are the *faces* of the solid, and, together, make its surface. The bounding lines of the solid are its *edges*.



A Rectangular Solid.

318. A *cube* is a rectangular solid bounded by six equal squares.

A cube each of whose edges is 1 inch long is a cubic inch. A cube each of whose edges is 1 foot long is a cubic foot, etc.



A Cube.

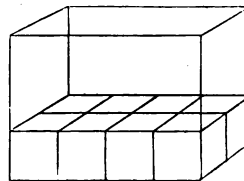
319. The *volume* of a solid is its contents reckoned in cubic units.

To find the Volume of a Rectangular Solid.

320. ILLUSTRATIVE EXAMPLE. What is the volume of a block of marble 4 ft. long, 2 ft. wide, and 3 ft. thick?

Explanation.—If the block is 4 feet long and 2 feet wide, its lower base must contain 4×2 , or 8 square feet (Art. 313). A solid 1 foot thick upon these 8 square feet will contain 8 cubic feet, and a solid 3 feet thick will contain 3 times 8 or 24 cubic feet.

ILLUSTRATION.



In the same way it can be shown that the volume of any rectangular solid is found by *multiplying the number of units in the length by the number of like units in the breadth, and this product by the number of like units in the thickness*. This is expressed, for brevity, as *multiplying together the length, breadth, and thickness*.

321. Oral Exercises.

a. How would you find the number of cubic inches in a cube 12 inches long, 12 inches wide, and 12 inches thick, or in 1 cubic foot?

b. How many cubic feet in a cube 3 feet long, 3 feet wide, and 3 feet thick, or in 1 cubic yard?

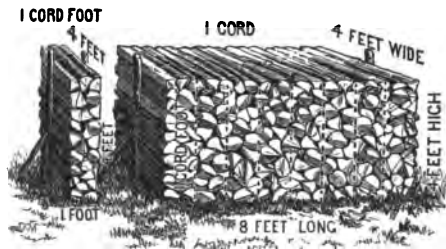
322. In measuring solids we employ the *cubic yard* (cu. yd.), *cubic foot* (cu. ft.), and *cubic inch* (cu. in.). These are the units of

Cubic Measure.

1728 cubic inches = 1 cubic foot.

27 cubic feet = 1 cubic yard.

128 cubic feet = 1 cord (cd.), used in measuring wood.



Wood is generally cut for the market into sticks 4 feet long, and laid in piles, so that the length of the sticks becomes the width of the pile. A pile 4 feet wide, 4 feet high, and 8 feet long, contains 1 cord.

One eighth of a cord is called 1 *cord foot*. 1 cord foot contains 16 cubic feet. (See illustration above.)

c. How many cords are there in a pile of wood 4 feet wide, 4 feet high, and 20 feet long? 32 feet long? 90 feet long?

d. What is the cost of a pile of wood 2 feet wide, 4 feet high, and 10 feet long, at \$8 a cord?

e. What must I pay for 3 cords of hard wood at \$9.50 a cord, and $\frac{1}{2}$ of a cord of pine at \$5 a cord?

MEASURES OF WEIGHT.

323. In weighing groceries and most other common goods, we use the *ton* (T.), *pound* (lb.), and *ounce* (oz.). These are the units of

Avoirdupois Weight.

16 oz. = 1 lb.
2000 lb. = 1 T.

NOTE I. In weighing some articles, as iron and coal at the mines, and goods on which duties are paid at the United States custom-houses, the *long ton* of 2240 lbs. is used. In this weight

28 lb. = 1 quarter (qr.).
4 qr. = 1 hundredweight (cwt).
20 cwt. = 1 T.

NOTE II. For apothecaries' weight, see Appendix, page 307.

NOTE III. The standard unit of weight is the **TROY POUND**. From this the other units of weight are derived.

NOTE IV. A cubic foot of water weighs 1000 ounces avoirdupois.

324. In weighing silver, gold, precious stones, etc., we use the *pound*, *ounce*, *pennyweight* (pwt.), and *grain* (gr.). These are the units of

Troy Weight.

24 gr. = 1 pwt.
20 pwt. = 1 oz.
12 oz. = 1 lb.

325. Comparison of Weights.

175 lb. Troy = 144 lb. av.
170 oz. " = 192 oz. av.
7000 gr. " = 1 lb. av.

Which is heavier, a pound Troy or a pound avoirdupois? an ounce Troy or an ounce avoirdupois?

326. Oral Exercises.

- a. How many ounces in 1 lb. avoirdupois? in 2 lb. 1 oz.?
- b. How many ounces in 1 lb. Troy? in 4 lb. 5 oz.?
- c. Change 60 gr. to pennyweights; 90 pwt. to ounces.
- d. How many ounces in 3 pounds of silver?
- e. What is the value of a gold chain weighing $1\frac{1}{2}$ ounces at 90¢ a pwt.?
- f. At 80¢ a pound for camphor, what is the cost of an ounce?
- g. How many more pounds in a long ton than in a common ton?

MEASURES OF CAPACITY.

327. In measuring liquids we use the *gallon* (gal.), *quart* (qt.), *pint* (pt.), and *gill* (gi.). These are the units of

Liquid Measure.

4 gi. = 1 pt.

2 pt. = 1 qt.

4 qt. = 1 gal.

NOTE I. A pint of water weighs about a pound avoirdupois.

NOTE II. The standard unit for liquid measure is the GALLON.

NOTE III. The standard unit for dry measure is the BUSHEL.

NOTE IV. In buying and selling grain and many other kinds of produce, the bushel is reckoned at a certain number of pounds. Thus, potatoes have 60 pounds to a bushel and corn has 56 pounds to a bushel.

328. In measuring dry articles, as grain, small fruits, seeds, etc., we use the *bushel* (bu.), *peck* (pk.), *quart*, *pint*, and *gill*. These are the units of

Dry Measure.

4 gi. = 1 pt.

2 pt. = 1 qt.

8 qt. = 1 pk.

4 pk. = 1 bu.

329. Comparison of Liquid and Dry Measures.

<i>Liquid Measure.</i>		Cu. In.	<i>Dry Measure.</i>		Cu. In.
1 quart	=	57½	1 quart	=	67½
1 gallon	=	231	1 bushel	=	2150.42

330. Oral Exercises.

a. How many half-pint tumblers can a person fill with 1 gallon of jelly?

b. How many quart measures can be filled with 1 bushel of cranberries?

c. What does a vender receive for 1 peck of peanuts which he sells at 5 cents a pint?

d. Which is larger, 1 quart of milk, or 1 quart of berries?

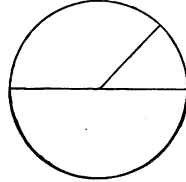
e. How many pints in a bushel? How many gills in a gallon?

f. I bought 3 bushels of pears for \$2 a bushel, and sold them at 10 cents a quart; what did I gain by the sale?

CIRCULAR AND ANGULAR MEASURES.

331. A plane surface bounded by a line every point of which is equally distant from a point within, called the centre, is a *circle*.

332. The bounding line of a circle is the *circumference*. Any part of the circumference is an *arc*.



A Circle.

333. The circumference of a circle is divided into 360 equal arcs, called *degrees* ($^{\circ}$), each degree into 60 *minutes* ($'$), and each minute into 60 *seconds* ($''$). These are the units of

Circular Measure.

60 seconds = 1 minute.

60 minutes = 1 degree.

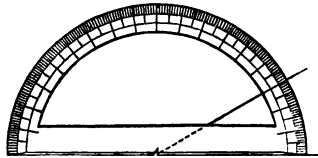
360 degrees = 1 circumference.

NOTE. As the circumference of every circle has 360 degrees, the length of the degree differs in different circles.

334. A degree of the circumference of the earth at the equator is about 69.16 common miles in length.

335. A minute of the circumference of the earth at the equator is a *geographical* or *nautical mile*, and equals about 1.15 common miles.

336. If the centre of a circle is placed at the vertex of an angle, the arc included between the sides is the measure of the angle. Thus, if the arc contains 30 degrees, the angle is called an angle of 30 degrees. (See illustration.)



NOTE. An angle of one degree has always the same size, but the arc that measures it differs in different circles.

337. Oral Exercises.

a. How many degrees are there in a semi-circumference? in $\frac{1}{2}$ of a circumference, or a quadrant? in $\frac{1}{4}$ of a circumference, or a sextant?

b. The torrid zone is 47° wide. How would you find its width in nautical miles? in common miles?

c. Through how many degrees does the hour hand of a clock move in 3 hours? in 1 hour? in 2 hours?

d. Through how many degrees does the minute hand of a clock move in 5 minutes of time? in 1 minute? in a quarter of an hour? in half an hour?

e. How many degrees in a right angle?

f. How long does it take the hour hand of a clock to move through a right angle? How long does it take the minute hand?

g. The hour and minute hand of a clock form an angle of how many degrees at 3 o'clock? at 4 o'clock? at 10 o'clock? at 7 o'clock? at 12 o'clock?

MEASURES OF TIME.

338. In measuring time we employ the *century*, *year*, *month* (mo.), *week* (w.), *day* (d.), *hour* (h.), *minute* (m.), and *second* (s.). These are the units of

Time Measure.

60 seconds = 1 minute.

60 minutes = 1 hour.

24 hours = 1 day.

7 days = 1 week.

365 days }
or 52 weeks 1 day } = 1 common year (c. y.).

366 days = 1 leap year (l. y.).

100 years = 1 century (C.).

339. Any year is a leap-year *when the number denoting the year is divisible by 4 and not by 100, and when it is divisible by 400.* (See Appendix, page 307.)

Which of the following named years are leap-years :

1878 ? 1892 ? 1888 ? 1900 ? 2000 ? 1864 ? 1880 ?

340. The year begins with the first of January, and is divided into four *seasons* of three months each, as follows :

The *winter* months are December, January, and February.

The *spring* months are March, April, and May.

The *summer* months are June, July, and August.

The *autumn* months are September, October, and November.

341. April, June, September, and November have 30 days each. February has 28 days, in leap year 29. The other months have 31 days each.

342. Oral Exercises.

- a. What date is three months from Jan. 5 ? July 10 ?
- b. What date is 6 months from May 2 ? Feb. 11 ? Nov. 1 ?
- c. What months contain 30 days each ? 31 days each ?
- d. At 10 cents an hour for 5 hours of every working day, how much can you earn in 4 weeks ?
- e. 8 years and 9 months are how many months ?
- f. How many years are there in 100 mo. ? in 200 mo. ?
- g. What date is 30 days from May 5 ? from Apr. 4 ?
- h. How many days from May 3 to June 5 ?

Miscellaneous Measures.

343. Numbers.

12 units = 1 dozen.
 12 dozen = 1 gross.
 12 gross = 1 great gross.
 20 units = 1 score.

344. Paper.

24 sheets = 1 quire.
 20 quires = 1 ream.
 2 reams = 1 bundle.
 5 bundles = 1 bale.

NOTE. For other measures sometimes used, see Appendix, page 307.

SECTION XII.

COMPOUND NUMBERS.

345. In 2 feet 7 inches, how many inches? *Ans.* 31 inches.

The number 31 inches expresses a quantity by reference to a single integral unit. Such a number is a *simple number*.

346. The number 2 feet 7 inches expresses a quantity by reference to two units of different denominations. A number expressing a quantity by reference to two or more units of different denominations is a *compound number*.

The compound number 2 feet 7 inches expresses the same quantity that the simple number 31 inches does.

347. When the name of the units is given, the number is a *denominate number*. Thus, 31 inches and 2 feet 7 inches are both denominate numbers.

348. When the name of the unit is not given, the number is a *general number*. Thus, 31 is a general number.

NOTE. Denominate numbers are sometimes called *concrete numbers*, and general numbers are called *abstract numbers*.

Name a simple number ; a compound number ; a denominate number ; a general number. Is 5 feet 2 inches a denominate or general number ? a simple or compound number ? Is 25 a denominate or a general number ? a simple or a compound number ?

349. Written Exercises.

Write from memory the table for Long Measure, Square Measure, Cubic Measure, Liquid Measure, Dry Measure, Avoirdupois Weight, Troy Weight, Circular or Angular Measure, Numbers, Paper.

REDUCTION.

To change a Compound Number to a Simple Number.

350. ILLUSTRATIVE EXAMPLE. Change 2 bu. 3 pk. 4 qt. to quarts.

Explanation. — Since in 1 bushel there are 4 pecks, in 2 bushels there are 2 times 4, or 8 pecks, which with 3 pecks added are 11 pecks. Since in 1 peck there are 8 quarts, in 11 pecks there are 11 times 8 quarts, etc.

WRITTEN WORK.
2 bu. 3 pk. 4 qt.
4

11 pk.

8

92 qt. *Ans.*

Rule.

351. To change a compound number to a simple number of a lower denomination: *Multiply the number of the highest denomination by the number of units it takes of the next lower denomination to make one of that higher, and to the product add the given number of the next lower denomination. Multiply this sum in like manner, and so proceed till the given number is changed to units of the required denomination.*

352. Examples for the Slate.

1. Change 4 T. 350 lb. 8 oz. to ounces.
2. What is the value of 2 lb. 8 oz. of gold at \$ 20 an ounce?
3. Change 3 rd. 4 yd. 1 ft. to feet.
4. What will it cost to fence both sides of a road 26 rd. 6 ft. long, at 22¢ a foot?
5. How many square feet are there in an acre?
6. Change 3 sq. m. 35 A. to acres.
7. In a cubic yard, how many cubic inches?
8. What shall I receive for 25 gal. 3 qt. of milk at 7 cents a quart?
9. Mr. Russell sold 4 bu. 1 pk. 2 qt. of cherries at 12 cents a qt.; what did he receive for them?

10. If the pulse beats 80 times in 1 minute, how many times will it beat in a common year?

11. If a child sleeps $\frac{1}{3}$ of his time, how many hours will he sleep in 5 years, allowing for 1 leap year?

12. How many minutes were there in the first century?

13. The Tropic of Cancer is $23^{\circ} 30'$ north of the equator. What is the distance in geographical miles? in common miles?

353. Changing numbers to numbers of lower denominations is called *reduction descending*.

For other examples in reduction descending, see page 171.

To change a Simple Number to a Compound Number.

354. ILLUSTRATIVE EXAMPLE. Change 4354 feet to rods, yards, etc.

WRITTEN WORK.

$$\begin{array}{r} 3) 4354 - 1 \text{ ft. Rem.} \\ 5\frac{1}{2}) \overline{1451} \\ \underline{2} \quad \underline{2} \\ 11) \overline{2902} - \frac{2}{3} \text{ yd.} = 4\frac{1}{3} \text{ yd. Rem.} \\ \underline{263} \end{array}$$

Ans. 263 rd. $4\frac{1}{3}$ yd. 1 ft., or
263 rd. 4 yd. 2 ft. 6 in.

Explanation. — Since 3 ft. make a yard, in 4354 ft. there are as many yards as there are 3's in 4354, which are 1451, and 1 ft. remains.

Since $5\frac{1}{2}$ yards make a rod, in 1451 yards there are as many rods as there are times $5\frac{1}{2}$ in 1451, which are 263, and $\frac{2}{3}$ yd., or $4\frac{1}{3}$ yd., remain, etc.

Rule.

355. To change a simple number to a compound number of higher denominations: *Divide the given number by the number of units it takes of its denomination to make one of the next higher. Set aside the remainder, and divide, as before, the quotient thus obtained; and so proceed till the required denomination is reached. The last quotient with the several remainders is the number sought.*

356. Examples for the Slate.

Change to compound numbers:

- | | |
|--------------------|----------------------------|
| (14.) 3268 yards. | (20.) 9328 lb. of soap. |
| (15.) 4687 feet. | (21.) 19547 oz. of salt. |
| (16.) 9687 sq. rd. | (22.) 9321 pwt. of silver. |
| (17.) 5692 sq. yd. | (23.) 2089 gr. of gold. |
| (18.) 4791 sq. in. | (24.) 5087 qt. of berries. |
| (19.) 53684" | (25.) 1127793 minutes. |

26. What will 20 old silver dollars weigh in oz. pwt. etc., each dollar weighing $412\frac{1}{2}$ grains?

27. The trade-dollar weighs 420 grains. What will 20 trade-dollars weigh?

28. How many miles is it through the earth from pole to pole, the distance being 41707308 feet?

29. In a certain pasture 973 quarts of berries were picked in one week. How many bushels were picked?

357. Changing numbers to numbers of higher denominations is called *reduction ascending*.

For other examples in reduction ascending, see page 171.

REDUCTION OF DENOMINATE FRACTIONS.

To change a Denominate Fraction to Integers of Lower Denominations.

358. ILLUSTRATIVE EXAMPLE I. Change $\frac{5}{8}$ rd. to yards, feet, and inches.

WRITTEN WORK.

$$\begin{aligned}\frac{5}{8} \text{ of } \frac{1}{2} \text{ yd.} &= 4\frac{7}{8} \text{ yd.} \\ \frac{7}{8} \text{ of } 3 \text{ ft.} &= 1\frac{3}{4} \text{ ft.} \\ \frac{3}{4} \text{ of } 12 \text{ in.} &= 9 \text{ in.}\end{aligned}$$

Ans. 4 yd. 1 ft. 9 in.

Explanation. — We first change $\frac{5}{8}$ of a rod to yards, and have $4\frac{7}{8}$ yards for the result. We then change $\frac{7}{8}$ of a yard to feet, and have $1\frac{3}{4}$ feet for the result. $\frac{3}{4}$ of a foot is 9 inches. Ans. 4 yd. 1 ft. 9 in.

359. ILLUSTRATIVE EXAMPLE II. Change 0.62 rd. to yards and feet.

WRITTEN WORK.			<i>Explanation.</i> — We first change 0.62 of a rod to yards, and have 3.41 yards. We next change 0.41 of a yard to feet, and have 1.23 feet. <i>Ans.</i> 3 yd. 1.23 ft.
0.62 rd.	or	0.62 rd.	
$5\frac{1}{2}$		5.5	
<u>310</u>		<u>310</u>	
31		310	
<u>3.41 yd.</u>		<u>3.410 yd.</u>	
3		3	
<u>1.23 ft.</u>		<u>1.23 ft.</u>	
<i>Ans.</i> 3 yd. 1.23 ft.			

360. From the preceding operations we derive the following

Rule.

To change a fraction of one denomination to integers of lower denominations: *Change the fraction, as far as possible, to an integer of the next lower denomination. If a fraction occurs in the result, proceed with it as with the first fraction, and so continue as far as required.*

361. Examples for the Slate.

Change to units of lower denominations:

- | | |
|--|------------------------------------|
| (30.) $\frac{3}{4}$ of a rod. | (36.) $\frac{7}{8}$ of a cu. yard. |
| (31.) $\frac{1}{4}$ of a mile. | (37.) 0.15625 of a gal. |
| (32.) $\frac{1}{2}$ of a sq. mile. | (38.) 0.6 of a bushel. |
| (33.) $\frac{1}{2}$ of $\frac{3}{4}$ of an acre. | (39.) $\frac{5}{8}$ of a degree. |
| (34.) $\frac{1}{2}$ of a ton. | (40.) $\frac{3}{4}$ of a c. year. |
| (35.) 0.875 of a lb. Troy. | (41.) 0.75 of a l. year. |
42. At 20¢ a foot, what is the cost of $\frac{3}{4}$ of an acre of land?
43. In $\frac{3}{4}$ of an ounce of Dover's powder, how many doses of 5 grains each?
44. How many planks 8 inches wide will cover the roadway of a bridge $\frac{1}{2}$ of a mile long, each plank reaching from side to side?

To change Integers of Lower Denominations to a Fraction of a Higher.

362. ILLUSTRATIVE EXAMPLES. (I.) Change 5 oz. 6 pwt. 16 gr. to the fraction of a pound. (II.) Change 2 pk. 6 qt. to the decimal of a bushel.

WRITTEN WORK.

(I.)	(II.)
16 gr. = $\frac{16}{24}$ pwt. = $\frac{2}{3}$ pwt.	8) 6 qt.
$6\frac{2}{3}$ pwt. = $\frac{20}{3 \times 48}$ oz. = $\frac{5}{36}$ oz.	4) 2.75 pk.
$5\frac{1}{3}$ oz. = $\frac{16}{3 \times 16}$ lb. = $\frac{1}{3}$ lb. <i>Ans.</i>	0.6875 bu. <i>Ans.</i>

Explanation (I.). — Since 24 grains make a pennyweight, 16 gr. are $\frac{16}{24}$ pwt., or $\frac{2}{3}$ pwt., which, added to the 6 pwt. given, are $6\frac{2}{3}$ pwt.

$6\frac{2}{3}$ pwt. = $\frac{20}{3}$ pwt. Since 20 pwt. make an ounce, $\frac{20}{3}$ pwt. equals $\frac{1}{3}$ as large a part of an ounce, or $\frac{1}{36}$ oz., etc.

Explanation (II.). — Since 8 qts. make a peck, 6 qts. are equal to 0.75 pk., which, added to the 2 pecks given, are 2.75 pecks. Since 4 pks. make a bushel, 2.75 pk. are equal to 0.6875 bu. *Ans.* 0.6875 bu.

363. From the preceding operations we derive the following

Rule.

To change integers of lower denominations to a fraction of a higher denomination: *Change the number of the lowest given denomination to a fraction of the next higher. Unite this fraction with the number of that higher denomination. Change, in like manner, the number thus formed, and so continue as far as required.*

45. Change 1 qt. 0 pt. 1 gi. to the fraction of a gallon.
46. Change 242 rd. 2 yd. to the fraction of a mile.
47. What part of a rod is 4 yd. 0 ft. $4\frac{1}{2}$ in.?
48. What part of an acre is 81 sq. rd. 24 sq. ft.?
49. What part of a cu. yd. is 13 cu. ft. 864 cu. in.?
50. What part of a year are the three winter months?
51. Change to the decimal of a mile 87 rd. 10 ft.

52. Change to the decimal of an acre 135 sq. rd. 54 sq. ft.

53. Regarding a year as 12 months of 30 days each, what decimal of a year is 6 mo. 18 d. ? 8 mo. 24 d. ? 5 mo. 27 d. ?

For other examples in reduction of denominate fractions, see page 171.

ADDITION.

364. The operations upon compound numbers are similar to those upon simple numbers, the principal difference being that in operations upon compound numbers we use *irregular scales*, instead of the scale of tens. No special rules, therefore, are necessary for addition, subtraction, multiplication, and division.

365. ILLUSTRATIVE EXAMPLES. (I.) What is the sum of $11^{\circ} 4' 58''$, $37^{\circ} 30' 27''$, and $27^{\circ} 24' 54''$?

(I.) *Explanation.* — (I.) We write these numbers so that units of the same denomination shall be expressed in the same column. Adding the seconds, we have $139''$. Dividing $139''$ by 60 ($60'' = 1'$), we have $2' 19''$. We write the $19''$ under the line in the seconds' place. Adding the $2'$ with the minutes of the given numbers, and dividing the sum by 60 ($60' = 1^{\circ}$), we have $1^{\circ} 0'$. We write $0'$ under the line in the minutes' place. Adding the 1° with the degrees of the given number, we have 76° . *Ans.* $76^{\circ} 0' 19''$.

WRITTEN WORK.

$11^{\circ} 4' 58''$	
$37^{\circ} 30' 27''$	
$27^{\circ} 24' 54''$	
$Ans. 76^{\circ} 0' 19''$	

(II.)			(III.)			
bu.	pk.	qt.	m.	rd.	yd.	ft.
85	3	7	3	192	4	2
9	2	5		316	0	1
98	0	6	5	76	4	2
2	3	1				
<i>Ans.</i> 196 bu. 2 pk. 3 qt.			<i>Ans.</i> 9 m. 265 rd. $3\frac{1}{2}$ yd. 2 ft.			
			<i>or</i> 9 m. 265 rd. 4 yd. 0 ft. 6 in.			

NOTE. Change any denominate fraction which occurs in an answer, or in an example, to units of the lower denominations given. (See examples 56, 57, and 58.)

366. Examples for the Slate.

54. What are the contents of three barrels which contain respectively, 45 gal. 2 qt., 42 gal. 3 qt., and 47 gal. 1 qt. ?

55. How much land in three lots which contain as follows : 7 A. 83 sq. rd. 31 sq. ft., 15 A. 146 sq. rd., 22 A. 52 sq. rd. 18 sq. ft., and 5 A. 9 sq. rd. ?

56. What is the length of three roads measuring respectively 15 m. 87 rd., 28 m. 40 rd., and $35\frac{1}{4}$ miles ?

57. To $8^{\circ} 17' 32''$ add 4.735° and $\frac{1}{8}^{\circ}$.

58. Add together 7 d. 6 h., $6\frac{3}{4}$ d., and 0.375 of a week.

367. ILLUSTRATIVE EXAMPLE II. To $\frac{3}{8}$ of a gallon add $\frac{1}{4}$ of a quart.

WRITTEN WORK.

$\frac{3}{8}$ gal. = $\frac{3}{8}$ of 4 qt. = $1\frac{3}{8}$ qt.

$\frac{1}{4}$ qt.

Ans. $1\frac{7}{8}$ qt.

Explanation.— That these fractions may be added they must first be expressed in the same denomination. They may be so expressed by changing $\frac{3}{8}$ gal. to quarts, etc.

59. Add $\frac{3}{4}$ of a quart to $\frac{1}{2}$ of a bushel.

60. Add $45\frac{3}{4}$ rods to $\frac{1}{2}$ of a mile.

61. Add $54\frac{1}{2}$ pounds to $\frac{1}{4}$ of a ton.

Perform such examples in exercises 206–208, page 171, as the teacher may indicate.

SUBTRACTION.

368. ILLUSTRATIVE EXAMPLE. What is the difference between 5 rd. 3 yd. 1 ft. and 1 rd. 4 yd. 2 ft. ?

WRITTEN WORK.

5 rd. 3 yd. 1 ft.

1 4 2

Ans. 3 rd. $3\frac{1}{2}$ yd. 2 ft.

or 3 rd. 4 yd. 0 ft. 6 in.

Explanation.— We write these numbers as in simple subtraction, and subtract first the 2 feet of the subtrahend. As we have but 1 foot in the minuend, we cannot now take 2 feet away. So we change 1 of the 3 yards (leaving 2 yards) to feet. This 1 yard equals 3 feet. We add the

3 feet to the 1 foot, making 4 feet. Subtracting 2 feet from 4 feet we have 2 feet left, which we write as part of the remainder.

As we have but 2 yards left in the minuend, we cannot now take 4 yards away, so we change 1 of the 5 rods to yards. This equals $5\frac{1}{2}$ yards, which, added to 2 yards, make $7\frac{1}{2}$ yards. Subtracting 4 yards from $7\frac{1}{2}$ yards, we have $3\frac{1}{2}$ yards left, etc.

369. Examples for the Slate.

(62.)			(63.)			(64.)		
bu.	pk.	qt.	oz.	pwt.	gr.	°	'	"
5	3	2	6	10	13	35	47	28
2	1	7	3	15	18	19	54	48

65. 1 m. 80 rd. 2 yd. less 315 rd. 3 yd. equals what ?

66. What is the difference between 5 ft. 6 in. and $\frac{3}{4}$ rd. ?

67. A man who had $\frac{1}{4}$ of a square mile of woodland sold $5\frac{1}{2}$ square rods. How much had he left ?

68. A man having $\frac{3}{4}$ of a pound of silver ore, gave away $3\frac{1}{2}$ pennyweights. How much had he left ?

69. What is the difference between 0.378 of a day and 44.55 of a minute ?

70. Cape Horn is in $55^{\circ} 58' 4''$ south latitude, and the Cape of Good Hope is in $34^{\circ} 22'$ south latitude. Which is farther south, and how much ?

The difference of latitude between places on opposite sides of the equator is found by *adding* the latitudes. The difference of longitude between places on opposite sides of the first meridian is found by *adding* the longitudes. If their sum exceeds 180° , the difference of longitude equals 360° minus that sum.

For a table of longitudes, see page 159.

What is the difference of longitude between

71. Albany and Chicago ? 73. Rome and New York ?

72. Berlin and Paris ? 74. San Francisco and Calcutta ?

75. What is the difference in latitude between Philadelphia $39^{\circ} 57'$ north latitude, and Buenos Ayres $34^{\circ} 3'$ south latitude ?

To find the Number of Years, Months, and Days from one Date to another.

NOTE. The following method of finding the time is generally used in computing interest.

370. ILLUSTRATIVE EXAMPLE I. What is the time in years, months, and days from Jan. 11, 1877, to May 5, 1881?

Explanation. — From Jan. 11, 1877, to Jan. 11, 1881, is 4 years; from Jan. 11, 1881, to April 11, 1881, is 3 months; from April 11 to April 30 is 19 days, and from April 30 to May 5 is 5 days more. *Ans.* 4 y. 3 m. 24 d.

Rule.

371. To find the difference in time between two dates: *First find the number of entire years between the two dates, then the number of calendar months remaining, and lastly, the remaining days.*

372. Oral Exercises.

a. How many years, months, and days are there from Feb. 3, 1875, to Oct. 17, 1878?

b. How many years, months, and days are there from Sept. 25, 1874, to Jan. 4, 1882?

c. Mozart was born Jan. 27, 1756, and died Dec. 5, 1791; at what age did he die?

d. Goethe died March 22, 1832, and Bryant was born Nov. 3, 1794; what was Bryant's age when Goethe died?

373. ILLUSTRATIVE EXAMPLE II. How many days are there from Nov. 12, 1875, to March 10, 1876?

Explanation. — There are 18 days remaining in November, 31 days in December, 31 in January, 29 in February, and 10 in March.

$$18 + 31 + 31 + 29 + 10 = 119. \quad \text{Ans. 119 days.}$$

e. How many days from March 7 to July 1, 1878?

f. How many days from Oct. 9 to Feb. 11, 1877?

g. How many days from January 15 to August 7, 1875?

374. A Table showing the Number of Days

From any Day of	To the corresponding Day of the following											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January ..	365	31	59	90	120	151	181	212	243	273	304	334
February.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August ...	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October ..	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

NOTE. In leap years, if the last day of February is included in the time, a day must be added to the number obtained from the table.

Find from the table above the number of days

- h.* From April 19 to June 19. *j.* From Dec. 5 to Feb. 5.
i. From Jan. 1 to March 4. *k.* From Oct. 12 to Feb. 15.

Perform such examples of exercises 209 and 210, page 171, as the teacher may indicate.

MULTIPLICATION.

375. ILLUSTRATIVE EXAMPLE. How much land is there in 4 gardens, each containing 13 sq. rd. 72 sq. ft. ?

WRITTEN WORK.

13 sq. rd. 72 sq. ft.

4

53 sq. rd. 15½ sq. ft. *Ans.*

carried, are 53 sq. rd.

Explanation. — Multiplying 72 sq. ft. by 4, we have 288 sq. ft. for a product, which equals 1 sq. rd. and 15½ sq. ft. We write the 15½ sq. ft. and carry the 1 sq. rd. to the square rods in the product. 13 sq. rd. multiplied by 4 are 52 sq. rd., which, with the 1 sq. rd. carried, are 53 sq. rd. *Ans.* 53 sq. rd. 15½ sq. ft.

376. Examples for the Slate.

76. How much syrup will 7 jars contain if each jar holds 1 pt. 3 gi. ?

77. How much wheat is contained in 5 bins if each bin contains 7 bu. 4 pk. 3 qt. ?

78. If a car runs 18 m. 149 rd. in half an hour, how far will it run in 7 hours ?

Perform such examples in exercises 211 and 212, page 171, as the teacher may indicate.

DIVISION.

377. ILLUSTRATIVE EXAMPLE. Divide $47^{\circ} 18' 36''$ by 11.

WRITTEN WORK.

11) $47^{\circ} 18' 36''$

$4^{\circ} 18' 36''$ Ans.

Explanation. — Dividing 47° by 11, we have 4° for a quotient, with a remainder of 3° . We write the 4° under the line, and change the 3° remaining to minutes, obtaining $180'$. Adding $180'$ to the $18'$ in

the dividend, we have $198'$. Dividing $198'$ by 11, we have $18'$ for a quotient, etc.

378. Examples for the Slate.

79. A farmer brought 5 bu. 3 pk. of corn to mill. How much corn did the miller take as toll, if he took $\frac{1}{8}$ part ?

80. If 65 A. 125 sq. rd. be divided into 50 house-lots, what is the size of each ?

81. How long will it take to travel 1 mile, at the rate of 75 miles in 10 h. 18 min. ?

82. Among how many men may 624 gal. 3 qt. be divided, that each man may receive 12 gal. 3 qt. ?

NOTE. Change both numbers to quarts before dividing.

83. How many bins, each containing 5 bu. 3 pk., will be required to hold 885 bu. 2 pk. of potatoes ?

84. If a man walks 3 m. 264 rd. in one hour, how long will it take him to walk 23 m. 273 rd. ?

Perform such examples in exercises 213 to 215, page 171, as the teacher may indicate.

LONGITUDE AND TIME.

379. As the earth turns upon its axis once in 24 hours, it follows that $\frac{1}{24}$ of 360° , or 15° of longitude, must pass under the sun in 1 hour, and $\frac{1}{60}$ of 15° , or $15'$, must pass under the sun in 1 min. of time, and $\frac{1}{60}$ of $15'$, or $15''$, must pass under the sun in 1 sec. of time. Hence the following

TABLE.

A difference of 15° in longitude	}	makes	{	A difference of 1 hour in time.
A difference of $15'$ in longitude				A difference of 1 minute in time.
A difference of $15''$ in longitude	}	makes	{	A difference of 1 second in time.

380. From the table above we derive the following

Rule.

To find the difference of longitude between any two places when the difference of time is known: *Multiply the difference of time between the two places, expressed in hours, minutes, and seconds, by 15. The product will express the number of degrees, minutes, and seconds required.*

NOTE. As the earth turns from west to east, midday occurs sooner in places east and later in places west of any given point. Hence the time shown by a clock is later in all places east, and earlier in all places west, of any given point than it is at that point.

381. Examples for the Slate.

What is the difference in longitude between two places, the difference in their time being

(85.) 4 h. 17 m. ?

(87.) 6 h. 12 m. 10 s. ?

(86.) 2 h. 9 m. ?

(88.) 1 h. 5 m. 25 s. ?

In what longitude from Greenwich is a place whose time compared with that of Greenwich is

(89.) 3 hours earlier ?

(91.) 1 hour 12 minutes later ?

(90.) 5 minutes later ?

(92.) 4 hours 8 minutes earlier ?

93. The time in St. Louis is 1 h. 5 min. $\frac{1}{3}$ s. slower than in New York; what is the difference in longitude between these places, and what is the longitude of St. Louis, that of New York being $74^{\circ} 0' 3''$ west?

94. A and B sailed together from San Francisco. A kept his watch by San Francisco time, and B set his by the sun every day. After 10 days, A's watch was 4 hours 39 minutes faster than B's: in what longitude were they then, the longitude of San Francisco being $122^{\circ} 26' 15''$ west?

382. From Art. 379 we may also derive the following

Rule.

To find the difference in time between any two places when the difference in longitude is known: *Divide the difference in longitude, expressed in degrees, minutes, and seconds, by 15. The quotient will express the number of hours, minutes, and seconds required.*

383. The names of a few important cities are given below, with the longitude of each from Greenwich.

Places.	Longitudes.	Places.	Longitudes.
Albany.....	$73^{\circ} 44' 53''$ W.	New Orleans	$90^{\circ} 7'$ W.
Boston.....	$71^{\circ} 3' 30''$ W.	New York ...	$74^{\circ} 0' 3''$ W.
Berlin.....	$13^{\circ} 23' 43''$ E.	Paris.....	$2^{\circ} 20' 22''$ E.
Calcutta.....	$88^{\circ} 19' 2''$ E.	Philadelphia	$75^{\circ} 10'$ W.
Chicago	$87^{\circ} 35'$ W.	Rome (Italy)	$12^{\circ} 27' 14''$ E.
London.....	$0^{\circ} 5' 38''$ W.	San Francisco	$122^{\circ} 26' 15''$ W.
Montreal.....	$73^{\circ} 25'$ W.	Washington	$77^{\circ} 2' 48''$ W.

384. Using the longitudes given above, find the difference in time between

95. Albany and Boston. 97. Montreal and New Orleans.
96. London and New York. 98. Philadelphia and Chicago.

When it is noon in Washington, what is the time

- | | |
|-------------------------|--------------------|
| 99. In Philadelphia ? | 103. In Rome ? |
| 100. In New Orleans ? | 104. In Berlin ? |
| 101. In Chicago ? | 105. In Paris ? |
| 102. In San Francisco ? | 106. In Calcutta ? |

MENSURATION OF SURFACES AND SOLIDS.

385. Oral Exercises.

a. How many square feet are there in the top of a table that is 7 feet long and 3 feet wide ? (Art. 313.)

b. How many square yards are there in a concrete walk $16\frac{1}{2}$ feet long and 4 feet wide ?

c. How do you find the area of any rectangle or square ?

386. From Art. 313, it follows that when the area and one dimension of a rectangle or a square are given, the other dimension is found *by dividing the number of units of area by the number of units in the given dimension.*

d. There are 15 square yards in a piece of carpeting 5 yards long ; what is its width ?

e. What must be the length of a walk $2\frac{1}{2}$ feet wide to contain 17 square feet ?

f. How many cubic feet will a box contain that measures on the inside 7 feet in length, 3 feet in width, and 2 feet in height ? (Art. 320.)

g. How do you find the volume of any rectangular solid ?

387. From Art. 320, it follows that when the volume and two dimensions of a rectangular solid are given, the other dimension is found *by dividing the number of units of volume by the product of the number of units in each of two given dimensions.*

h. What must be the depth of a cistern 5 feet long and 4 feet wide to contain 80 cubic feet?

i. What must be the height of a room 6 yards long and 5 yards wide to contain 90 cubic yards?

j. A box 4 inches square must be how deep to contain a quart dry measure?

Examples for the Slate.

388. SQUARES AND RECTANGLES.

107. How many yards of carpeting 1 yard wide will cover a floor 17 feet long and 15 feet wide?

108. How many yards of carpeting 27 inches wide will be required to cover the same floor?

109. What must I pay for laying a sidewalk 5 rods long and 5 feet wide at 90¢ per square yard?

110. If one side of a square field is 4 rd. 8 ft. long, how many square feet are there in the field?

111. What must I pay for a building lot in St. Louis, 90 feet long and 2 rods wide, at \$1.75 per square foot?

112. My building lot contains 1 quarter of an acre, is rectangular, and measures on the street 90 feet, how far back does it extend?

113. What must I pay for a quarter of an acre of land at 20¢ per square foot?

114. How many more square rods are there in a field 42 rods square than in a 10-acre lot?

115. My neighbor's garden is 2 rods square, and mine contains 2 square rods; what is their difference in size?

116. How many acres were covered by the main Centennial building in Philadelphia, which was 1880 feet long and 464 feet wide?

117. What would it cost to make the floor of the above-named building, the boards costing \$37 per thousand feet, square measure, and the work costing 25¢ per hundred, square measure?

389. GOVERNMENT LANDS.

Before being brought into market, the public lands of the United States are usually divided by parallels and meridians into townships, each being as nearly as possible six miles square. Each township is divided in the same way into 36 sections, and each section into 4 quarter-sections. The township and sections are numbered and referred to special meridians and base lines, so as to be easily designated and pointed out on government maps.

N					
6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36
S					

A Township.

- a. How many square miles in a township ? in a section ?
 b. How many acres in a section ? in a quarter-section ?

118. What must I pay for the N. W. quarter of section No. 9 of township 5 North, 20 West, meridian Michigan, at \$ 2.50 an acre ?

390. RECTANGULAR SOLIDS.

119. How many cubic inches are there in a beam 4 ft. 5 in. long, 8 in. wide, and 4 in. thick ?

120. What is the weight of a block of Quincy granite 15 ft. long, $1\frac{1}{4}$ ft. wide, and 6 in. high, if 1 cubic foot weighs 165 pounds ?

121. How high must a block of freestone be to contain 84 cubic feet, if its length is $4\frac{1}{2}$ ft. and its width 3 ft. ?

122. If a bin contains $11\frac{2}{3}$ cubic yards, and its height is 2 ft., what is the area of its base ?

123. There being $112\frac{1}{2}$ cubic feet in a shaft of marble which is 27 in. square at each end, what is its length ?

391. Wood Measure.

124. If a pile of wood is 3 ft. 8 in. high and 4 ft. wide, how long must it be to contain 1 cord?

125. At \$6 a cord, what is the cost of a pile of wood 33 ft. long, 8 ft. 10 in. high, and 4 ft. wide?

126. If wood is cut in lengths of $3\frac{1}{2}$ ft. and piled to a height of 4 ft., how long must the pile be to contain 1 cord?

127. On measuring what I bought for a cord of wood, I found it 8 feet long, 4 feet wide, and only 3 feet 8 inches high. At \$5 a cord, how much money should be deducted from the original price?

392. LUMBER AND BOARDS.

Sawed timber and boards, when 1 inch or less in thickness, are generally reckoned by the square foot of surface measure. When more than 1 inch in thickness, they are reckoned in proportion to their thickness. Thus,

2000 sq. ft., 1 inch or less in thickness,	= 2000 ft., board measure,
2000 sq. ft., $1\frac{1}{2}$ inches thick,	= 3000 ft., board measure,
2000 sq. ft., 2 inches thick,	= 4000 ft., board measure,

and so on.

128. How many feet of boards $\frac{3}{4}$ of an inch thick will be required to make a fence 2 rods long and 3 feet high?

129. How many feet, board measure, are there in a piece of square timber 10 in. wide, 6 in. thick, and 9 ft. long?

130. How many feet, board measure, are there in 200 pieces of scantling, each 18 ft. long, 4 in. wide, and 2 in. thick?

131. How many feet in 8 boards, each 15 ft. long, 8 in. wide, and $1\frac{1}{2}$ in. thick?

132. How many feet, board measure, in a plank 24 ft. long, 3 in. thick, 11 in. wide at one end, and 16 in. wide at the other?

NOTE. First find the *average width*, which equals one half the sum of the widths at the ends.

133. At \$ 22 a thousand, what is the cost of 20 boards, each 18 ft. long, 1 in. thick, 20 in. wide at one end, and 17 in. wide at the other ?

134. Arthur bought wood for Sorrento carving, each piece being 2 feet long and $\frac{1}{8}$ of an inch thick, as follows: white holly, 12 inches wide at 8¢ a foot, board measure; black walnut, 18 inches wide at 6¢; ebony, 9 inches wide at 25¢; red cedar, 14 inches wide at 10¢. What was the cost of the whole ?

393. CAPACITY OF CISTERNS, BINS, ETC.

135. I have a cask that contains 2 cu. ft.; how many quarts of berries will it hold ? (See Art. 329.)

136. How many gallons of water will a cistern hold that is 3 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep ?

137. If a jar weighs 10 pounds when empty and 74 pounds when full of water, what is its capacity in cubic feet ? How many gallons will it hold ? (See page 141, Note IV.)

138. How many bushels of wheat can be put into a bin 8 ft. long, 3 ft. 2 in. wide, and 2 ft. 3 in. deep ?

In measuring bulky fruits and vegetables, as apples and potatoes, the measures are heaped. Heaped measures fill about $\frac{1}{4}$ more space than the even measures.

139. If 24 bushels of wheat can be put into a certain bin, how many bushels of apples might be put into the same bin ?

140. How many bushels of beets can be put into a barrel that holds 47 gallons ?

141. What is the difference in inches between $\frac{3}{4}$ of a bushel and 1 cubic foot ?

NOTE. The difference being so slight, for rough estimates of the contents of bins, etc., it is sufficiently accurate to call every cubic foot $\frac{3}{4}$ of a bushel, even measure, or $\frac{1}{4}$ of a bushel, heaped measure.

142. A box whose capacity is 50 cubic feet, will contain how many bushels of rye ? how many bushels of pears ?

394. General Review, No. 3.

143. Change 5 m. 42 rd. 8 ft. to feet.
144. Change 4865 gr. to Troy pounds, ounces, etc.
145. Change $\frac{3}{4}$ cu. yd. to feet and inches.
146. What cost 12 bu. 2 pk. of plums at 6¢ a pint?
147. What cost 2 qt. $1\frac{1}{2}$ pt. of oil at \$1.12 a gallon?
148. Change 41' 42" to the decimal of a degree.
149. Change $2\frac{3}{8}$ yd. to the decimal of a rod.
150. What part of an acre is 116 r. 3 ft. 6 in. + 17 r. 2 ft.?
151. Add 0.44 c. y. to 25 d. 5 h. 4 m.
152. Divide 12 A. less 7 A. 16 r. by 9.
153. Change 2 lb. av. to integers of Troy weight.
154. How many square feet in a garden 4 rd. long and 1 rd. 15 ft. wide?
155. How many cu. ft. of space in a cellar measuring on the inside of the wall 5 yd. 1 ft. in length, 4 yards in width, and 10 feet in depth?
156. What must be the depth of a cistern to contain 420 gallons of water, the base being a square covering $12\frac{1}{4}$ sq. ft.?
157. When a cistern 4 feet high is full of water, what weight is supported by every square inch of the base? (See page 141, Note IV.)
158. How many bricks 4 in. by 8 in. will be required to pave a court 20 ft. long and 10 ft. wide? Find the cost at \$9 per M.
159. Divide 0.006 by 0.06, multiply the quotient by 0.05, and divide that product by 0.005.
160. Change 0.0625 to a common fraction in smallest terms?
161. How many yards of carpeting $\frac{3}{4}$ of a yard wide must be bought to cover a floor 13 feet square, no allowance being made for matching and no breadth to be divided?
162. What is the difference of time in two places whose longitudes differ $7^{\circ} 8' 4''$?
163. When the difference of time between two places is 3 h. 4 m. 6 s., what is the difference of longitude?
164. How many days from Jan. 5, 1876, to March 3, 1877?

395. Miscellaneous Examples.

165. If eggs are worth 30 cents a dozen, and 10 weigh a pound, what are eggs worth by the pound?

166. If I burn 30 lbs. of coal a day, and buy my coal by the long ton, at \$7 a ton, what is the cost of my coal for December?

167. How many furrows, each 20 inches wide, will be made in ploughing lengthwise a lot of land which is 6 rd. 1 ft. wide?

168. A quantity of silver weighed 4 lb. 10 oz. 3 pwt. before refining, and 3 lb. 11 oz. 2 pwt. 9 gr. afterwards; what weight was lost in the process?

169. How many square feet on the top and sides of a box that is 3 ft. long, 2 ft. wide, and 2 ft. 6 in. high?

170. What will be the cost of fencing a lot of land 20 rods by 26 rods at 25 cents a foot?

171. Change $\frac{3}{4}$ of a great gross to units of lower denominations.

172. Divide an angle of $20^{\circ} 4' 5''$ by 9.

173. Dr. Smith's wagon-wheel, which is 3 ft. 4 in. in circumference, turns round 200 times in going from his house to the post-office; how far does he live from the post-office?

174. If a bird can fly 1° in 1 h. 8 m. 15 s., in what time can it fly around the world at the same rate?

175. What is the cost of 137 gal. 2 qt. of vinegar at 50 cts. per gal.?

176. How many bushels of grain will a bin contain which is 10 ft. long, 8 ft. wide, and 5 ft. deep?

177. What is the cost of oil-cloth to cover a floor 12 feet by $16\frac{1}{2}$ feet, at 75 cents per square yard?

178. A farmer divided one half of his estate of 350 A. 140 rd. equally between his two daughters, and the balance, after setting off $17\frac{3}{4}$ A., equally between his two sons. What was the share of each son and daughter?

179. How many yards of carpeting $\frac{3}{4}$ yd. wide will cover a floor 18 ft. sq.?

180. If a cotton-mill can make 1200 yds. of cloth per hour, how many yards could be made by working 10 hours a day from July 7th to January 4th, allowing for 26 Sundays?

181. Change 15 lb. 8 oz. Av. to pounds and ounces Troy.

182. How many sq. ft. does the surface of a block contain, which is 3 ft. long, 2 ft. wide, and 6 ft. thick?

183. When 2 dozen grape-vines can be bought for \$6.50, what is the cost of each vine?

184. From a pile of wood 58 ft. long, 4 ft. high, and 4 ft. wide, was sold at one time $3\frac{3}{8}$ cords, at another $2\frac{1}{4}$ cords. What is the remainder worth at \$4 a cord?

185. I have a shed which measures on the inside 18 ft. 7 in. by 8 ft. by 10 ft. in height. How many cords of wood can be put in it?

186. A man purchased 75 cords of wood for \$360; he sold the following lots, $10\frac{1}{2}$ cd., 15 cd., and $11\frac{3}{4}$ cd., all at \$5 per cord. What did he gain on what he sold?

187. What would be the cost of sawing the remainder of the 75 cords, at \$1 a cord?

188. How many gals. of water will be contained in a tank 3 ft. square, if the water is 4 ft. 3 in. deep?

189. At 15 cents per pound, what was the cost for lead, 5 lbs. to the sq. ft., to line the above tank, it being 5 feet deep?

190. What must I pay for a dozen silver spoons, each weighing 2 oz. $9\frac{1}{2}$ pwt., at \$1.50 per ounce?

191. Add $\frac{3}{4}$ of the month of February, 1876, to $\frac{1}{2}$ of the days from March 21st to June 17th, 1877.

192. How much carpeting $\frac{3}{4}$ yd. wide will cover the top and sides of a block 3 ft. long, 8 inches wide, and 6 inches high?

193. Estimate the cost of feeding a pair of oxen through the winter of 1879 and 1880, if 1 ox weighed 1772 lbs. and the other 1431 lbs., and hay was \$13.75 per ton, and the oxen were allowed $\frac{1}{10}$ of their weight in hay each day.

194. How many paving-stones 6 in. by 8 in. will be required to pave a street 27 rods long by 50 ft. wide?

195. At 9 o'clock P. M. in Boston, what is the time in Paris?

196. If a druggist sells 1 gross 2 doz. powders a day, how many will he sell from the 19th of Dec., 1877, to 15th Mar., 1878, deducting 12 Sundays?

197. In what time will a vessel go through a strait 2 miles long, if she is carried ahead by tide 30 feet a minute, by wind 25 feet a minute, and by steam 100 feet a minute? In what time can she go through the strait against wind and tide?

396. Questions for Review.

Repeat the table of LONG MEASURE. Draw a line an inch long. Hold your hands a foot apart. What do you think the height of your school-room to be? In some convenient place mark off and walk 100 feet, counting your steps as you walk, and find their average length. By counting your steps, find how far you live from school. What is the standard unit of length?

How is an angle formed? Upon what does its size depend? What is a right angle? a rectangle? a square? area? How do you find the area of a rectangle or a square? Illustrate. Repeat the table of SQUARE MEASURES. From what are the units of square measure derived? What is the principal unit of land measure?

What is a rectangular solid? a cube? How many faces has a cube? how many edges? How do you find the volume of any rectangular solid? Illustrate. Repeat the table of CUBIC MEASURE. From what are the units of cubic measure derived?

Repeat the table of LIQUID MEASURES; of DRY MEASURES. Which is larger, 1 quart liquid measure, or 1 quart dry measure? What is the standard unit of liquid measure? of dry measure? How many cubic inches are there in a gallon? in a bushel? How do we ascertain the WEIGHT of anything? Repeat the table of Avoirdupois weights; of Troy weights. By which would you buy iron? silver? salt? emeralds? flour? What is a long ton? How many grains Troy make a pound Avoirdupois? Which is heavier, 1 lb. Avoirdupois, or 1 lb. Troy? 1 oz. Avoirdupois, or 1 oz. Troy? What is the standard unit of weight?

What is a circle? the circumference? an arc? Repeat the table of CIRCULAR MEASURES. Are all degrees of the same length? What

is the length of a degree of the circumference of the earth at the equator? What is a nautical mile? What is its length in English miles?

How is an angle measured? Are all angles of one degree of the same size?

Repeat the table of TIME. How do you know what years are leap years? Name the months which contain 30 days each; name the months which contain 31 days each.

What is a COMPOUND NUMBER? a denominate number? a general number? How do the units of different denominations in compound numbers increase?

Give the rule for REDUCTION DESCENDING; for REDUCTION ASCENDING. How do you change a denominate fraction to integers of lower denominations? How do you change integers of lower denominations to the fraction of a higher?

How are compound numbers ADDED, SUBTRACTED, MULTIPLIED, and DIVIDED?

How do you find the number of years, months, and days between two dates? (Art. 371.) How do you find the time in days between two dates? When the difference in time between two places is given, how do you find their difference in longitude? When the difference in longitude is given, how do you find the difference in time?

Find the area of the top of your desk. Draw a square 1 inch each way; $\frac{1}{2}$ inch each way. What part of the first square is the second? Difference between 5 square inches and 5 inches square? When the length of one side of a rectangle is given in feet, and the other in rods, how do you find the surface? When the area and one dimension are given, how do you find the other?

How are the PUBLIC LANDS of the United States divided?

When the volume and two dimensions of a rectangular solid are given, how do you find the third?

How is wood generally cut for market? How many cubic feet are there in 1 cord? How would you estimate the contents of SAWED TIMBER and BOARDS? How do you find the average width of a board that decreases regularly in width from end to end?

How are bulky fruits and vegetables measured? How does a HEAPED MEASURE compare in bulk with an even measure? A cubic foot is equal to what part of a bushel, even measure? What part of a bushel, heaped measure?

397. DRILL TABLE No. 7.

	A	B	C			D		
1.	T.	lb.	4 T.	625 lb.	12 oz.	2 T.	1428 lb.	6 oz.
2.	l. T.	lb.	5 l. T.	12 cwt.	3 qr.	18 cwt.	2 qr.	7 lb.
3.	lb.*	pwt.	9 lb.	8 oz.	5 pwt.	3 lb.	7 oz.	10 pwt.
4.	m.	ft.	10 m.	200 rd.	4 yd.	34 rd.	3 yd.	2 ft.
5.	sq. m.	sq. rd.	1 sq. m.	560 A.	4 sq. rd.	48 A.	9 sq. rd.	
6.	cu. yd.	cu. in.	8 cu. yd.	4 cu. ft.	900 cu. in.	6 cu. yd.	15 cu. ft.	1506 cu. in.
7.	cd.	cu. ft.	100 cd.	2 cd. ft.	14 cu. ft.	92 cd.	6 cd. ft.	12 cu. ft.
8.	gal.	gi.	25 gal.	3 qt.	0 pt.	4 gal.	2 qt.	1 pt.
9.	bu.	pt.	9 bu.	3 pk.	1 qt.	5 bu.	1 pk.	2 qt.
10.	circ.	(')	1 circ.	90°	40'	280°	2'	28"
11.	c. y.	hours	3 c. y.	4 d.	12 h.	2 c. y.	7 d.	18 h.
12.	l. y.	min.	1 l. y.	65 d.	18 h.	7 d.	20 h.	5 min.
13.	rd.	in.	6 m.	310 rd.	2 yd.	15 rd.	11 ft.	8 in.
14.	A.	sq. yd.	5 A.	29 sq. rd.	66 sq. ft.	18 sq. rd.	206 sq. ft.	9 sq. in.
15.	pk.	pt.	7 pk.	2 qt.	1 pt.	6 qt.	1 pt.	2 gi.
16.	(°)	(")	54°	51'	45"	18°	36'	54"
17.	oz.*	gr.	5 lb.	10 oz.	2 pwt.	11 oz.	18 pwt.	20 gr.
18.	yd.	in.	3 m.	41 rd.	2 yd.	5 yd.	1 ft.	4 in.
19.	sq. rd.	ft.	5 sq. rd.	29 sq. yd.	4 sq. ft.	40 sq. yd.	8 sq. ft.	9 sq. in.
20.	qt.†	gi.	3 qt.	0 pt.	2 gi.	2 qt.	1 pt.	2 gi.
21.	w.	min.	28 w.	3 d.	12 h.	3 w.	5 d.	18 h.
22.	sq. yd.	sq. in.	21 sq. yd.	0 sq. ft.	12 sq. in.	4 sq. yd.	3 sq. ft.	36 sq. in.
23.	d.	sec.	284 d.	13 h.	9 min.	169 d.	19 h.	42 min.
24.	sq. ft.	sq. in.	11 sq. rd.	4 sq. ft.	90 sq. in.	3 sq. rd.	204 sq. ft.	10 sq. in.
25.	gross	units	12 gross	7 doz.	2	8 gross	3 doz.	9

* Troy.

† Liquid.

DRILL TABLE No. 7
(continued).

	E		
1.	7 lb.	8 oz.	
2.	4 cwt.	3 qr.	11 lb.
3.	11 oz.	14 pwt.	
4.	5 yd.	1 ft.	
5.	2 sq. rd.	4 sq. yd.	
6.	14 cu. ft.	329 cu. in.	
7.	18 cd. ft.	14 cu. ft.	
8.	3 qt.	1 pt. (liquid)	
9.	4 pk.	7 qt.	1 pt.
10.	98'	14"	
11.	348 d.	3 h	
12.	21 h.	10 min.	
13.	2 ft.	9 in.	
14.	9 sq. yd.	110 sq. in.	
15.	1 pt.	3 gi. (dry)	
16.	68'	58"	
17.	22 pwt.	23 gr.	
18.	2 ft.	3 in.	
19.	9 sq. yd.	8 sq. ft.	
20.	1 pt.	3 gi. (liquid)	
21.	23 h.	41 min.	
22.	27 sq. ft.	28 sq. in.	
23.	48 min.	18 sec.	
24.	178 sq. ft.	108 sq. in.	
25.	1 gross	8 doz.	10

398. Exercises upon the Table.

196. Change five A to B.
197. Change E to units of the lowest denomination in the example.
198. Change D to units of the lowest denomination in the example.
199. Change 3284 B to A.
200. Change 182687 B to units of higher denominations.
201. Change $\frac{3}{4}$ A to B.
202. Change 0.4627 A to B.
203. Change the numbers of lower denominations in D to a fraction of the highest.
204. Change the numbers of lower denominations in C to a decimal of the highest.
205. What part of A is E?
206. Add C, D, and E.
207. Add $\frac{3}{4}$ A to D.
208. Add 0.5784 A to E.
209. Take E from D.
210. Take D from C.
211. Multiply C by 6.
212. Multiply E by 15.
213. Divide C by 10.
214. Divide D by 7.
215. Divide D by 4 of the lowest denomination in the example.

SECTION XIII.

THE METRIC SYSTEM OF WEIGHTS
AND MEASURES.

399. The metric system of weights and measures, now used in the greater part of Europe and coming into use in the United States, is derived from the standard **METER**.

NOTE. The word **METER** means a *measure*. The standard meter is a certain bar of platinum carefully preserved at Paris. Copies of this bar, made with the utmost precision, have been procured and are carefully preserved by the nations that have adopted the Metric System. The standard meter of the United States is such a copy, and it is kept at Washington. The meter-sticks made for ordinary use are copies of the standard meter.

MEASURES OF LENGTH.

400. The standard unit of length in the metric system is the *meter*.

NOTE. The teacher should show the pupil a meter and its subdivisions. If none can readily be obtained, one can easily be made from the decimeter represented on the next page. This meter may be divided into decimeters and centimeters. From this measure the pupils can easily make their own of paper or wood.

401. One tenth of a meter is a *déc'i-meter*.

NOTE. The prefix **deci-** means *one tenth of*.

402. One hundredth of a meter is a *cén'ti-meter*.

NOTE. The prefix **centi-** means *one hundredth of*.

403. One thousandth of a meter is a *mîl'li-meter*.

NOTE. The prefix **milli-** means *one thousandth of*.

404. Exercises on the Meter and its subdivisions.

a. How many meters long is the room? How many meters wide?

b. How many decimeters long is the table?

c. How many decimeters wide is the door?

d. How many centimeters long and wide is your slate? the window-pane? etc.

e. How many millimeters apart are two lines on a sheet of writing-paper?

f. How many millimeters thick is your slate-frame? your ruler? etc.

g. How many millimeters are there in one centimeter?

h. How many centimeters are there in one decimeter?

i. How many decimeters are there in one meter?

j. How many millimeters are there in one decimeter? in one meter?

k. How many centimeters are there in 37 millimeters, and how many millimeters remain?

l. How many decimeters are there in 84 centimeters, and how many centimeters remain?

m. How many meters are there in 347 centimeters, and how many centimeters remain?

n. In measuring the length of the room, did you find it to be an exact number of meters long?

o. If not, how many decimeters do you find in the remainder? Do you find an exact number of decimeters?

p. If there is still a remainder, how many centimeters do you find in it?



To write Numbers in the Metric System.

405. To express a length in meters and parts of a meter, we write whole meters in the units' place, decimeters in the tenths' place, centimeters in the hundredths' place, and millimeters in the thousandths' place.

Thus, if a room is found to be 8 meters 6 decimeters 9 centimeters long, we write :

Length of the room = 8.69 meters.

2 decimeters 3 centimeters 5 millimeters is written :

0.235 meters.

406. The abbreviations used in writing expressions of length are: For meters, *m*; for decimeters, *dm*; for centimeters, *cm*; and for millimeters, *mm*.

407. Lengths may be expressed in other denominations as well as in meters, *by putting the decimal point at the right of the place of the required denomination, and writing the proper name or abbreviation after the figures.*

Thus, 0.235^m may be written 2.35^{dm}, 23.5^{cm}, or 235^{mm}.
So also 728^{mm} may be written 72.8^{cm}, 7.28^{dm}, or 0.728^m.

408. Exercises in reading Numbers.

Read the following :

a. 5 ^m	e. 5.926 ^m	i. 6.58 ^{dm}
b. 47 ^m	f. 36 ^{dm}	j. 3.4 ^{cm}
c. 3.9 ^m	g. 428 ^{cm}	k. 43.7 ^{cm}
d. 4.21 ^m	h. 23 ^{mm}	l. 2.5 ^{mm}

409. Examples for the Slate.

Change the following to meters :

(1.) 1 ^{dm}	(4.) 1 ^{cm}	(7.) 1 ^{mm}
(2.) 13 ^{dm}	(5.) 38 ^{cm}	(8.) 48 ^{mm}
(3.) 214 ^{dm}	(6.) 529 ^{cm}	(9.) 3675 ^{mm}

Multiples of the Meter.

410. Besides the meter and its subdivisions, there are longer measures, which are multiples of the meter.

411. The *dēka-meter* is ten times as long as the meter.

NOTE. The prefix **deka-** means *tenfold*.

412. The *hēkto-meter* is a hundred times as long as the meter.

NOTE. The prefix **hekto-** means *a hundredfold*.

413. The *kilo-meter* is a thousand times as long as the meter.

NOTE. The prefix **kilo-** means *a thousandfold*.

414. The *mýria-meter* is ten thousand times as long as the meter.

NOTE. The prefix **myria-** means *ten thousandfold*.

NOTE. Of these longer measures, the kilometer is used in measuring distances on roads, canals, rivers, etc. The other measures are much less frequently used; the myriameter hardly ever.

415. Exercises on the Multiples of the Meter.

a. Measure off a string ten meters long. What name is given to the length of this string?

NOTE. The string may be used in measuring distances. For this purpose it will be well to make knots at the end of each meter.

b. Measure in dekameters and meters the length and breadth of the school-yard; of a garden; of a field, etc.

c. Measure off in the street, or other convenient place, a distance of 10 dekameters. What name is given to this distance?

d. Walk from the beginning to the end of the distance thus measured off, and count your paces. How many of your paces make a hektometer?

e. How many of your paces would make a kilometer?

f. How many kilometers from your home to the school-house?

- g. How long does it take you to walk a kilometer?
 h. How many kilometers can you walk in an hour?
 i. If 1500 of your paces make a kilometer, how many make a dekameter?

416. To express distances in meters and multiples of a meter, we write meters in the units' place, dekameters in the tens' place, hektometers in the hundreds' place, and so on.

417. To express a distance in kilometers, we write kilometers in the units' place, and then hektometers, dekameters, and meters will be written in the tenths', hundredths', and thousandths' places respectively.

Thus, if the distance from one town to another is found to be 9780 meters, the usual form of writing would be 9.78 kilometers.

NOTE. The greatest distances are usually expressed in kilometers. Thus, the distance of the earth from the sun is about 149000000 kilometers.

418. The abbreviations used in writing are: For the dekameter, *Dm*; for the hektometer, *Hm*; and for the kilometer *Km*.

419. Table of Long Measure.

10 millimeters (mm)	= 1 centimeter (cm).
10 centimeters	= 1 decimeter (dm).
10 decimeters	= 1 meter (m).
10 meters	= 1 dekameter (Dm).
10 dekameters	= 1 hektometer (Hm).
10 hektometers	= 1 kilometer (Km).
10 kilometers	= 1 myriameter (Mm).

420. Oral Exercises.

Read the following:

a. 123 ^m	d. 42 ^{Dm}	g. 49 ^{Km}
b. 497.6 ^m	e. 36.7 ^{Dm}	h. 593.7 ^{Km}
c. 346 ^{Hm}	f. 57.5 ^{Hm}	i. 6000 ^{Km}

421. Examples for the Slate.

Change the following to meters :

- | | | |
|--------------------------|--------------------------|---------------------------|
| (10.) 425 ^{Dm} | (13.) 94.6 ^{Hm} | (16.) 0.72 ^{Km} |
| (11.) 35 ^{Hm} | (14.) 9.24 ^{Km} | (17.) 0.073 ^{Hm} |
| (12.) 23.5 ^{Km} | (15.) 39.7 ^{Dm} | (18.) 0.05 ^{Km} |

Addition, Subtraction, Multiplication, and Division of Metric Numbers.

422. ILLUSTRATIVE EXAMPLE. Change to meters and add 14.83^{Dm}, 75.6^{Hm}, and 948^{cm}.

WRITTEN WORK.	Explanation.
14.83 ^{Dm} = 148.3 ^m	When these expressions have been changed to meters, they are all of the same denomination, and the sum is found in the same way as in the addition of simple numbers.
75.6 ^{Hm} = 7560.	
948 ^{cm} = 9.48	
<u>7717.78^m</u>	

423. Numbers expressing metric measures and weights are added, subtracted, multiplied, and divided by the same rules as apply to simple numbers.

424. Examples for the Slate.

19. Add 5.6^m, 24.07^m, 30.5^m, and 7.508^m.
20. Express as meters and add 582^{cm}, 6428^{dm}, and 495^{mm}.
21. Express as meters and add 369^{Dm}, 4073^{Hm}, and 5^{Km}.
22. Add 48.06^m, 709.63^m, 3708.9^m, 800.9^m, and express the answer in kilometers.
23. If 7^{Km} be taken from 42^{Km}, how many meters remain?
24. From 87.04^m take 42^{cm}.
25. The distance round a certain park is 2.58 kilometers. How many meters will a man go who rides around it six times?
26. A school-boy walked one third around the above park in 12 minutes. How many meters did he walk in 1 minute?
27. How many kilometers in 36.68^m × 2004?
28. Divide 38.07^m by 4 and by 3, and add the answers.
29. Ellen's hoop is 3.6^m around. How many times will it turn in rolling a distance of 1.08^{Km}?

MEASURES OF SURFACE.

425. The units used in measuring surfaces are squares, each having sides equal to a unit of long measure.

Thus, a *square meter* is a square having sides one meter long; a *square decimeter* is a square having sides one decimeter long; etc.

426. Exercises.

a. How many square decimeters in a square meter? Illustrate by drawing a square meter on the blackboard or on the floor and dividing it into square decimeters.

b. How many square centimeters in a square decimeter? Illustrate by drawing a square decimeter on your slate and dividing it into square centimeters.

c. How many square meters in a square dekameter?

427. The square dekameter, when used as a unit of land measure, takes a special name, and is called an *ar*. One hundredth of an *ar*, which is one square meter, is called a *centar*. A hundred *ars*, equal to one square hektometer, is called a *hektar*.

428. Square Measure.

100 square millimeters (sq mm) = 1 square centimeter (sq cm).

100 square centimeters = 1 square decimeter (sq dm).

100 square decimeters = 1 square meter (sq m) = 1 centar (ca).

100 square meters = 1 square dekameter = 1 ar (a).

100 square dekameters = 1 square hektometer = 1 hektar (Ha).

100 square hektometers = 1 square kilometer (sq Km).

429. As the units of square measure form a scale of hundreds, in writing numbers expressing surface two decimal places must be allowed for each denomination.

Thus, 45^{sq m} 4^{sq dm} 86^{sq cm} are written 45.0486^{sq m}; and 7^{Ha} 6^a 5^{ca} are written 706.05^a.

430. Examples for the Slate.

30. How many square meters of carpeting will be required to carpet a room 5.3^m long and 4.5^m wide ?

31. How many meters of carpeting 0.7^m wide will be required to carpet a room 4^m long and 3.5^m wide ?

32. What is the cost of polishing the surface of a rectangular piece of marble 2.8 meters long and 1.2 meters wide, at \$2.50 per sq. meter ?

33. In a piece of land 15^m long and 14.5^m wide are how many square meters or centars ? how many ars ?

34. Express the following in ars and add them : 1.3 hektars, 155.5 ars, 43 hektars, 26 centars.

35. A had 6 hektars, 7 ars, 9 centars of land, and sold 0.2 of it at \$54 an ar. How much did he receive for what he sold ?

MEASURES OF VOLUME.

431. The units used in measuring cubic contents, or volume, are cubes, each having its edges equal to a unit of long measure.

Thus, the *cubic meter* is a cube having edges one meter long ; a *cubic decimeter* is a cube having its edges one decimeter long ; etc.

432. Exercises.

a. How many cubic decimeters in a cubic meter ?

b. How many cubic centimeters in a cubic decimeter ? Illustrate by means of a cubical block having edges one decimeter long, marked off into centimeters.

433. The cubic meter, when used as a unit of measure for wood and stone, takes a special name, and is called a *ster*.

434. The cubic decimeter, when used as a unit of liquid or dry measure, is called a *liter*.

435. Cubic Measure.

1000 cubic millimeters (cu mm) = 1 cubic centimeter (cu cm).

1000 cubic centimeters = 1 cubic decimeter (cu dm) = 1 liter.

1000 cubic decimeters = 1 cubic meter (cu m) = 1 ster.

436. Wood Measure.

10 decisters (ds) = 1 ster (s).

10 sters = 1 dekaster (Ds).

437. As the units of cubic measure form a scale of thousands, in writing numbers expressing volume three decimal places must be allowed for each denomination.

Thus, $427^{\text{cu m}}$ $29^{\text{cu dm}}$ $3^{\text{cu cm}}$ are written 427.029003^{cu m}.

438. As the units of wood measure form a scale of tens, only one decimal place is needed for each denomination.

Thus, 7 dekasters 5 sters 6 decisters are written 75.6 sters.

439. Examples for the Slate.

36. Express the following in cubic meters and add them :
7 cu. meters 40 cu. decimeters ; 5 cu. meters 3 cu. decimeters
19 cu. centimeters ; 25 cu. centimeters 49 cu. millimeters.

37. How many cubic meters of earth must be removed to dig a cellar 14.5^m long, 4.6^m wide, and 2.3^m deep ?

38. At \$1.25 a cubic meter, what will it cost to dig a trench 75.5^m long, 2.2^m wide, and 1.8^m deep ?

39. How many loads of earth, each filling 2.25^{cu m}, will fill a space 15.4^m long, 12^m wide, and 4.5^m deep ?

40. If a cubic centimeter of gold is worth \$12.50, what is the value of a brick of gold 2.4^{cm} long, 1.3^{cm} wide, and 0.75^{cm} thick ?

41. If I burn 27 sters of wood in the three winter months, what must be the length of a pile 1 meter wide and $\frac{3}{4}$ meter high to last a month, and what will it cost at \$2.25 a ster ?

MEASURES OF CAPACITY.

440. The primary unit of measure for all substances that can be poured into a dish or box is the *liter*.

441. A liter is equal in volume to one cubic decimeter.

442. Larger and smaller measures are derived from the liter in the same way that longer and shorter measures are derived from the meter, that is, by taking decimal multiples and subdivisions.

443. Liquid and Dry Measures.

	1 milliliter (ml)	= 1 cu cm.
10 milliliters	= 1 centiliter (cl).	
10 centiliters	= 1 deciliter (dl).	
10 deciliters	= 1 liter (l)	= 1 cu dm.
10 liters	= 1 dekaliter (Dl).	
10 dekaliters	= 1 hektoliter (Hl).	
10 hektoliters	= 1 kiloliter (Kl).	= 1 cu m.

NOTE. The milliliter is employed in computations, but rarely, if ever, in actual measurements. Chemists and druggists use cubic centimeters instead of milliliters.

444. Examples for the Slate.

42. If one hektoliter of kerosene costs \$ 20, what is the price of a liter ?

43. What must be paid for 2.5 liters of milk each day for a week, at 7 cents a liter ?

44. From a vessel containing 1 hektoliter of syrup, 25 liters were drawn out. How many liters remained ?

45. How many hektoliters of oats can be put into a bin that is 2^m long, 1.3^m wide, and 1.5^m deep ?

46. What must be the length of a bin 1 meter wide and 1 meter deep, to contain 4500 liters of grain ?

WEIGHTS.

445. The primary unit of weight is the *gram*.

446. A gram is the weight of one cubic centimeter of pure water at the temperature of 4 degrees centigrade (= 39.2 degrees Fahrenheit), at which temperature water has its greatest density.

447. Larger and smaller weights are derived from the gram by taking decimal multiples and subdivisions.

448. *Weights.*

10 milligrams (mg)	= 1 centigram (cg).	
10 centigrams	= 1 decigram (dg).	
10 decigrams	= 1 gram (g)	= wt. of 1 cu cm of water.
10 grams	= 1 dekagram (Dg).	
10 dekagrams	= 1 hektogram (Hg).	
10 hektograms	= 1 kilogram (K)	= wt. of 1 cu dm of water.
10 kilograms	= 1 myriagram (Mg).	
10 myriagrams	= 1 quintal (Q).	
10 quintals	= 1 metric ton (T.)	= wt. of 1 cu m of water.

NOTE I. The gram, kilogram, and metric ton are the only units used in actual weighing, except by jewellers, druggists, and those who weigh very small or very expensive articles, like gold or powerful medicines.

NOTE II. The kilogram is generally called the kilo. The kilo is the unit of weight for weighing common articles, such as sugar, tea, etc.

NOTE III. The metric ton is used to weigh very heavy articles, like hay, coal, etc.

449. *Examples for the Slate.*

47. At \$0.60 a kilo for honey, what is the cost of 5.15 kilos?

48. At \$11 per T. for coal, what will the coal cost to keep a fire one week if 30 kilos are burnt each day?

49. What weight of mercury will a vessel contain whose capacity is 10 cu cm, mercury being 13.5 times as heavy as water?

50. If marble is 2.7 times as heavy as water, what is the weight of a pedestal 1 meter square at each end and 2 meters high?

450. Table of Equivalents.

The equivalents here given agree with those that have been established by Act of Congress for use in legal proceedings and in the interpretation of contracts.

1 inch = 2.540 centimeters.	1 centimeter = 0.3937 inch.
1 foot = 3.048 decimeters.	1 decimeter = 0.328 foot.
1 yard = 0.9144 meters.	1 meter = 1.0936 yds. = 39.37 in.
1 rod = 0.5029 dekameters.	1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.	1 kilometer = 0.62137 mile.
1 sq. inch = 6.452 sq. centimeters.	1 sq. centimeter = 0.1550 sq. inch.
1 sq. foot = 9.2903 sq. decimeters.	1 sq. decimeter = 0.1076 sq. foot.
1 sq. yard = 0.8361 sq. meter.	1 sq. meter = 1.196 sq. yards.
1 sq. rod = 25.293 sq. meters.	1 ar = 3.954 sq. rods.
1 acre = 0.4047 hektar.	1 hektar = 2.471 acres.
1 sq. mile = 2.590 sq. kilometers.	1 sq. kilometer = 0.3861 sq. mile.
1 cu. inch = 16.387 cu. centimeters.	1 cu. centimeter = 0.0610 cu. inch.
1 cu. foot = 28.317 cu. decimeters.	1 cu. decimeter = 0.0353 cu. foot.
1 cu. yard = 0.7645 cu. meter.	1 cu. meter = 1.308 cu. yards.
1 cord = 3.624 sters.	1 ster = 0.2759 cord.
1 liquid quart = 0.9463 liter.	1 liter = 1.0567 liquid quarts.
1 gallon = 0.3785 dekaliters.	1 dekaliter = 2.6417 gallons.
1 dry quart = 1.101 liters.	1 liter = 0.908 dry quart.
1 peck = 0.881 dekaliter.	1 dekaliter = 1.135 pecks.
1 bushel = 3.524 dekaliters.	1 hektoliter = 2.8375 bushels.
1 ounce av. = 28.35 grams.	1 gram = 0.03527 ounce av.
1 pound av. = 0.4536 kilogram.	1 kilogram = 2.2046 pounds av.
1 ton (2000 lbs.) = 0.9072 met. ton.	1 metric ton = 1.1023 tons.
1 grain Troy = 0.0648 gram.	1 gram = 15.432 grains Troy.
1 ounce Troy = 31.1035 grams.	1 gram = 0.03215 ounce Troy.
1 pound Troy = 0.3732 kilogram.	1 kilogram = 2.679 pounds Troy.

451. To change numbers in the metric system to equivalents of the old system :

Examples.

51. In 48 meters how many feet ?
52. If you travel 50 kilometers in a day, how many miles do you travel ?
53. Change 18 hektars of land to acres.
54. How many inches long is an insect that is 5.2 centimeters long ?
55. How many pounds av. are there in 85.6 kilos of salt ?
56. How many gallons are there in 24 kiloliters ?
57. In 20 metric tons how many tons ?

452. To change numbers in the old system to equivalents of the metric system :

Examples.

58. Change 25 miles to kilometers.
59. In 200 acres are how many hektars ?
60. How many liters will a cistern hold that measures on the inside 5 feet in length, 4 feet in width, and 4 feet in height ?
61. In 3 rods how many meters ?
62. Change 18 qt. 1 pt. to liters.
63. In 1 lb. 7 oz. 18 pwt. of gold, how many grams ?
64. What is the weight of a barrel of flour (196 lbs.) in kilograms ?

453. Approximate Equivalents.

The equivalents here given are accurate enough for most purposes, and are easy to remember.

A decimeter	= 4 inches.	A ster	= $\frac{1}{4}$ of a cord.
A meter	= $\left\{ \begin{array}{l} 3 \text{ ft. } 3\frac{3}{4} \text{ in.,} \\ \text{or } 1\frac{1}{10} \text{ yards.} \end{array} \right.$	A liter	= $\left\{ \begin{array}{l} 1.06 \text{ liquid qt.,} \\ \text{or } \frac{1}{10} \text{ of a dry qt.} \end{array} \right.$
A dekameter	= 2 rods.	A dekaliter	= 1 peck and 1 qt.
A kilometer	= $\frac{1}{5}$ of a mile.	A hektoliter	= $2\frac{1}{2}$ bushels.
An ar	= $\left\{ \begin{array}{l} 4 \text{ sq. rods,} \\ \text{or } \frac{1}{10} \text{ of an acre.} \end{array} \right.$	A gram	= $15\frac{1}{2}$ grains.
A hektar	= $2\frac{1}{2}$ acres.	A kilogram	= $2\frac{1}{2}$ pounds av.
		A metric ton	= 2200 pounds av.

SECTION XIV.

PERCENTAGE.

454. Find $\frac{9}{100}$ of 500 men. *Ans.* 45 men.

A number found by taking a number of hundredths of another number is a *percentage* of that number.

455. The number of which a number of hundredths is taken to find a percentage is the *base* of that percentage.

In the above example what number is the percentage? the base?

456. If a person having \$2000 should gain a sum equal to $\frac{10}{100}$ of it, how much would he then have?

$$2000 \times \frac{10}{100} = 200; 2000 + 200 = 2200. \quad \text{Ans. \$ 2200.}$$

The sum of the base and percentage is the *amount*.

457. If a person having \$2000 should lose $\frac{10}{100}$ of it, how much would he have left?

$$2000 \times \frac{10}{100} = 200; 2000 - 200 = 1800. \quad \text{Ans. \$ 1800.}$$

The part of the base left after a percentage is taken away is the *remainder*.

In the above examples what number is the amount? the remainder?

458. The number of hundredths which the percentage is of the base is the *rate per cent*, generally called the *per cent*. Thus, $\frac{7}{100}$ of anything is 7 per cent of it.

NOTE. Per cent is a contraction of the Latin *per centum*, and means *by the hundred*.

459. Oral Exercises.

- a. Find 1 per cent of 600; 7 per cent; 20 per cent.
- b. Find 10 per cent of \$250; 5 per cent; 50 per cent.

To express a given Per Cent.

460. The sign % is used for the words per cent. Thus, 5 % means 5 per cent.

461. Any per cent may be expressed as a common fraction, as a decimal, or with the sign for per cent, %. Thus,

1 per cent	may be expressed	$\frac{1}{100}$, 0.01, or 1%.
6 per cent	" "	$\frac{6}{100}$, 0.06, or 6%.
$7\frac{1}{2}$ per cent	" "	$\frac{7\frac{1}{2}}{100}$, 0.07 $\frac{1}{2}$, or 7 $\frac{1}{2}$ %.
100 per cent	" "	$\frac{100}{100}$, 1.00, or 100%.
120 per cent	" "	$\frac{120}{100}$, 1.20, or 120%.
$\frac{1}{4}$ per cent	" "	$\frac{1}{400}$, 0.00 $\frac{1}{4}$, or $\frac{1}{4}$ %.

Exercises.

462. Express the following in the three forms given above :

- a. 2 per cent. c. $7\frac{3}{10}$ per cent. e. 175 per cent.
 b. 5 per cent. d. 200 per cent. f. $\frac{1}{2}$ per cent.

463. Express the following as common fractions, and change them to their smallest terms :

- g. 5 %. k. 50 %. o. 4 %. s. $6\frac{1}{4}$ %.
 h. 10 %. l. 100 %. p. 75 %. t. $8\frac{1}{3}$ %.
 i. 20 %. m. $12\frac{1}{2}$ %. q. 90 %. u. $83\frac{1}{3}$ %.
 j. 25 %. n. $16\frac{2}{3}$ %. r. $37\frac{1}{2}$ %. v. 125 %.

To find the Complement of a given Per Cent.

464. What is the difference between 100 % and 25 % ?

The difference between 100 % and any given per cent less than 100 % is the *complement* of the given per cent.

465. Oral Exercises.

- a. What is the complement of 75%? 40%? 60%? $33\frac{1}{3}$ %? $6\frac{1}{4}$ %? 15%?
 b. What is the complement of $62\frac{1}{2}$ %? 16%? $37\frac{1}{2}$ %? 18%? $87\frac{1}{2}$ %? 72%?

To change a Common Fraction to a Per Cent.

466. ILLUSTRATIVE EXAMPLE. What per cent of a number is $\frac{1}{4}$ of it?

WRITTEN WORK.

Explanation. — Since any number equals 100% of itself, $\frac{1}{4}$ of the number must equal $\frac{1}{4}$ of 100%, or 25%. *Ans.* 25%.

$$\begin{array}{r} 4) 100 \\ \hline 25 \end{array} \text{ Ans. 25\%.}$$

467. Oral Exercises.

- a. What per cent of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$? $\frac{1}{13}$? $\frac{1}{14}$? $\frac{1}{15}$? $\frac{1}{16}$? $\frac{1}{17}$? $\frac{1}{18}$? $\frac{1}{19}$? $\frac{1}{20}$?
- b. What per cent is $\frac{3}{4}$? $\frac{2}{3}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{2}{5}$? $\frac{3}{10}$? $\frac{4}{10}$? $\frac{5}{10}$? $\frac{6}{10}$? $\frac{7}{10}$? $\frac{8}{10}$? $\frac{9}{10}$? $\frac{1}{10}$? $\frac{2}{10}$? $\frac{3}{10}$? $\frac{4}{10}$? $\frac{5}{10}$? $\frac{6}{10}$? $\frac{7}{10}$? $\frac{8}{10}$? $\frac{9}{10}$?
- c. What per cent is $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{1}{2}$? $\frac{3}{8}$? $\frac{5}{8}$?

The Base and Rate per cent being given, to find the Percentage, Amount, or Remainder.

468. Oral Exercises.

- a. What is 7% of \$300?
- Solution.* — 7 per cent of \$300 is $\frac{7}{100}$ of \$300, or \$21. *Ans.* \$21.
- b. What is 20% of 80 trees? of 300 words? of 90? 60? 240?
- c. What is 10% of 80 days? $12\frac{1}{2}\%$ of 80 days? 25%? 40%? 50%? 90%?
- d. What is 6% of \$100? of \$200? of \$1.50? of \$2.50? of \$500? of \$12.50?
- e. What is the amount of \$40 + 5% of \$40? \$16 + 25% of \$16?
- f. What is the amount of \$100 + 7% of \$100? \$60 + 50% of \$60?
- g. What remains of an income of \$500 after 40% of it is spent? after 25% is spent? 10%? 15%? 30%? 80%?

469. ILLUSTRATIVE EXAMPLE I. A had \$500. If by trading he gained a sum equal to 25 % of his money, what was his gain? How much money did he then have?

WRITTEN WORK.

Base,	\$ 500	
Per cent,	0.25	
Percentage, \$ 125	A's gain.	
\$ 625	Amount.	

470. ILLUSTRATIVE EXAMPLE II. B had \$800. If by trading he lost 12 % of his money, what was his loss? How much money did he then have?

WRITTEN WORK.

Base,	\$ 800	
Per cent,	0.12	
Percentage, \$ 96	B's loss.	
\$ 704	Remainder.	

In the examples above, A's amount and B's remainder might have been considered as a percentage of the base and obtained directly thus:

(I.)	
Base, \$ 500	
Per cent, 1.25	
\$ 625	A's amount.

(II.)	
Base, \$ 800	
Per cent, 0.88	
\$ 704	B's remainder.

Explanation. — (I.) The money A had in trade was 100 % of itself. Adding to this the 25 % gain, his amount was 125 % or 125 hundredths of \$500, equal to \$625.

(II.) The money B had in trade was 100 % of itself. Having lost 12 % of this, he had remaining 88 % or 88 hundredths of \$800, equal to \$704.

471. From the operations above we derive the following

Rules.

1. To find the percentage: *Multiply the base by the rate per cent.*
2. To find the amount: *Add the percentage to the base, or multiply the base by 1 plus the rate per cent.*
3. To find the remainder: *Subtract the percentage from the base, or multiply the base by 1 minus the rate per cent.*

472. Examples for the Slate.

What is

- (1.) 12 % of \$ 940 ? (6.) 85 % of $16\frac{1}{2}$ pounds ?
- (2.) 25 % of \$ 250.60 ? (7.) 75 % of 120 % of 486800 ?
- (3.) 62 % of 2000 men ? (8.) $37\frac{1}{2}$ % of 4000 feet.
- (4.) $\frac{3}{4}$ % of \$ 28.80 ? (9.) 100 % of \$ 6000 + 75 % of \$ 6000 ?
- (5.) 120 % of 75 days ? (10.) 100 % of 10800 - 13 % of 10800 ?
11. If 5 % of the price of goods is deducted for cash, what deduction is made from a bill of \$ 25.40 ?
12. If a piece of rubber hose 146 feet long shrinks 10 % when wet, what is its length when wet ?
13. What is 25 % of 125 % of 75 % of 50 % of 384 inches ?
14. A farmer paid for shearing 104 sheep 4 % of what he received for the wool ; the fleeces averaged 5 pounds each, and sold at 40¢ a pound. What did he pay for shearing ?

The Percentage and Rate per cent being given, to find the Base.

473. ILLUSTRATIVE EXAMPLE. 750 bushels is 25 % of what number ?

WRITTEN WORK.

$$\frac{750 \times 100}{25} = 3000. \quad \text{Ans. 3000 bu.}$$

Explanation. — Since 25 % of the number sought is 750, 1 % of the number sought is $\frac{1}{25}$ of 750, and 100 % of the number sought, or the number

itself, is 100 times $\frac{1}{25}$ of 750, or 3000. *Ans.* 3000 bu. Hence the following

Rule.

To find the base when the percentage and rate per cent are given: *Divide the percentage by the numerator of the rate per cent, and multiply the quotient by 100.*

NOTE. Since 750 divided by 25 and multiplied by 100 equals 750 divided by 0.25 (which is the rate per cent), the form of written work given below may be used instead of that above :

$$\text{WRITTEN WORK.} \quad \frac{750}{0.25} = 3000. \quad \text{Ans. 3000 bu.}$$

Here the work is done by *dividing the percentage by the rate per cent.* This rule agrees with formula 4, page 192.

474. Examples for the Slate.

(15.) \$48 is 3% of what number? 10% of what number?

(16.) 436 days is 24% of what number? 8% of what number?

(17.) \$31.35 is 5% of what number? 15% of what number?

(18.) \$300 is $7\frac{1}{2}\%$ of what number? 5% of what number?

(19.) \$220.50 is 105% of what number? 75% of what number?

(20.) $\frac{4}{5}$ is 25% of what number? $\frac{1}{3}\%$ of what number?

21. The number of children of school age in a certain town is 1275; if this is 20% of the whole population, what is the whole population?

22. I drew out 25% of my deposits in a bank; of this I have spent \$468.72, which is 9% of what I drew out. What did I draw out? What remains in the bank?

23. If \$240 is 20% more than some number, what is that number?

NOTE. Since \$240 is 20% more than the number sought, it must be 120% of the number sought, etc. Hence when the amount is given instead of the percentage, *divide by 100 plus the numerator of the rate per cent, and multiply by 100.*

24. \$1860 is 25% more than what number?

25. A sold a horse for \$225, which was 5% more than he paid for it. What did he pay for it?

26. A grocer sold tea for 115% of its cost, and made 9 cents per pound. What did it cost a pound?

27. If \$450 is 10% less than some number, what is that number?

NOTE. Since \$450 is 10% less than the number sought, it must be 90% of the number sought, etc. Hence when the remainder is given instead of the percentage, *divide by 100 minus the numerator of the rate per cent, and multiply by 100.*

28. \$1000 is 4% less than what number?

29. Having lost 40% of my money, I have \$750 left. How much had I at first?

30. A son is 15 years old, which is $62\frac{1}{2}\%$ less than his father's age. What is his father's age?

31. The daily attendance upon a school is 558, which is 7% below the number belonging. What is the number belonging?

32. After the wages of a workman were reduced $7\frac{1}{2}\%$, he received \$3.70 a day. What were his wages before they were reduced?

33. By assessing a tax of $\frac{3}{4}\%$ on the valuation, a town raised \$75000. What was the valuation?

The Percentage and Base being given, to find the Rate per cent.

475. ILLUSTRATIVE EXAMPLE. If a pupil is absent from school 6 days in a term of 75 days, what per cent of the time is he absent?

WRITTEN WORK. *Explanation.* — If he is absent 6 days in 75 days, he is absent $\frac{6}{75}$ of the time. $\frac{6}{75}$ changed to hundredths is 0.08, or 8%. *Ans.* 8%.

75) 6.00
0.08, or 8% *Ans.*

476. From the example above may be derived the following

Rule.

To find the rate per cent when the base and percentage are given: *Divide the percentage by the base, carrying the division to hundredths.*

477. Examples for the Slate.

34. What per cent of \$104 is \$26? is \$52? is \$18.20?

35. What per cent of \$3 is 12¢? is \$3.75? is 1¢?

36. What per cent of a dozen is a score?

37. Out of 300 words, Charles spelled 280 correctly; Mary 284, Sarah 268, and Dwight 272. What per cent of the words did each spell correctly?

38. The surface of the earth contains about 144 million square miles of water, and about 53 million square miles of land. What per cent of the entire surface of the earth is water?

39. From a cask containing 120 gal. of oil, 6 gal. 2 qt. leaked out. What % was lost?

478. The operations in percentage, illustrated above, may be expressed by the following formulas:

1. $\text{Percentage} = \text{Base} \times \text{Rate}.$
2. $\text{Amount} = \text{Base} \times (1 + \text{Rate}).$
3. $\text{Remainder} = \text{Base} \times (1 - \text{Rate}).$
4. $\text{Base} = \text{Percentage} \div \text{Rate}.$
5. $\text{Rate} = \text{Percentage} \div \text{Base}.$

For additional examples in percentage, see page 253.

PROFIT AND LOSS.

479. Oral Exercises.

a. How much money is gained by selling goods at 25% above cost, the cost being \$8? \$10? \$1.60?

b. How much money is lost on goods which cost \$24, by selling them at a loss of 25%? 50%? 12½%?

c. At what price must paper which cost \$2 a ream be sold to gain 10%? 20%? 25%? 50%? 100%?

d. At what price must hats which cost 80¢ be sold to lose 10%? 5%? 25%? 50%? 12½%?

e. What must have been paid a pound for nutmegs if by selling them at \$1.00, there is a gain of 25%? 33½%? 10%?

f. What was the cost of gloves which sold for \$1.00 a pair at a loss of 20%? 50%? 33½%? 25%?

g. What per cent is gained if goods costing 10¢ a yard are sold for 11¢? 12¢? 15¢? 20¢?

h. What per cent would be lost if goods costing 15¢ a yard were sold for 12¢ ? 10¢ ? 9¢ ?

i. A drover bought cows at \$25 a head, and paid \$7 each to get them to market. If he sold them at \$40 a head, what per cent did he gain ?

j. What is the cost of goods when a gain of 20¢ a yard in selling is 10% of the cost ? 5% ? 8% ? 50% ? 12½% ?

k. What was the length of a piece of cloth before shrinking, if when shrunk 6 inches, it was shortened 1% ? 2% ? 3% ? 4% ?

480. The difference between the cost of goods and the price at which they are sold is a *profit* or a *loss*.

481. Profit and loss may be reckoned as percentage, the cost being taken as the base. Hence *the rules of percentage already illustrated apply to profit and loss*.

482. Examples for the Slate.

40. A farm which cost \$6842 was sold at a gain of 16%. What was received for it ?

41. A lot of coal was bought for \$750. For what must it be sold to gain 33½% ?

42. If 2000 reams of paper were bought for \$1500, at what price per ream must it be sold to gain 40% ?

43. A merchant sold a cargo of wheat at 12½% profit, and gained \$746.25. What was the cost ?

44. By selling a farm for \$2760, a man gained on the cost 15%. What was the cost ?

45. What was my property worth 5 years ago, if it has increased 150%, and is now worth \$17500 ?

46. A man sold a picture for \$275 at a loss of 16⅔%. What did he pay for it ?

47. If I pay 45¢ a pound for tea, and sell it at 56¢, what per cent do I gain ?

48. What was the original value of a share in a bridge, which, selling at an advance of 35%, brings \$780 ?

49. What is the per cent of gain if goods which cost \$7500 sell at a gain of \$1875?

50. A grocer sold 280 barrels of apples for \$708.40. If he paid \$1.40 per barrel for the apples, and 44¢ a barrel for transportation, what per cent did he gain?

51. A merchant bought carpetings at 85¢, \$1.20, and \$1.50 a yard. At what prices must he sell them to make 20% profit?

52. If \$1000 be paid for goods of which one half sells for \$640, and the remainder for \$300, what is the per cent of loss?

53. Bought paper at \$1.75 per ream, and sold it at 20 cents per quire. What per cent did I gain?

54. A dealer bought 10 gross of combs at \$12.50 a gross. If he sold 50 of the combs at 20 cents apiece and the rest at 18 cents apiece, what per cent did he gain?

55. If 150 beeves are bought at the rate of \$42.50 each, and 30 at the rate of \$45.00 each, and the lot is sold for \$10300, what per cent is gained?

COMMISSION.

483. One person is sometimes employed to buy goods or collect money for another, and is allowed for the service a percentage on the amount he lays out or collects. This percentage is called *commission*.

484. A person employed to transact business for another is an *agent* or *factor*.

485. A person who sends goods to another for sale is a *consignor*, and the person to whom the goods are sent is a *consignee*.

486. The remainder, after the commission and other charges of a sale are deducted, is the *net proceeds*.

487. Commission being a percentage, of which the money expended or received is the base, *the rules of percentage already illustrated apply to commission*.

488. Examples for the Slate.

56. At 1% commission, what is the commission on the sale of 4750 pounds of sugar at $7\frac{1}{2}$ cents per pound?

57. A factor in Mobile purchased for the Pacific Mills \$90000 worth of cotton at $1\frac{1}{4}$ % commission. What was the bill for cotton and commission?

58. If an auctioneer sells on a commission of 8%, 14 chairs at \$1.25, 1 table for \$10, and a miscellaneous lot for \$53.79, what is his commission, and what sum will be due the person for whom he makes the sale?

59. A lawyer collected 25% of an account of \$680, charging 5% commission. What was his commission, and what sum should he pay over?

60. What is the commission on the sale of 200 yards of cloth at \$4.80 per yard, 6% being paid for selling, and $2\frac{1}{2}$ % for guaranteeing payment?

61. What are the net proceeds from the sale of 1250 barrels of flour at \$5.50 per barrel, charges for freight and storage being 40¢ per barrel, commission for selling being 2%, and for guaranteeing payment $1\frac{1}{4}$ %?

62. An architect charged \$139.75 for plans and for superintending the building of a house. If his commission was $2\frac{1}{2}$ %, what was the cost of the house, including his commission?

63. What is the per cent of commission when an agent reserves to himself \$270.00 of \$9270, sent him to invest?

489. ILLUSTRATIVE EXAMPLE. What part of a remittance of \$328.25 will remain to be invested after 1% of the investment has been deducted?

Solution.—The remittance contains both the investment and the commission upon it. The commission being 1% of the investment, the remittance must be 101% of the investment. Hence $\$328.25 \div 1.01 = \325 , the investment.

64. I send to my agent at Havana \$1224. What part of this sum will remain to invest in sugars, after deducting his commission of 2% on what he lays out?

65. How many barrels of flour at \$5 each can a factor purchase with a remittance of \$2575, after deducting his commission of 3%?

66. A real estate broker received \$2593.75 for the purchase of land. Reserving $3\frac{3}{4}\%$ commission on the purchase, what number of acres of land could he purchase at \$125 per acre?

67. If \$109.65 is sent to an agent to purchase 2000 pounds of sugar at $5\frac{3}{4}$ cents per pound, and to pay his commission on the purchase, what % is the commission?

68. An agent sold 62 lawn-mowers at \$20 each, and 18 at \$15 each. If, after deducting his commission, he remitted \$1057 to the manufacturer, what was the % of his commission?

69. Find the balance of the following account of sales:

*Account of Sales of Forty-nine Tubs Butter,
Sold on account of Messrs. H. M. Ball & Son,*

By COON, BRO., & CO.

1877.						\$	¢
Mar. 13.	33 Tubs...	2095*	339†	1756‡	25¢		
" 14.	8 " ...	551	84	467	26¢		
" 15.	8 " ...	533	84	449	28¢		
CHARGES:							
Paid Freight and Cartage.....						\$20.89	
Commission and Guarantee, 4%...							
PHILADELPHIA, April 15, 1877.						Balance.....	

490. Written Exercises.

a. Supplying names and dates, write an account of the sales given in Example 58.

b. In the same way write an account of the sales given in Example 61.

* Gross weight.

† Weight of tubs.

‡ Net weight.

STOCKS, DIVIDENDS, AND BROKERAGE.

491. An association of individuals formed for the purpose of transacting business is a *company* or *partnership*.

492. An association of individuals authorized by law to transact business under a company name, to hold property and be liable for debts in that name as an individual would be, is a *corporation*.

493. When a corporation is formed for transacting business, the persons forming the corporation subscribe what money is needed for conducting the business. This money is called *capital stock*. This stock is divided into *shares*, usually of \$ 100 each.

494. The owners of the stock are *stockholders*. As evidence of their ownership, they hold papers called *certificates of stock*. The stockholders form the corporation and elect *directors*, who are responsible for the business transacted.

495. A sum levied upon a stockholder to help meet the expenses or losses of the business is an *assessment*.

496. The gain upon the capital of a corporation is divided among the stockholders. Gain thus divided is called a *dividend*.

Each stockholder's part of the dividend is the same per cent of his stock that the whole dividend is of the capital.

497. Stocks may be bought and sold like other property. Persons who make a business of buying and selling stocks are called *stock-brokers*. The commission paid to a broker is called *brokerage*.

NOTE I. When a share of stock will sell at its nominal value, it is *at par*; when for more than its nominal value, it is *above par*, or at a *premium*; when for less than its nominal value, it is *below par*, or at a *discount*.

NOTE II. The market values of stocks are "quoted" daily in the principal newspapers, at given per cents of their values. When a stock is quoted at 90, it is worth 90% of its face or nominal value; it is then 10% below par. When quoted at 105, stock is worth 105% of its face or nominal value; it is then 5% above par.

498. *The rules of percentage already illustrated apply to stocks, dividends, and brokerage.*

Examples for the Slate.

499. The following quotations are taken from a daily paper:

Sales of Stock this day at the Brokers' Board.

70 Chicago, Burlington, & Quincy R. R.	103 $\frac{3}{4}$
150 Burlington & Mo. R. R. in Neb.	43 $\frac{1}{2}$
\$5000 Atchison, Topeka, & Santa Fé 7's, 1st mortgage.....	88 $\frac{1}{4}$

AT AUCTION.

15 Bates Manufacturing Co. 80 $\frac{3}{4}$	8 American Watch Co..... 90 $\frac{1}{4}$
12 Neptune Insurance Co.. 122 $\frac{3}{4}$	5 Metropolitan Bank..... 92 $\frac{3}{4}$
5 Maverick Bank 150 $\frac{3}{4}$	40 Boston & Albany R. R.... 125
10 N. England Bank 135 $\frac{3}{4}$	36 Nashua & Lowell R. R.... 94 $\frac{1}{4}$

At the above quotations, what is the cost

70. Of 3 shares in the Maverick bank, and 7 in the Metropolitan?

71. Of \$2000 Atchison, Topeka, and Santa Fé 7's?

72. Of 8 shares in the Bates Manufacturing Co., and 7 in the Neptune?

73. Of 75 shares in the Burlington and Missouri, including $\frac{1}{4}$ % brokerage on the par value?

NOTE. Brokerage is usually $\frac{1}{4}$ %, and reckoned on the par value. It is thus reckoned in this book, unless otherwise specified.

At the above quotations, what is the cost, with brokerage,

74. Of 10 shares Boston and Albany R. R., and 25 Nashua and Lowell?

75. Of 15 shares in the Chicago, Burlington, and Quincy R. R., 5 shares in the American Watch Co., 40 shares in the New England Bank, and 12 shares in the Neptune Insurance Co.?

76. What is the value of 7 shares in a gold company's stock at $4\frac{1}{4}\%$ above par, the original value being \$200 per share?

77. A dividend of 3% having been declared by a gas company, what should a stockholder receive who owns 700 shares, the par value of each share being \$100?

78. A broker sold a lot of stock for \$2250, which was 10% below par. What was the par value?

79. When stock, originally worth \$30 per share, sells for \$45, at what % above par does it sell?

500. At present, 1878, paper currency is below par. The value of gold as compared with it is given from day to day in the newspapers.

80. When gold is quoted at $102\frac{1}{2}$, how much paper currency can be bought for \$200 in gold, no allowance being made for brokerage?

81. If the passage to Liverpool is \$125 in gold, and gold is at $103\frac{1}{2}$, what shall I pay in "greenbacks" for two tickets?

82. Wishing to send to Ireland 6 pounds sterling, valued at \$4.86 each in gold, what shall I pay for them in "greenbacks," gold being at $102\frac{1}{2}$, and brokerage $\frac{1}{4}\%$?

83. When gold is quoted at 103, what per cent of a gold dollar is the value of a 1-dollar bill?

INSURANCE.

501. A, owning a house, agrees to pay B a certain percentage on its value, B on his part agreeing to pay A the whole value of the house in case it should within a limited time be destroyed by fire. Such a contract is a contract of *insurance*; and A's house is said to be *insured*.

502. *Insurance* is security against loss.

503. *Fire insurance* is security against loss of buildings or goods by fire; *marine insurance* is security against loss of ships or cargoes at sea; *accident insurance* against

loss by accident in travelling or otherwise; *health insurance* secures a stated allowance during sickness, and *life insurance* secures a certain sum to one's heirs or assigns in case of death.

504. The parties that insure are called *insurers* or *underwriters*.

505. The written contract that binds the parties is the *policy*.

506. The sum paid for insurance is the *premium*.

NOTE I. When property is insured, the valuation or amount insured is generally made less than the value of the property.

NOTE II. Policies are renewed yearly, or at stated periods, and the premium is paid in advance.

507. The premium is a PERCENTAGE of which the sum insured is the base. Hence *the rules of percentage already illustrated apply to insurance*.

508. Examples for the Slate.

84. What is the insurance on \$1500 worth of goods at $\frac{3}{4}\%$, including \$1 for the policy?

85. What amount is paid for insurance on $\frac{3}{4}$ of a store valued at \$15600 at $\frac{5}{8}\%$, including \$1 for the policy?

86. A merchant insured a cargo from Liverpool worth 2000 pounds at a premium of $1\frac{1}{2}\%$. What was the premium, the pound being valued at \$4.86?

87. A merchant insured \$3600 worth of goods in one company at $1\frac{1}{2}\%$ premium, and \$2500 worth in another at $1\frac{1}{4}\%$ premium. What was the cost, including \$1 for each policy?

88. A druggist paid \$125 for the insurance of a lot of goods in transportation. If the face of the policy was \$10000, what was the rate of insurance?

89. Jan. 1, 1876, a person took out a health policy, paying \$1.50 on the first day of each month. March 2, 1877, he was disabled by sickness, and received \$12 a week for 3 weeks. How much did he receive more than he paid out for premiums?

The yearly rates of life insurance depend upon the age of the person when he begins to insure, younger persons paying less per year than older persons, because they are likely to live longer. Thus A, being 35 years old, pays \$109.50 a year for a policy of \$5000, while B, who is 40 years old, pays \$131.50 a year for a policy of the same amount. The number of years that a person of a given age is likely to live is called his *expectation of life*.

90. At the age of 38, I secured a policy upon my life for \$5000, paying the first year \$122.55, including \$1 for the policy. What was the premium paid upon \$1000?

91. Jan. 1, 1868, a man took out a policy on his life for \$3000, in favor of his wife, paying \$21.30 on \$1000 yearly. If the man died Feb. 15, 1878, how much did the widow receive more than had been paid in premiums?

TAXES.

509. The citizens of a town or city or the members of a society usually meet the expenses of their government or society by a sum assessed on their property, their income, their business, or their persons. Such a sum is called a tax.

510. A tax on the person of a citizen is called a *poll tax*. A tax on property is called a *property tax*. A tax on annual income is called an *income tax*.

511. Movable property, such as money, stocks, cattle, ships, etc., is called *personal property*. Immovable property, as lands, houses, etc., is called *real estate*.

512. Officers appointed to estimate the value of property and to apportion the sum to be raised among the individuals are called *assessors*.

513. A property tax is reckoned at a certain per cent on the estimated value of each person's property, or at a given number of mills or cents on \$1, \$100, or \$1000.

514. An income tax is reckoned at a fixed per cent on the net income of a person after certain deductions have been made.

515. ILLUSTRATIVE EXAMPLE. The whole amount to be raised for State, county, and town taxes in a certain town is \$10600. The property of the town is valued at \$1250000, and there are 300 polls, each taxed \$2. What is the tax on \$1? What is the tax of E. Stiles, who has \$4000 worth of real estate and \$1000 worth of personal property, and who pays 1 poll tax?

WRITTEN WORK.

$$\begin{array}{r}
 \$10600 \\
 \quad 600 \\
 125|0000) \underline{1|0000} \\
 \quad 0.008 \\
 \quad \underline{5000} \\
 \quad \$40.
 \end{array}$$

$$\$40 + \$2 = \$42$$

$$\text{Ans. } \$0.008; \$42.$$

Explanation.—\$10600 less the amount of poll taxes leaves \$10000 to be levied on \$1250000, which is 8 mills on \$1. If E. Stiles pays 8 mills on \$1, on \$5000 he will pay 5000 times 8 mills, or \$40. \$40 plus his poll tax of \$2 is \$42. *Ans.* 8 mills on \$1; \$42 tax.

516. From the above may be derived the following rules for assessment of taxes:

I. To find the rate of the property tax: *Deduct from the whole amount to be raised the amount of the poll taxes, and divide the remainder by the amount of taxable property.*

II. To find each person's tax: *Multiply each person's taxable property by the rate, and to the product add his poll tax.*

517. Examples for the Slate.

92. The tax levied by a certain town is \$46800; the valuation of the town is \$3600000, and there are 1800 polls, at \$1 each. What is the tax on \$1? What is the tax of A, who has \$15000, and who pays a poll tax of \$1?

93. The valuation of a school district is \$48000. A tax of \$120 is levied for the repairs upon a school-house. What is the tax on \$1? What is assessed upon a person having \$3500 of taxable property?

94. What is the net tax in a town whose taxable property is \$430000, the rate 12 mills on the dollar, 5% of the tax assessed being paid for collecting?

95. The school-tax of a certain town being \$5625, at the rate of $3\frac{1}{2}$ mills on the dollar of taxable property, what is the taxable property?

96. The amount of money to be raised by taxes in the town of H is \$212093.20; the taxable property is \$11522400; there are 3350 polls, each taxed \$1.40. Find the tax on \$1.

NOTE. Assessors commonly construct a table giving the tax on convenient amounts of property at the determined rate.

518. TAX TABLE

Showing the tax on various sums, at the rate of 18 mills on \$1.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
\$1	\$0.018	\$10	\$0.18	\$100	\$1.80	\$1000	\$18	\$10000	\$180
2	0.036	20	0.36	200	3.60	2000	36	20000	360
3	0.054	30	0.54	300	5.40	3000	54	30000	540
4	0.072	40	0.72	400	7.20	4000	72	40000	720
5	0.090	50	0.90	500	9.00	5000	90	50000	900
6	0.108	60	1.08	600	10.80	6000	108	60000	1080
7	0.126	70	1.26	700	12.60	7000	126	70000	1260
8	0.144	80	1.44	800	14.40	8000	144	80000	1440
9	0.162	90	1.62	900	16.20	9000	162	90000	1620

97. Find by the above table the tax on \$4250.

NOTE. Find the tax on \$4000, \$200, and \$50 separately, and add the results.

Find by the above table the tax

- | | |
|----------------------|-----------------------|
| 98. Of A on \$3000. | 102. Of E on \$9068. |
| 99. Of B on \$2800. | 103. Of F on \$6555. |
| 100. Of C on \$7850. | 104. Of G on \$5687. |
| 101. Of D on \$1565. | 105. Of H on \$10793. |

CUSTOMS OR DUTIES.

519. The expenses of the national government are met in part by taxes upon imported goods; these taxes are called *customs* or *duties*.

NOTE I. A tax called *tonnage* is laid upon a vessel according to the weight she is estimated to carry.

NOTE II. Places are established by government for the collection of customs or duties; these places are called *ports of entry*. Each port of entry has a *custom house*, which is in charge of an officer who collects the customs; this officer is called the *collector of customs*.

520. A duty proportioned to the quantity of goods imported, is a *specific duty*. Thus a duty of 30¢ a pound on yarn is a *specific duty*.

NOTE. In estimating specific duties, an allowance is made (1) for waste, or impurities, called *draft*; (2) for the weight of boxes, casks, etc., called *tare*; (3) for the waste of liquids, called *leakage*; (4) for the breaking of bottles, called *breakage*.

521. The weight of goods, before allowances are made, is called *gross weight*; and the weight, after all allowances are made, is called *net weight*.

522. A duty proportioned to the cost of goods in the country from whence they are imported, is an *ad valorem* duty. Thus a duty of 15% on iron castings is an *ad valorem* duty.

NOTE. A list of a ship's cargo containing a description of each package of goods imported, with the price in the currency of the country from whence imported, must be exhibited to the collector. Such a list is called an *invoice* or *manifest*. When no invoice is received, the value of the goods is determined by appraisement.

523. Examples for the Slate.

106. What is the duty at 5¢ a gallon, on 238 hogsheads of molasses, 60 gallons in a hogshead?

107. What is the duty at 30 cents a gallon on 25 barrels of spirits of turpentine, 32 gallons in a barrel, leakage 2%?

108. At 15%, what is the duty on 75 boxes of tin, 112 lbs. in each box, invoiced at 7¢ a pound, tare 10 pounds a box?

109. What is the duty at $2\frac{1}{2}$ cents a pound on 13 boxes of raisins, 24 lbs. in a box, tare $6\frac{1}{4}$ lbs. a box?

110. At 25%, what is the duty on 100 dozen watch-crystals invoiced at \$1.50 a dozen, breakage 3%?

111. At 36%, what is the duty on 200 tons of bar-iron (2240 lbs. to a ton), invoiced at $2\frac{1}{4}$ ¢ a pound, tare 5%?

112. At $3\frac{1}{2}$ ¢ a pound and 10% ad valorem, what is the duty on 7147 lbs. of steel, invoiced at 20 cents a pound, damage being 8%?

113. What is the cost at the store of 2556 lbs. of sugar bought in Havana for \$148.92, on which is paid \$35.75 for freight and carting, and $2\frac{1}{4}$ ¢ a pound for duties, after deducting 15% for tare?

524. General Review, No. 4.

114. Change $\frac{1}{3}$ to a per cent.

115. Represent $1\frac{1}{10}$ % decimally.

116. Change $106\frac{1}{4}$ % to a common fraction in its smallest terms.

117. What is $\frac{1}{4}$ per cent of \$56.49?

118. \$700 is 140% of what number?

119. If a percentage is \$540 and the rate 3%, what is the base?

120. 25% of a certain number exceeds 10% of it by \$75. What is that number?

121. A schooner formerly valued at \$7500 has depreciated 20%. What is her present value?

122. Find the cost of goods which sell for \$120 at a gain of 25%.

123. What per cent is 125 of 1200?

124. What commission must be paid for collecting \$17380 at $3\frac{1}{2}$ per cent?

125. What amount of stock can be bought for \$9682, allowing $\frac{1}{4}$ per cent brokerage?

126. What is the value of 20 shares bank stock, at $8\frac{1}{2}$ per cent discount, the par value of each share being \$150?

127. How many shares of stock at 35% advance on a par value of \$100 can be bought for \$1215?*

128. Insurance was effected on the ship Susan, to Cadiz and back, for \$10000 at 2%, and on her return cargo, worth \$7500, at $1\frac{1}{2}$ %. What was the amount of premium, including \$1 for policy?

129. What insurance may be covered by a premium of \$28 at $\frac{1}{4}$ %?

130. What is the insurance premium at $\frac{1}{2}$ % on $\frac{3}{4}$ of a house worth \$6000?

131. What is the duty, at 12¢ a lb. and 10% ad valorem, on 20 bags of wool, each containing 115 lbs., valued at 42 cts. per lb.?

525. Miscellaneous Examples.

132. A man paid for a house \$4500, for repairs \$157.50, and then sold it for 18% above the entire cost. What did he receive for it?

133. I bought 100 railroad shares at $116\frac{1}{2}$ and sold them at $120\frac{1}{2}$. What did I gain, the par value being \$100?

134. A mason sold 75 barrels of lime at 27% profit, and gained \$40.50. What was the cost per barrel?

135. A broker bought 48 shares of \$50-stock at $9\frac{1}{2}$ % discount and sold them at $2\frac{1}{4}$ % premium. How much did he make?

136. What amount of current money will be given in exchange for \$450 of that which is at 5% discount?

137. If I buy 10 shares of stock originally worth \$100 each at 18% above par, and sell it at 7% below par, what do I lose?

138. A cotton-mill valued at \$175000 is insured for $\frac{3}{4}$ of its value by two companies, the first taking $\frac{3}{8}$ of the risk at 0.9%, the second the remainder at $\frac{7}{8}$ %. What is the total cost of the premium?

* See Art. 499, note.

139. A school-house was insured for \$15500 at $2\frac{3}{4}\%$, \$1.50 being paid for the policy and survey. What was the entire expense for insurance?

140. If the school-house named above was lost by fire, what was the net loss to the insurance company?

141. Suppose I buy 20 shares of stock originally worth \$50 a share, at 10% discount, and sell at a premium of 8%, what do I make?

142. A merchant sold some iron for \$278, and made 15%. What should he have sold it for to make 25%?

143. When 75 shares of stock originally worth \$100 a share sell for \$7556.25, at what per cent above par does it sell?

144. If a company takes an accident risk of \$8000 at $1\frac{1}{4}\%$, and reinsures one half of it in another company at $1\frac{1}{2}\%$, what will they gain if no accident occurs?

145. After losing 11% of his apples, a dealer has 133.5 bbls. of apples left; if they cost him \$2.50 per bbl., for what must they be sold per bbl. that he may lose nothing upon his purchase?

146. A broker bought insurance stock at 80, and sold it at 112. What per cent did he make upon his investment?

147. A broker sold 19 shares of stock for \$1389.85, which was at $4\frac{1}{2}\%$ above par. What was the brokerage at $\frac{1}{2}\%$ on the par value?

148. A factory is insured at the rate of \$2 on \$100. If the premium, with \$1 for the policy, is \$241, and the insurance is upon $\frac{3}{4}$ of the value of the property, what is the value of the property?

149. When an insurance stock, originally \$100 per share, is quoted at $102\frac{3}{4}$, how many shares can be bought for \$8815, brokerage $\frac{1}{8}\%$?

150. If a watch sells for \$60 at a loss of 22%, what should it have sold for to gain 30%?

151. The capital of a gas company is \$200000, and the net earnings are \$10746. What rate of dividend can the com-

pany declare, reserving a surplus of \$2746 to meet future demands?

152. A vessel brought into port 12000 melons. 8% proved worthless, 10% of the remainder sold for 18¢ apiece, and the rest for 12½¢ apiece. What was received for the whole?

153. At the sale of a piano, 20% was deducted from the retail price, and 5% of the balance for cash payment. If the retail price was \$750, and the wholesale price \$475, for what per cent advance upon the wholesale price was it then sold?

154. A regiment of 1000 men was reduced to 850 by sickness and battle, the loss by sickness being 50% as great as by battle. What was the entire per cent of loss? what by sickness? by battle?

155. I sold 250 lbs. of fish, gaining thereby \$3.75, which was 42½% of the cost. What was the cost? For how much a pound was the fish sold?

156. A grain dealer's sales amounted in one year to \$75000; ⅔ of his receipts were for wheat, on which he made 10% profit, and the balance for other grains, on which he made 20% profit. What was the cost of the whole stock?

157. A broker bought stock at 8% premium, and sold it at 9% discount, and lost \$510. How many shares originally worth \$100 each did he buy?

158. Two horses were sold for \$144 each; on one there was a gain of 20%, and on the other a loss of 20%. How much was the gain or loss on both?

159. What is the cost of 5 hhds. of molasses containing in all 2074 gallons, which was bought in Porto Rico at 42¢ a gallon, and on which is paid \$45.75 for freight and carting, and 5¢ a gallon for duty, after deducting 12% for leakage?

160. A certain corporation wishing to increase its stock without multiplying the number of its shares, assessed the stockholders 40% on the par value of their stock, which was \$500 per share. What was the par value of the stock after the assessment was made?

SECTION XV.

SIMPLE INTEREST.

526. A had the use of \$ 300 of B's money for a year. At the end of the year he paid B for its use a sum equal to 7% of the money borrowed. What did he pay for its use?

Ans. \$ 21.00.

527. Money paid for the use of money is *interest*.

528. The money for the use of which interest is paid is the *principal*.

529. The sum of the principal and interest is the *amount*.

In the above example, what is the interest? the principal? the amount?

530. Interest is reckoned at a certain per cent of the principal. It is, therefore, a percentage of which the base is the principal.

531. The number of hundredths of the principal taken in finding the interest for one year is the *rate per cent per annum*, usually called the *rate*.

NOTE. When a rate of interest is given, it is understood to be the rate per year, unless a different time is stated.

532. The rate of interest established by law is the *legal rate*. Interest at a rate higher than the legal rate is *usury*.

NOTE. Debts of all kinds draw interest from the time they become due, but not before, unless it is so specified. Interest on interest unpaid when due is sometimes, though not usually, allowed.

533. Interest on the principal alone is *simple interest*.

NOTE. The laws regulating rates of interest are frequently changed, but the following is a table compiled from official sources in 1877.

534. Table of Legal Rates of Interest.

When two rates are given in this table, any rate not exceeding the highest is allowed, if agreed upon in writing.

States.	Rate %.	States.	Rate %.	States.	Rate %.	States.	Rate %.
Ala.	8	Ill.	6	Mo.	6 10	S. C.	7 Any
Ark.	6	Ind.	6 10	Montana.	10	Tenn.	6 10
Arizona.	12 Any	Iowa.	6 10	N. H.	6	Texas.	8 10
Cal.	10	Kan.	7 12	N. J.	7	Utah.	10 Any
Conn.	7	Ky.	6 8	N. Y.	7	Vt.	6
Colo.	10 Any	La.	6	N. C.	6 8	Va.	6
Dak.	12 Any	Maine.	6 Any	Neb.	10 12	W. Va.	6
Del.	6	Md.	6	Nev.	10 Any	W. T.	10 Any
D. C.	6 10	Mass.	6 Any	Ohio.	6 8	Wis.	7 10
Fla.	8 Any	Mich.	7 10	Or.	10 12	Wy.	12 Any
Ga.	7 12	Minn.	7 12	Penn.	6		
Idaho.	10	Miss.	6 10	R. I.	6 Any		

NOTE I. In this book, when no rate is mentioned or implied, 6% is understood.

NOTE II. In reckoning interest, it is customary to consider a year to be 12 months, and a month 30 days.

535. In reckoning the months and days between two dates, *take the entire calendar months as months, and then the exact number of days remaining.* (See Art. 371.)

NOTE. In computing interest for short periods of time, it is customary to take the exact number of days.

536. Oral Exercises.

What is the interest

- Of \$100 for 1 year at 7%? for 2 years at 3%?
- Of \$300 for 2 years at 6%? at 8%? at 11%? at 12%?
- Of \$400 for $3\frac{1}{2}$ years at 4%? at 10%? at 7%? at 8%?
- Of \$40 for 3 years at 10%? at 5%? at 7%? at 6%?
- What part of a year's interest is the interest on any sum of money for 6 mo.? 2 mo.? 3 mo.? 4 mo.? 1 mo.?
- At 5%, what is the interest of \$600 for 1 year? for 6 mo.? 3 mo.? 4 mo.? 2 mo.?
- At 9%, what is the interest of \$100 for 1 year? for 1 mo. or 30 days? for 6 days? for 1 day? for 5 days?
- What is the amount of \$100 for 4 years 6 months at 8%?
- What is the amount of \$100 for 1 year 4 months at 5%?
- What is the amount of \$200 for 3 years 3 months at 10%?

METHODS OF COMPUTING INTEREST.

TO THE TEACHER. Two methods of computing interest are given in the following pages ; but the teacher is advised to have pupils use but one. The method by aliquot parts will be found on page 308 of the Appendix.

GENERAL METHOD.

537. ILLUSTRATIVE EXAMPLE. Find the interest of \$840 for 4 y. 3 mo. 5 d. at 8%.

WRITTEN WORK.

$$\$840 \times 0.08 \times 4 = \$268.80$$

$$\begin{array}{r} 7 \\ \$840 \times 0.08 \times 95 = 17.73 \\ \hline 360 \\ 3 \end{array} \quad \text{Ans. } \$286.53$$

Explanation.—The interest of \$840 for 1 year at 8% is \$840 × 0.08. The interest for 4 years is 4 times as much, or \$268.80.

3 mo. 5 d. equal 95 days. The interest of \$840 for 1 year being \$840 × 0.08, the interest for 1 day is $\frac{1}{360}$ of this (Art. 534, Note II.), and for 95 days it is 95 times as much, or \$17.73, which, added to \$268.80, makes \$286.53, the entire interest.

538. From the example above may be derived the following

Rule.

1. To find the interest at any per cent for any number of years: *Multiply the principal by the rate for 1 year, and that product by the number of years.*

2. To find the interest for months and days: *Change the months to days (Art. 535) and take as many 360ths of a year's interest as there are days in the given time.*

539. This rule may be expressed by the formula :

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Number of years.}$$

540. Examples for the Slate.

1. What is the interest of \$720 for 3 y. 7 mo. 6 d. at 8%?
2. Of \$472.30 for 2 y. 2 mo. 12 d. at 4%?
3. Of \$400.50 for 3 y. 10 mo. 24 d. at 10%?
4. Of \$84.80 for 5 y. 3 mo. 20 d. at 6%?
5. Of \$116.20 for 2 y. 10 mo. 16 d. at 7%?

SIX PER CENT METHOD.

541. Oral Exercises.

a. At 6 %, what part of the principal is the interest for 1 year? for 2 months?

b. If the interest for 2 months is 0.01 of the principal, what part of the principal is the interest for any number of months?
Ans. One half as many hundredths of the principal as there are months.

c. At 6 %, what is the interest of \$500 for 2 mo.? for 4 mo.? 6 mo.? 8 mo.? 10 mo.? 5 mo.? 7 mo.? 15 mo.?

d. If the interest for 2 months, or 60 days, is 0.01 of the principal, what part of the principal is the interest for 6 days?

e. If the interest for 6 days is 0.001 of the principal, what part of the principal is the interest for any number of days?
Ans. One sixth as many thousandths of the principal as there are days.

f. At 6 % what is the interest of \$500 for 6 days? 1 day? 2 days? 3 days? 12 days? 18 days? 24 days?

542. ILLUSTRATIVE EXAMPLE. What is the interest of \$480 for 1 y. 3 mo. 7 d. at 6 %? at 7 %? What is the amount at 7 %?

WRITTEN WORK.

\$480	0.075
0.076 $\frac{1}{2}$	0.001 $\frac{1}{2}$
<hr/> 2880	<hr/> 0.076 $\frac{1}{2}$
3360	
80	
<hr/> 6) \$36.560 Int. at 6%.	
6.09	
<hr/> \$42.65 Int. at 7%.	
480.	
<hr/> \$522.65 Amt. at 7%.	

Explanation. — 1 y. 3 mo. equals 15 mo. The interest for 15 mo. at 6 % is 0.07 $\frac{1}{2}$, or 0.075 of the principal. The interest for 7 days is 0.001 $\frac{1}{2}$ of the principal. Hence the interest for 1 y. 3 mo. 7 d. at 6 % is 0.076 $\frac{1}{2}$ of the principal. 0.076 $\frac{1}{2}$ of the principal is \$36.56.

To find the interest at 7 %, we add to the interest at 6 % $\frac{1}{2}$ of itself, and have for the sum \$42.65.

\$480 + \$42.65 = \$522.65, the amount at 7 %.

Ans. \$36.56 ; \$42.65 ; \$522.65.

543. From the foregoing may be derived the following

Rule.

1. To compute interest at 6% : *Take 6 times as many hundredths as there are years, 1 half as many hundredths as there are months, and $\frac{1}{2}$ as many thousandths as there are days, and by this decimal multiply the principal.*

2. To find the interest at any rate other than 6% : *Having found the interest at 6%, increase or diminish that interest by adding or subtracting such part of itself as will give the interest at the required rate.*

3. To find the amount : *Add the principal to the interest.*

NOTE I. Observe that $1\% = \frac{1}{3}$ of 6% ; $2\% = \frac{1}{3}$ of 6% ; $3\% = \frac{1}{2}$ of 6% ; $4\% = 6\% - 2\%$; $5\% = 6\% - 1\%$; $7\% = 6\% + 1\%$; $7\frac{1}{2}\% = 6\% + (\frac{1}{2}$ of $6\%)$, etc.

NOTE II. It will often be more convenient to increase or diminish the principal before taking the interest instead of increasing or diminishing the interest. Thus, in the foregoing illustrative example we might add to \$480 $\frac{1}{2}$ of itself and then take 6% interest on \$560. This would be the same as the interest at 7% on \$480, which is \$42.65.

544. Examples for the Slate.

Find the interest on \$1 at 6%

- | | |
|------------------------|---------------------------|
| 6. For 1 y. 3 mo. 6 d. | 10. For 1 y. 1 mo. 10 d. |
| 7. For 4 y. 16 d. | 11. For 1 y. 8 mo. |
| 8. For 4 mo. 5 d. | 12. For 16 y. 8 mo. |
| 9. For 1 mo. 25 d. | 13. For 7 y. 10 mo. 18 d. |

At 6% what is the interest

14. Of \$300 for 2 y. 5 mo. ?
15. Of \$36.18 for 3 y. 7 d. ?
16. Of \$872.32 for 6 y. 2 mo. 16 d. ?
17. Of \$130.50 for 2 y. 9 mo. 13 d. ?
18. Of \$800.20 for 3 y. 4 mo. 12 d. ?
19. Of \$1000 for 3 y. 10 mo. 2 d. ?
20. Of \$25.50 for 1 y. 1 mo. 1 d. ?
21. Of \$400.37 for 2 y. 5 mo. 26 d. ?

What is the interest

22. Of \$837.36 for 3 y. 2 mo. at 7 %?
23. Of \$187.50 for 2 mo. 12 d. at 10 %?
24. Of \$1000 from Nov. 11, 1874, to Aug. 15, 1880, at 7 %?
25. Of \$130.16 from Feb. 7, 1874, to Dec. 1, 1878, at 8 %?
26. Of \$19.80 from Oct. 15, 1875, to April 19, 1876, at 5 %?
27. Of \$62.50 from Aug. 3, 1874, to April 11, 1875, at $7\frac{1}{2}$ %?

Find the amount

28. Of \$540 for 3 y. 6 mo. at 6%.
29. Of \$495.60 for 2 y. 2 mo. at 12%.
30. Of \$830 for 5 y. 4 mo. at 8%.
31. Of \$110.10 for 3 y. 5 mo. at 9%.
32. Of \$896 for 2 y. 6 mo. 15 d. at $6\frac{2}{3}$ %.
33. Of \$416 for 3 y. 16 d. at 7%.
34. Of \$720 for 3 y. 9 mo. 19 d. at 8%.
35. A note for \$150, dated July 5, 1872, was paid Mar. 17, 1874, with interest at 6%. What was the amount?
36. I gave my note to a person, Jan. 1, 1877, for \$387.20, with interest at 7% from date. What should I pay to discharge this note Oct. 20, 1877?
37. Chase and Fowle bought goods to the following amounts, agreeing to pay 7% interest from the date of purchase: July 8, 1876, \$470; July 28, \$235; Oct. 2, \$206. What will be the amount due Jan. 1, 1877?

Short Method for Days; Application of 6 per cent Method.

545. ILLUSTRATIVE EXAMPLE. What is the interest of \$126.80 for 93 days at 6%?

WRITTEN WORK.

\$126.80	
1.268	Int. for 60 d.
0.634	" " 30 d.
0.063	" " 3 d.
Ans. \$1.965	" " 93 d.

Explanation.—The interest at 6% for 60 days, or 2 months, is 0.01 of the principal. 0.01 of \$126.80 may be expressed by moving the decimal point two places towards the left; this gives \$1.268. The interest for 1 month, or 30

days, is $\frac{1}{10}$ of \$1.268, or \$.0634, and for 3 days it is $\frac{1}{10}$ of \$.0634, or \$.0063. Adding these interests, \$1.268 + \$.0634 + \$.0063 = \$1.965.
Ans. \$1.97.

546. From the foregoing may be derived the following

Rule.

1. *Find the interest for 60 days at 6% by taking 0.01 of the principal.*
2. *For other periods of time, Take convenient multiples or aliquot parts of the interest for 60 days.*

547. Examples for the Slate.

Find the interest of

- | | |
|--|---------------------------------|
| (38.) \$ 300 for 93 d. at 6%. | (40.) \$ 1000 for 33 d. at 10%. |
| (39.) \$ 250 for 95 d. at 7%. | (41.) \$ 280 for 127 d. at 12%. |
| (42.) \$ 270.80 from Aug. 20. to Oct. 30 at 8%. | |
| (43.) \$ 416.60 from Nov. 12, 1875, to Feb. 5, 1876, at 5%. | |
| (44.) \$ 1560.50 from Mar. 27, 1875, to June 7, 1875, at 9%. | |
| (45.) \$ 6000 from Nov. 15, 1875, to March 7, 1876, at 6%. | |

ACCURATE INTEREST.

NOTE. The above methods of performing examples in interest being based upon the supposition that a year equals 12 months of 30 days each, or 360 days, though in common use, are not exact. The government of the United States and that of Great Britain pay accurate interest.

548. To obtain accurate interest for months and days:
Find the exact number of days between the given dates, and take as many 365ths of a year's interest as there are days.

549. Examples for the Slate.

46. Find the accurate interest of \$2000 from Mar. 1 to Aug. 10 at 5%.

What is the accurate interest

- 47.** Of \$700 from May 7 to July 9 at $7\frac{1}{2}\%$?
- 48.** Of \$20000 from April 4 to July 7 at 7%?
- 49.** Of \$1000 from Nov. 15, 1875, to April 1, 1876, at 5%?

For additional examples in interest, see page 253.

PARTIAL PAYMENTS.

550. [DEMAND NOTE.]

\$470 $\frac{60}{100}$.*Cincinnati, Jan. 1, 1874.*

For value received, I promise to pay to the order of Charles Gleason, Four Hundred Seventy $\frac{60}{100}$ Dollars, on demand, with interest at 6 per cent.

Ephraim Flint.

551. The above is the written promise of one person, Flint, to pay another person, Gleason, or any one to whom Gleason may order it paid, a certain sum of money, \$470.60, for value received. Such a promise is called a **promissory note**, or simply a **note**.

552. The sum named in the note (as \$470.60 above) is the **face** of the note.

To discharge the interest and in part pay the above note a payment of \$94.13 was made Nov. 1, 1874. What balance then remained due ?

Ans. \$400.

Suppose the above balance of \$400 to remain on interest from Nov. 1, 1874, to Nov. 1, 1875, when a payment of \$224 was made, what sum then remained due ?

Ans. \$200.

553. Payments in part of a note or other debt, as the payments described above, are **partial payments**.

554. A record of the sum paid, with the date of the payment, is made upon the back of the note ; such a record is an **indorsement**.

The method adopted by the Supreme Court of the United States, and by most of the States, for computing interest in case of partial payments, requires (1.) *That a payment be applied first to discharge accrued interest, and then, if the payment is large enough, to reduce the principal.* (2.) *That no unpaid interest be added to the principal to draw interest.*

555. ILLUSTRATIVE EXAMPLE. A note for \$600, dated June 20, 1874, had payments indorsed upon it as follows:

Oct. 2, 1874, \$110.20.	May 23, 1876, \$125.25.
Feb. 29, 1876, 24.00.	Dec. 11, 1876, 113.20.

Find the balance due Jan. 21, 1877; interest 6%.

WRITTEN WORK.

Principal from June 20, 1874	\$ 600.00
Interest to Oct. 2, 1874 (3 mo. 12 d.)	10.20
Amount	610.20
First payment, Oct. 2, 1874	110.20
New principal from Oct. 2, 1874	500.00
Interest on \$500 to Feb. 29, 1876 (1 y. 4 mo. 27 d.)	42.25
Second payment, \$24 will not discharge interest.	
Interest on \$500 to May 23, 1876 (2 mo. 24 d.) *	7.00
Amount	549.25
Second and third payments, \$24 + \$125.25	149.25
New principal from May 23, 1876	400.00
Interest to Dec. 11, 1876 (6 mo. 18 d.)	13.20
Amount	413.20
Fourth payment	113.20
New principal from Dec. 11, 1876	300.00
Interest to Jan. 21, 1877 (1 mo. 10 d.)	2.00
Balance due Jan. 21, 1877 (Ans.)	\$ 302.00

556. The above is in accordance with

The United States Rule for Partial Payments.

1. *Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest; take from this amount a sum equal to the payment or payments.*

2. *With the remainder as a new principal, proceed as before, to the time of settlement.*

* Interest could be taken from Oct. 2, 1874, to May 23, 1876, at once, if it were seen that the payment was too small to discharge the interest to Feb. 29, 1876.

557. Examples for the Slate.

50. Oct. 12, 1873, I gave my note on demand, with interest at 6%, for \$480; Feb. 6, 1874, I paid \$120. What remained due Aug. 24, 1874?

51. I held a note for \$500, which bore interest at 6% from May 10, 1869; Sept. 16, 1870, I received \$140; July 28, 1872, I received \$50. What remained due Sept. 4, 1872?

52. June 15, 1873, George Rich borrowed of John Jones \$2000, and gave his note for the same, with interest at 8%. Aug. 27, 1874, a payment of \$1450 was made, and a new note given for the balance. For what sum was the new note given? Write the new note in proper form, dating it at Boston.

53. A note for \$1000, dated Oct. 5, 1874, was indorsed as follows: Dec. 8, 1874, \$125; May 12, 1875, \$316; Sept. 2, 1875, \$417. What balance was due March 9, 1876; interest 6%?

54. What balance will be due July 1, 1881, on a note of \$935 on interest from Sept. 1, 1875, and indorsed \$125.75, Jan. 15, 1876; \$250, March 25, 1877; \$300, May 10, 1877; interest being 6%.

(55.) \$425.

NEW YORK, *July 13, 1869.*

Six months after date I promise to pay A. Hyde & Co. Four Hundred Twenty-five Dollars, with interest at 6%; value received.

STEWART E. FRENCH.

Indorsements: Aug. 9, 1871, \$50; Nov. 17, 1872, \$150.

What was due July 12, 1873?

(56.) \$800.

ST. LOUIS, *July 15, 1870.*

For value received, We jointly and severally promise to pay H. Hooker, or order, Eight Hundred Dollars on demand, with interest at 7%; value received.

JAMES HOLLAND.

HENRY HOLLAND.

Indorsements: April 18, 1871, \$100; Dec. 31, 1872, \$70;

June 14, 1874, \$62.50.

What was due July 14, 1875?

558. ILLUSTRATIVE EXAMPLE.

\$ 800.

Philadelphia, July 7, 1876.

Three months after date I promise to pay Moses Pinnoch, or order, Eight Hundred Dollars, with interest at 6%; value received.

Stephen Ruggles.

Indorsements: Aug. 16, 1876, \$ 200; Oct. 8, 1876, \$ 480; Feb. 20, 1877, \$ 49.92. What balance was due July 1, 1877?

559. When partial payments are made upon notes on interest for short periods of time, as upon the above, interest is often computed by the following, called

The Merchants' Rule.

1. *Compute interest on the principal from the time it begins to draw interest to the time of settlement, and also on each payment from*

WRITTEN WORK OF EXAMPLE ABOVE.

Principal on interest from July 7, '76	\$800.00
Interest to July 1, '77 (11 mo. 24 d.)	47.20
Amount of note	847.20
Payment, Aug. 16, '76	200.00
Interest to July 1, '77 (10 mo. 15 d.)	10.50
Payment, Oct. 8, '76	480.00
Interest to July 1, '77 (8 mo. 23 d.)	21.04
Payment, Feb. 20, '77	49.92
Interest to July 1, '77 (4 mo. 11 d.)	1.09
	762.55
Balance due	Ans. \$84.65

the time it is made to the time of settlement.

2. *Take the difference between the sum of the principal and its interest and the sum of the payments and their interests; this difference will be the balance due.*

560. Examples for the Slate.

(57.) \$10000 $\frac{75}{100}$.

WASHINGTON, Oct. 3, 1875.

In two months from date I promise to pay to the order of Cyrus Parsons, at Suffolk Bank, Boston, Ten Thousand $\frac{75}{100}$ Dollars, with interest at 6%; value received.

IRA NORTON.

Indorsements: Nov. 5, 1875, \$672.41; Nov. 15, 1875, \$7682.42;

Nov. 16, 1875, \$437.98; Nov. 19, 1875, \$833.42.

What was the balance due on the above when it became due?

(58.) \$1200.

BALTIMORE, April 1, 1875.

One year from date, for value received, I promise to pay
B. F. Bryant, or order, Twelve Hundred Dollars, with interest
at 7%.

ISAAC C. FELLOWS.

Indorsements: April 12, 1875, \$161.08; July 19, 1875, \$224.14;

July 28, 1875, \$17.90; Jan. 29, 1876, \$100.25.

What was due on the above note April 1, 1876?

For annual interest, also for Vermont, New Hampshire, and Connecticut
rules for partial payments, with annual interest, see Appendix, pages 309
and 310.

PROBLEMS IN INTEREST.

To find the Time, having the Interest, Principal, and Rate given.

561. ILLUSTRATIVE EXAMPLE. In what time will \$480
on interest at 5% yield \$36 of interest?

WRITTEN WORK.

$$\$480 \times 0.05 = \$24.$$

$$\$36 \div \$24 = 1\frac{1}{2}.$$

$$1\frac{1}{2} \text{ yr.} = 1 \text{ yr. 6 mo.}$$

Explanation. — The interest of \$480 for 1
year at 5% is \$24.

Since \$480 at 5% yields \$24 of interest in
1 year, to yield \$36 it will require as many
years as there are times \$24 in \$36, which is
 $1\frac{1}{2}$. *Ans.* 1 yr. 6 mo.

562. From the above may be derived the following

Rule.

To find the time, having the principal, interest, and rate
given: *Divide the given interest by the interest of the prin-
cipal at the given rate for 1 year.*

This rule may be expressed by the formula:

$$1. \text{ Number of years} = \frac{\text{Interest}}{\text{Principal} \times \text{Rate}}$$

NOTE. It will often be found more convenient to divide by the interest
for 1 month or 1 day, in which case the answer will be in months or in days.

563. Examples for the Slate.

In what time will,

(59.) \$400 gain \$20 at 6%? (62.) \$3000 gain \$205 at 5%?

(60.) \$500 gain \$60 at 4%? (63.) \$408 gain \$170 at $7\frac{1}{2}\%$?

(61.) \$640 gain \$67.20 at 7%? (64.) \$450 gain \$192.30 at 8%?

65. In what time will \$280 amount to \$301 at 5%?

NOTE. To find interest, subtract \$280 from \$301.

66. How long must a note of \$7500 run to amount to \$7800 at 8%?

67. In what time will \$500 double itself at 1%? at 2%? at 3%? at 6%? at 10%?

To find the Rate, having the Interest, Principal, and Time given.

564. ILLUSTRATIVE EXAMPLE. The interest on \$200 for 10 mo. 24 d. was \$14.40; what was the rate %?

WRITTEN WORK.

Explanation. — The interest of \$200 for

\$200 \times 0.009 = \$1.80. 10 mo. 24 d. at 1% is \$1.80.

\$14.40 \div \$1.80 = 8. Since the interest at 1% on \$200 for

Ans. 8%.

10 mo. 24 d. is \$1.80, to yield \$14.40 the rate must be as many times 1% as there

are times \$1.80 in \$14.40, which is 8. Ans. 8%.

565. From the above may be derived the following

Rule.

To find the rate, having the interest, principal, and time given: *Divide the given interest by the interest of the principal for the given time at 1%.*

The above rule may be expressed by the formula:

$$2. \text{ Rate} = \frac{\text{Interest}}{\text{Principal} \times \text{Number of years}}$$

566. Examples for the Slate.

68. At what rate % will \$360 gain \$40.80 in 1 y. 5 mo.?

69. At what rate % will \$100 gain \$33 $\frac{1}{3}$ in 12 y. 6 mo.?

At what rate %

70. Will \$250 gain \$3.75 in 4 mo. ?

71. Will \$25 gain \$7.87½ in 3 y. 6 mo. ?

72. Will \$100 gain \$25 in 7½ y. ?

73. The amount of \$75 for 2 y. 6 mo. was \$78.75; what was the rate % ?

NOTE. To find the interest, deduct \$75 from \$78.75.

74. A note of \$50 on interest from Feb. 29, 1872, to Feb. 28, 1874, amounted to \$55.25; what was the rate % ?

75. When a note of \$1000 amounts to \$1058.33½ in 7 mo., what is the rate % ?

To find the Principal, having the Interest or Amount, the Time, and the Rate given.

567. ILLUSTRATIVE EXAMPLE I. What principal on interest at 6% for 3 y. 4 mo. will yield \$80 of interest ?

WRITTEN WORK.

$$1 \times 0.06 \times 3\frac{1}{3} = 0.20$$

$$\$80.00 \div \$0.20 = 400$$

Ans. \$400.

Explanation. — The interest of \$1 at 6% for 3 y. 4 mo. is \$0.20.

Since 1 dollar of principal at 6% in 3 y. 4 mo. yields 20 cents of interest, to yield \$80 of interest will require as many dollars of principal as there are times 20 cents in \$80, which is 400.

Ans. \$400.

568. ILLUSTRATIVE EXAMPLE II. What principal on interest at 10% for 2 y. 6 mo. will amount to \$478.50 ?

WRITTEN WORK.

$$1 \times 0.10 \times 2\frac{1}{2} = 0.25$$

$$\$1.25) \$478.50 \text{ (382.8}$$

375

1035

1000

350

250

1000 etc.

Explanation. — The interest of \$1 for 2 y. 6 mo. at 10% is \$0.25, and the amount of \$1 is \$1.25.

Since \$1 of principal at 10% in 2 y. 6 mo. amounts to \$1.25, to amount to \$478.50 will require as many dollars of principal as there are times \$1.25 in \$478.50, which is 382.8.

Ans. \$382.80

569. From the foregoing may be derived the following

Rules.

I. To find the principal, having the interest, the time, and the rate given: *Divide the given interest by the interest of \$1 for the given time and rate.*

II. To find the principal, having the amount, the time, and the rate given: *Divide the given amount by the amount of \$1 for the given time and rate.*

The above rules may be expressed by the formulas :

$$3. \text{ Principal} = \frac{\text{Interest}}{\text{Rate} \times \text{Number of years}}.$$

$$4. \text{ Principal} = \frac{\text{Amount}}{1 + \text{Rate} \times \text{Number of years}}.$$

570. Examples for the Slate.

What principal on interest

76. At 6% will gain \$15 in 2 years ?

77. At 5% will gain \$20 in 4 years ?

78. At 3% will gain \$76.50 in 2 y. 6 mo. ?

79. At 4% will gain \$1.705 in 7 mo. 15 d. ?

80. At 6% will gain \$4.128 in 11 mo. 14 d. ?

NOTE. $4.128 \div 0.057\frac{1}{3}$ (both changed to thirds of thousandths) equals 12.384 \div 0.172.

81. At 2% a month will gain \$24 in 60 days ?

82. At 6% will amount to \$870 in 7 y. 6 mo. ?

83. At 5% will amount to \$2072.25 in 30 d. ?

84. At 1% a month will amount to \$412 in 90 d. ?

85. What sum on interest $3\frac{1}{2}$ yrs. at $5\frac{1}{2}\%$ will amount to \$100 ?

86. What sum put upon interest Jan. 1, 1875, at 7% will amount to \$343.75, Feb. 1, 1877 ?

87. What principal put upon interest to-day at 5% will amount to \$206.25 in 7 mo. 15 d. ?

PRESENT WORTH AND DISCOUNT.

571. ILLUSTRATIVE EXAMPLE. If one person owes another \$214, to be paid 1 year hence, without interest, what sum should be paid to-day to discharge the debt, the current rate of interest being 7 per cent ?

WRITTEN WORK.

1.07) 214.00 (200
 214

Ans. \$200.

Explanation. — In justice to both parties, such a sum should be paid to-day as would, if put at interest at 7 %, in 1 year amount to \$214.

Since \$1 in 1 year at 7 % amounts to \$1.07, it would require as many dollars to amount to \$214 as there are times \$1.07 in \$214, which is 200. *Ans.* \$200.

572. A sum which will without loss to either party discharge a debt at a given time before the debt is due is the *present worth* of the debt.

573. A sum deducted from a debt or from a price is *discount*. The difference between the face of a debt and the present worth is the *true discount*.

What is the present worth in the example above? What is the true discount?

NOTE. — It will be seen that the present worth is the principal, the true discount is the interest, and the sum due at a future time is the amount. This subject is then an application of that illustrated in Art. 568.

574. From the illustrative example above may be derived the following

Rules.

I. To find the present worth: *Divide the given debt by the amount of \$1 for the given time and rate.*

II. To find the true discount: *Subtract the present worth from the face of the debt.*

575. Examples for the Slate.

The current rate of interest being 6%, what is the present worth and what is the true discount

88. Of \$ 27.50, due 1 year 8 months hence ?
89. Of \$ 100.96, due 8 months hence ?
90. Of \$ 200, due in 3 months ?
91. Of \$ 175.80, due in 9 months 20 days ?
92. Of \$ 661.37 $\frac{1}{2}$, due in 3 months 15 days ?
93. What is the present worth and true discount of \$ 1609.30, due in 10 months 24 days, current rate 5% ?
94. If a bill of \$ 600 is payable in 3 months after May 1, without interest, what sum will discharge it June 1, current rate of interest being 10% ?
95. Macomber & Earle sold goods to the amount of \$ 138.48 on 6 months' credit. For how much ready money could they afford to sell the same goods, the use of the money being worth to them 2% a month ?
96. A merchant bought goods to the amount of \$ 1574, one half payable in 3 months and the rest in 6 months, without interest. What sum would pay the debt at the time of purchase, rate 7% ?
97. A dealer bought \$ 1500 worth of grain on 6 months' credit, and sold it immediately for 10% advance. If with the proceeds he paid the present worth of the \$ 1500, rate 8%, what sum remained ?
98. A bookseller bought \$ 240 worth of books at a discount of 33 $\frac{1}{3}$ % on the amount of his bill, and 5% on the balance for present payment. He then sold the books on 3 months' time for the price at which they were billed to him. Money being worth 7%, and the purchaser discounting his own bill by true present worth at the time of purchase, what was the bookseller's gain ?

For other examples in present worth, see page 253.

BANK DISCOUNT.

576. Holding a note against James Peak for \$ 500, due in 4 months from April 1, without interest, and desiring the money on that day, I transfer the note to a bank, and allowing the bank to take interest on the sum named in the note for 4 months, and 3 days (\$ 10.25), receive from the bank the balance (\$ 489.75) in cash. The note is then said to be *discounted*.

NOTE. The sum named in the note is called the face of the note.

577. The interest upon the face of a note from the time it is discounted to the time it is due is *bank discount*.

What is the bank discount in the example given ?

578. The face of a note, less the discount, is the *proceeds, avails, or cash value* of the note.

What are the proceeds in the example given ?

579. The three days for which interest is taken beyond the specified time for paying a note are called *days of grace*.

NOTE I. A note is nominally due at the expiration of the time specified in the note, but it is not legally due till the expiration of the 3 days of grace. A note is said to mature when it is legally due.

NOTE II. The time when a note is nominally and when legally due is usually written with a line between the dates; thus, July 1/3.

NOTE III. When a note is given for months, calendar months are understood, and the note is nominally due on the day corresponding with its date; if the month in which it falls due has no corresponding day it is due on the last day of that month.

NOTE IV. Notes maturing on Sunday or on a legal holiday must be paid on the business day next preceding.

NOTE V. In computing bank discount, the more general custom is to reckon the time in days; hence, in the examples in bank discount which follow, the time is so reckoned, when dates are given.

580. ILLUSTRATIVE EXAMPLE. What is the bank discount of a note for \$400, payable in 90 days, discount at 7%? What are the proceeds?

WRITTEN WORK.

$$\begin{array}{r}
 \$400 \\
 0.0155 \\
 \hline
 6) 6.2000 \\
 1.0333 \\
 \hline
 \$7.2333
 \end{array}$$

$$\$400 - \$7.23 = \$392.77.$$

Explanation.—Bank discount is interest for the specified time and 3 days of grace.

The interest of \$400 for 93 days at 7% is \$7.23, the discount. \$400 less \$7.23 equals \$392.77, the proceeds of the note.

Ans. \$7.23 discount; \$392.77 proceeds.

581. From the above may be derived the following

Rules.

I. To find bank discount on a note due at a future time, without interest: *Compute interest on the face of the note from the time of discount to maturity (including the three days of grace).*

II. To find the proceeds of the note: *Subtract the discount from the face of the note.*

NOTE. When a note drawing interest is discounted, the discount is computed upon the *amount* of the note at the time of its *maturity*.

582. Examples for the Slate.

99. What is the bank discount of a note for \$750, payable in 30 days, discount 6%? What are the avails?

Find the bank discount and proceeds of a note

100. For \$1000, payable in 90 d., discount 7%.

101. For \$300, payable in 4 mo., discount 8%.

102. For \$700, dated Dec. 10, payable in 69 days, and discounted at date at 10%.

103. For \$500, dated Aug. 20, payable in 3 mo., and discounted at date at $7\frac{1}{2}\%$.

Find the bank discount and proceeds of a note

104. For \$290, dated Dec. 30, 1877, payable in 2 mo., and discounted at date at 9%.

105. For \$500, dated May 10, payable in 90 days, and discounted June 9 at 6%.

106. For \$256.84, dated Oct. 28, payable in 60 days, and discounted Nov. 12 at 12%.

107. For \$1200, dated Jan. 31, payable in 3 months, and discounted March 8 at 5%.

108. I bought a horse and carriage for \$324, for which I gave my note Nov. 5, payable in 1 year, with interest at 6%. What would be the avails of this note at a bank, Aug. 1, discount 7%? *

109. Find the bank discount and avails of the following note, discounted Feb. 12, 1876, at 10%.

\$4000.

SAN FRANCISCO, Nov. 7, 1875.

Six months from date, with interest at 10%, I promise to pay F. Eggleston & Co. Four Thousand Dollars; value received.

JAMES NOBLE.

583. ILLUSTRATIVE EXAMPLE. For what sum must a note be drawn, payable in 60 days, without interest, that the avails may equal \$591.60 when the note is discounted at a bank at 8%?

WRITTEN WORK.

Bank discount of \$1 for 63 d. = \$0.014
Avails of \$1 for 63 d. = 0.986
\$591.60 ÷ \$0.986 = 600

Ans. \$600.

Explanation. — The

bank discount of \$1 for 63 days at 8% is \$0.014; hence, the avails of \$1 discounted will be \$1 minus \$0.014, which equals \$0.986. Since the avails of \$1 are \$0.986, that the avails may be \$591.60 the note must be drawn for as many dollars as there are times \$0.986 in \$591.60, which is 600. Ans. \$600.

* See Art. 581, note.

584. From the foregoing may be derived the following

Rule.

To find the face of a note which discounted at a bank will yield given proceeds: *Divide the given proceeds by the proceeds of 1 dollar for the given rate and time, with 3 days of grace.*

NOTE. To find the face of the note when the discount is given: *Divide the given discount by the discount of \$1 for the given rate and time, with 3 days of grace.*

585. Examples for the Slate.

110. For what sum must a 30 days' note, without interest, be drawn that the avails at 6% discount may be \$80?

111. For what must a 4 months' note, without interest, be drawn that when discounted at a bank it may yield \$489.75 at 6% discount?

112. What must be the face of a note given for 90 days, without interest, that the avails at a bank may be \$1469, discount being 8%?

113. What was the face of a note given for 45 days, not bearing interest, on which the discount at 9% was \$11.40?

586. Miscellaneous.

114. What difference does it make in the avails of a note for \$200, payable without interest in 18 months, whether it be reckoned by true or by bank discount, rate 8%?

115. What will be the difference between the true and the bank discount of a note for \$9171, due May 9, 1878, and discounted Jan. 15, 1878, at 6%?

\$500.

RICHMOND, Oct. 5, 1876.

For value received, I promise to pay Charles Towle, or order, Five Hundred Dollars in three months. JAMES ALLEN.

116. What cash must be paid to discharge the above note at its date by true present worth, rate of interest 6%?

117. What would be the avails of it at a bank, Dec. 5, 1876?

118. What would be the amount of it, March 17, 1877 ?
119. What would be the true discount of it, Nov. 5, 1876 ?
120. What would be the bank discount of it, Nov. 5, 1876 ?

For other examples in bank discount, see page 253.

COMMERCIAL DISCOUNT.

587. Business men are usually allowed a deduction for making cash payment for goods purchased on time. Notes also not bearing interest are discounted by the deduction of a certain per cent, not wholly depending upon the time. Such a deduction is called *business* or *commercial discount*.

588. Examples for the Slate.

121. A merchant bought a lot of goods amounting to \$124, on 30 days' credit; 5% discount on the price was allowed for making payment at the time of purchase. What was paid ?

122. A man having bought a bill of goods amounting to \$468.20 on 6 months' time, cashed the bill for 10% off. What did he pay ?

123. What is the cash value of a bill of cloth amounting to \$347.20, on the face of which a discount of 6% is made, and on the balance another of 5% ?

124. What is the difference between discounting a bill of \$1000 at $33\frac{1}{3}\%$ and taking 10% off from the remainder, and discounting the whole bill at $43\frac{1}{3}\%$?

125. A person paid \$1.14 per yard for goods after a discount of 5% had been made upon the invoice price. What was the invoice price ?

NOTE. Since 5% had been deducted, 95% remained.

126. What was the invoice price of a lot of French plate-glass for which I paid \$39 per pane after a discount of 40% had been made ?

127. If from the retail price of a book 20% is deducted, and a discount of 10% is made upon the balance, and then the book sells for \$1.33, what is the retail price ?

COMPOUND INTEREST.

589. A sum of \$ 500 was loaned at 7%, interest payable annually. At the end of the first year the interest for that year was added to the principal, and upon the amount as a new principal the interest was reckoned for the second year. The amount for the second year formed a new principal, upon which interest was reckoned for the next six months, at the end of which time the note, with interest, was paid. What was the amount then due? What was the interest gained?

WRITTEN WORK.

Principal	\$ 500.
Interest for 1st year	<u>35.</u>
Amount, or 2d principal	535.
Interest for 2d year	<u>37.45</u>
Amount, or 3d principal	572.45
Interest for 6 months	<u>20.0357</u>
Amount	\$ 592.49 <i>1st Ans.</i>
1st principal	<u>500.</u>
Interest	\$ 92.49 <i>2d Ans.</i>

590. Interest upon both interest and principal, the sum of the two forming a new principal for specified periods of time, is *compound interest*.

In the example above the interest is compounded annually. It may be compounded semi-annually, or for any period of time agreed upon.

591. From the operation above may be derived the following

Rule.

To compute compound interest:

1. Find the amount of the given principal for the first period of time. With this as a new principal, find the

amount for the second period of time, and so continue for the whole time. The last amount is the amount required.

2. The last amount minus the given principal is the compound interest.

592. Examples for the Slate.

At compound interest, what is the amount

128. Of \$200 for 3 years at 6%?

129. Of \$350.50 for 4 years at 5%?

130. Of \$2000 for 3 years 11 months at 6%?

131. Of \$2000 for 1 y. 6 mo. at 7%, interest compounded semi-annually?

NOTE. Take interest at $3\frac{1}{2}\%$ for three intervals of time.

132. What is the compound interest of \$40 for 1 y. 2 mo. at 6%, interest compounded semi-annually?

133. What is the compound interest of \$900 for 1 y. 1 mo. at 6%, interest compounded quarterly?

593. The work of computing compound interest may be shortened by the use of the following

TABLE,

Showing the amount of \$1 at compound interest from 1 year to 10 years, at 3, 4, $4\frac{1}{2}$, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.	6 per cent.	7 per cent.
1.	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2.	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3.	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4.	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5.	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6.	1.194052	1.265319	1.302260	1.340096	1.418519	1.500730
7.	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8.	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9.	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10.	1.343916	1.480244	1.552969	1.628895	1.790848	1.967151

594. ILLUSTRATIVE EXAMPLE. What is the compound interest of \$1000 for 2 y. 4 mo. at 7%?

WRITTEN WORK.

Amount of \$1 at 7% for 2 years . . .	\$1.1449
	<u>1000</u>
Amount of \$1000 for 2 years . . .	1144.90
	<u>1.02½</u>
Amount of \$1144.90 for 4 mo. }	
Amount of \$1000 for 2 y. 4 mo. }	1171.6143
	<u>1000.</u>
Compound interest	\$171.61 Ans

NOTE. In the above operation, the amount of \$1000 for 2 years is first found, and the amount for the months is then obtained by multiplying by 1.02½. It would be equally well to find the amount of \$1 for the entire time, and then multiply that amount by 1000.

595. Examples for the Slate.

Using the preceding table, find the amount at compound interest

134. Of \$200 for 2 y. 4 mo. at 7%.
135. Of \$580 for 7 y. 10 mo. at 6%.
136. What is the compound interest of \$300 for 3 y. 2 mo. 6 d. at 8%, interest payable semi-annually?
137. What is the compound interest of \$380 for 1 y. 10 mo. 22 d. at 6%, interest payable semi-annually?
138. If at the age of 25 years, a person puts \$1000 on interest, compounding annually at 6%, what will be the amount due him when he is 40 years old?

NOTE. First find by the table the amount for 10 years, then find the amount of that amount for 5 years more.

For additional examples in compound interest, see page 253.

AVERAGE OR EQUATION OF PAYMENTS.

596. ILLUSTRATIVE EXAMPLE. A debtor owes to one person the following sums at the dates specified: Oct. 1, \$262; Oct. 10, \$220; Nov. 6, \$250. At what date may he pay the total of these items without loss of interest to either party?

Interest Method.

WRITTEN WORK.				<i>Explanation.</i> — To do this example, we may suppose all the items to be paid at the earliest date at which any item becomes due, viz. Oct. 1. This would involve a loss to the debtor of interest on \$220 from Oct. 1 to Oct. 10 (9 days), and on \$250 from Oct. 1 to Nov. 6 (36 days).
Due.	Items.	Days.	Interest.	
Oct. 1,	\$ 262	0		
“ 10,	220	9	\$ 0.66	
Nov. 6,	250	36	3.00	
1 day's int. of 732 = 0.244) 3.66 (15				
Oct. 1 + 15 d. = Oct. 16. <i>Ans.</i>				

The interest of \$220 for 9 days at 12%* is \$0.66

“ “ “ 250 “ 36 “ “ 12% is 3.00

Total interest . . . \$3.66

That no loss may result, the total of the items, \$732, should be paid as many days after Oct. 1 as will be required for \$732 at 12% to gain \$3.66 of interest. To find this time, we divide \$3.66 by the interest of \$732 for 1 day at 12% (Art. 562, note), and have for a quotient 15.

15 days after Oct. 1 is Oct. 16. *Ans.* Oct. 16.

597. The process of finding the time when the payment of several items, due at different times, may be made at once, without loss of interest to either party, is *average*, or *equation of payments*.

598. The date at which several sums due at different times may be paid at once is the *average date* or *equated time of payment*.

* Any per cent may be taken, but 12 per cent (1% a month) is taken for convenience, the interest then being for every month 0.01 of the principal, and for every 3 days 0.001 of the principal.

599. From the foregoing operation may be derived

Rule I

To find the average time for the payment of several sums due at different times :

- * 1. *Select some convenient date ; for example, the earliest date at which any item matures.*
2. *Compute the interest on each item from the selected date to the date of its maturity.*
3. *Add the interests thus found ; divide their sum by the interest of the sum of the items for one day ; the quotient will express the number of days from the selected date to the average date of payment.*
4. *Add this number to the selected date ; the result will be the average date required.*

600. The foregoing illustrative example performed by

The Product Method.

WRITTEN WORK.

Days.	Products.
$0 \times 262 =$	00
$9 \times 220 =$	1980
$36 \times 250 =$	9000
	732) 10980 (15

Oct. 1 + 15 d. = Oct. 16. *Ans.*

Explanation. — To do this example by the product method, we select some date, for example the earliest date at which any item becomes due, and suppose all the items to be paid at this date. This would involve a loss to the debtor of interest on \$ 220 for 9 days, and on \$ 250 for 36 days.

The interest on \$ 220 for 9 days equals the interest on \$ 1 for 1980 days ; the interest on \$ 250 for 36 days equals the interest on \$ 1 for 9000 days, which together equals the interest on \$ 1 for 10980 days, but \$ 732 is the sum to be paid, and the time required for the interest on this sum to equal the interest on \$ 1 for 10980 days will be $\frac{1}{732}$ of 10980 days, which is 15 days.

15 days after Oct. 1 is Oct. 16. *Ans.* Oct. 16.

601. From the preceding operation may be derived

Rule II

To find the average date for the payment of several sums due at different dates :

1. *Select some convenient date ; for example, the earliest date at which any item matures.*
2. *Multiply the time each item has to run by the number of dollars in the item.*
3. *Divide the sum of the products thus obtained by the number of dollars in the sum of the items ; the quotient will express the time from the selected date to the average date of payment.*
4. *Add this time to the selected date ; the result will be the average date required.*

602. Proof.

Find the sum of the interests on all items due BEFORE the average date, from the date at which they are severally due to the average date ; also find the sum of the interests on all items due AFTER the average date from that date to the dates at which they are severally due. If these sums are equal, or differ by less than half a day's interest on the sum of all the items, the result is correct.

NOTE I. The examples in this book are performed by the interest method, which has the advantage of brevity when the accountant uses interest tables. The pupil will perform the work by either or by both methods, as directed by the teacher.

NOTE II. Any date may be selected from which to average an account. The last day of the month previous to the earliest day at which any item becomes due is a convenient date.

NOTE III. When any item contains cents, if less than 50, disregard them, if 50 or more, increase the units of dollars by \$ 1.

NOTE IV. When a quotient contains a fraction of a day, if less than $\frac{1}{2}$, disregard it; if $\frac{1}{2}$ or more, call it 1 day.

603. Examples.

139. What is the average date for paying three bills due as follows: March 31, \$400; April 30, \$300; May 30, \$200?

140. What is the average date of maturity of three notes of \$800 each, due respectively Nov. 5, Dec. 8, and Feb. 3?

141. What is the average date of maturity of the following items of account, viz., \$900 due Sept. 10; \$2250.48 due Oct. 21; and \$1049.65 due Oct. 28?

142. Find the equated time for paying \$430 due in 5 months; \$270 due in 9 months; and \$300 due in 8 months?

143. Average the above, having the first item due in 3 months, the others in 9 months each.

144. A gentleman purchased a farm for \$3600, agreeing to pay \$600 down, and the remainder in five equal semi-annual instalments. At what time may the whole be paid at once?

145. When shall a note to settle the following account be made payable?

J. R. INGERSOL

To R. FISH & CO., Dr.

1876.		\$
April 10	To Mdse on 30 days' credit.....	200
May 16	" " " 60 " "	300
June 3	" " " 30 " "	520
July 18	" Cash	250

NOTE. First find at what time each item falls due by adding the time of credit to the date of the item.

146. What is the equated date of maturity of the following?

V. M. HURON

To COLTON IRON CO., Dr.

1876.		\$
Mar. 11	To Mdse on 30 days' credit.....	438
" 29	" " " 30 " "	254
Feb. 29	" " " 60 " "	145
May 8	" " " 60 " "	300
June 12	" " " 30 " "	159

AVERAGE OF ACCOUNTS.

NOTE. Younger pupils may omit this subject.

604. ILLUSTRATIVE EXAMPLE. What is the average date of maturity of the following account?

Dr. PHILIP ARCHER in Acct. with E. GRANGER.				Cr.			
1877.		\$		1877.		\$	
Mar. 18	To Mdse	250		Apr. 1	To Cash.....	700	
" 30	" "	600		" 20	" Real Estate	300	

WRITTEN WORK.

Dr.				Cr.			
Due.	Items.	Days.	Interest.	Due.	Items.	Days.	Interest.
1877.				1877.			
March 18.	\$ 250			April 1.	\$ 700	14	\$ 3.27
" 30.	600	12	\$ 2.40	" 20.	300	33	3.30
	<u>850</u>		<u>2.40</u>		<u>1000</u>		<u>6.57</u>
					850		2.40
					1 day's int. of 150 = 0.05	0.05	4.17
							<u>83</u>

March 18 + 83 d. = June 9. Ans. June 9, 1877.

Explanation. — The payment of all these items at the earliest date, March 18, would involve a loss, at 1% a month, to the debtor of \$2.40 of interest, and to the creditor of \$6.57, or \$4.17 more to the creditor than to the debtor.

Now, as the balance of the account, \$150, is due from the creditor, he may, to avoid loss of interest, defer payment of the balance as many days after March 18 as will be required for \$150, at 1% a month, to gain \$4.17 of interest, which is 83 days. 83 days after March 18 is June 9. Ans. June 9, 1877.

In this case it will be seen that the balance of the account and of the interest are on the *same* side of the account.

605. Suppose, on the contrary, the first item in the foregoing account to be \$500, instead of \$250, what would then be the average date of its maturity?

Explanation. — In this case the loss of interest to both debtor and creditor is the same as before, but the balance of the account, \$100,

is due from the *debtor*, who, to cancel the excess of interest lost by the creditor, should pay the balance of the account as many days before March 18 as will be required for \$100, the balance, to gain \$4.17 of interest, which is 125 days. 125 days *before* March 18, 1877, is Nov. 13, 1876. *Ans.* Nov. 13, 1876.

In this case the balance of the account and of the interest are on *opposite* sides of the account.

606. From the above illustrations we derive the following

Rule.

To find the average or equated time for the settlement of an account when there are both debit and credit items :

1. *Find the interest on the several items of the account from the earliest date at which any item becomes due to their several maturities.*

2. *Find the balance of interest of the debit and credit sides of the account, also the balance of the items.*

3. *Divide the balance of interest by the interest of the balance of the items for one day. The quotient will be the time in days between the selected date and the average time of settlement.*

4. *Count this time FORWARD from the selected date, when the balance of the account and of the interest are on the same side of the account, and BACK when on opposite sides. The result will be the date of settlement.*

NOTE I. When settlement takes place after the equated time of payment, interest on the balance is charged ; when before the equated time, discount is allowed.

NOTE II. The balance due on an account at any day selected for settlement may be, and usually is, found without averaging the account, by computing the interest of the items on each side of the account from their several dates of maturity to the day of settlement. The interests so found on each side of the account are then added to that side. If any item matures *after* the day of settlement, the *discount* is computed and added to the *opposite* side of the account (which is equivalent to subtracting it from the side on which it occurs). When the two sides of the account have been so increased, their difference is the balance due.

607. Examples.

147. At what date can the balance of the following ledger account be paid without loss to either party?

Dr.		EDWIN C. CASTLETON.				Cr.	
1877.		\$	¢	1877.		\$	¢
April 1	To Mdse...	1000	00	April 14	By Mdse.....	1393	00
July 8	" Cash...	118	98	Aug. 10	" Real Estate	94	33

148. What is the average date of maturity for the following account?

Dr.		RUDE, ALDRICH, & CO.				Cr.	
1876.		\$	¢	1876.		\$	¢
Apr. 5	To Sundries on 2 mo. ...	400	00	June 1	Mdse on 60 d.	250	00
Aug. 5	" Mdse " 1 " ...	600	00	July 8	Mdse " 30 d.	700	00
" 15	" Mdse " 1 " ...	200	00	Aug. 13	Cash.....	200	00

149. Find the average date of maturity of the following account:

Dr.		EARL INGALLS.				Cr.	
1878.		\$	¢	1878.		\$	¢
Jan. 6	To Mdse. on 30 d.	600	00	Jan. 1	[on 90 d. By Real Estate	500	00
Feb. 7	" " " 60 "	420	00	Mar. 16	" Cash	300	00

150. Average the following:

Dr.		CHARLES RAYMOND.				Cr.	
1876.		\$	¢	1876.		\$	¢
Aug. 20	To Mdse, 60 d.	173	15	Aug. 25	By Mdse, 30 d.	500	00
Oct. 14	" Cash	314	68	Sept. 12	" Mdse, 30 d.	102	10
" 18	" Cash	230	00				
" 30	" Mdse, 1 mo.	81	25				

151. Average the following account:

Dr.		WILLIAM SMITH.				Cr.	
1877.		\$	¢	1877.		\$	¢
Jan. 6	To Mdse, 3 mo.	339	92	Jan. 1	By Bal. of acct....	361	20
" 25	" Mdse, 30 d.	582	20	" 15	" Real Estate, }	4000	00
Feb. 21	" Mdse, 3 mo.	85	12		3 mo.		
May 29	" Mdse, 2 "	2200	00	Feb. 7	" Mdse, 2 mo.	580	00

SECTION XVI.

EXCHANGE.

608. To avoid the risk and expense of sending money to make payments in distant places, merchants and others make use of drafts or bills of exchange. What these are, and how they are used, will best be shown by an example.

Suppose that J. G. Ames, in Boston, wishes to pay \$200 to William Smith, in New Orleans. He may pay the money to a banker, James A. Dupee, in Boston, who will write an order on his correspondent, George Flint, a banker in New Orleans, in the following form :

<div style="border: 1px solid black; padding: 5px; width: 100px;"> \$ 200. </div>	<div style="text-align: right;"> <i>Boston, July 18, 1877.</i> </div>
<div style="border: 1px solid black; padding: 5px; width: 100px;"> <i>Thirty days after sight,</i> <i>or order, Two Hundred</i> <i>same to the account of</i> </div>	<div style="text-align: right;"> <i>pay to William Smith,</i> <i>Dollars, and place the</i> <i>Your obedient servant,</i> <i>James A. Dupee.</i> </div>
<div style="border: 1px solid black; padding: 5px; width: 100px;"> <i>To George Flint, Esq.,</i> <i>New Orleans.</i> </div>	<div style="text-align: right;"> <i>Accepted Aug. 4, 77.</i> <i>George Flint.</i> </div>

609. Such a written order for the payment of money is a **draft**, or **bill of exchange**. The method of making payments by drafts or bills of exchange is **exchange**.

Ames will take this draft and send it to Smith, who, when he receives it, will present it to Flint for acceptance. If Flint is willing to obey the order and pay the money, he writes the word "Accepted" across the face of the draft, adds the date, and signs his name. In due time Smith gets the money from Flint, gives up the draft, and the transaction is complete.

610. The person who makes and signs a draft is the *drawer*. The person to whom it is addressed is the *drawee*. The drawee when he accepts the draft becomes the *acceptor*. The person to whom the draft is payable is the *payee*.

In the case described above, who is the drawer? the drawee? the acceptor? the payee?

611. If the payee wishes to transfer the draft to another person, he writes his own name across the back of the paper; this is called an *indorsement*, and the payee then becomes an *indorser*. The person to whom the draft is so transferred is an *indorsee*. If the indorsee wishes to transfer the draft to a third person, he also writes his name under that of the former indorser. He thus becomes a second indorser; and there may be a third indorser, a fourth, and so on indefinitely.

612. The person who holds the draft at any time (the payee or the last indorsee) is called the *holder*.

The holder looks for payment first to the acceptor, and then to the indorsers in their order. Each indorser is liable to pay the draft when the acceptor and previous indorsers have failed to do so. To avoid becoming liable, an indorser may write over his name the words "Without recourse."

Drafts may be "at sight" or "on time"; bankers charge less for the latter than for the former, the difference in price being equivalent to a discount for the given time.

When our exports to another country, England for example, exceed in value our imports from that country, more money is due to us from the English merchants than is due to them from our merchants. The larger sum due us in England will make it easy for us to buy bills of exchange on England. They will be plenty here, and the price of them will fall. If they can be bought for less than their face, they are at a *discount*, or *below par*.

On the contrary, when the value of the goods imported from England exceeds the value of those sent to England, more money is due

to the English merchants from us than is due to us from them. The smaller sums due us in England will then make it difficult for us to buy bills of exchange on England, and the price of them will rise. If they cost more than their face, they are at a *premium*, or *above par*.

613. Bills of exchange are either *foreign bills* or *inland bills*. Foreign bills are those which are drawn or are payable in a foreign country ; and for this purpose each of the United States is foreign to the others. Inland bills are drawn and payable in the same State.

614. Examples for the Slate.

1. What is the cost in Philadelphia of a draft on San Francisco for \$800 at 1% premium?

2. What is the cost of a draft on Detroit for \$2500 at $\frac{1}{2}$ % premium?

3. What is the cost of a draft on New York for \$700 at 12 days after sight, premium $\frac{1}{2}$ %?

4. What is the cost of a sixty days' draft on New York for \$2000 at 2% discount?

5. I bought a bill on Chicago for \$700 at a discount of $\frac{1}{2}$ %. What did I pay?

615. ILLUSTRATIVE EXAMPLE. What is the face of a draft on New York bought in St. Louis for \$8820, when the discount is 2%?

WRITTEN WORK.

$$\$1 - 2\% \text{ of } \$1 = \$0.98, \text{ cost of } \$1.$$

$$\$8820 \div \$0.98 = 9000. \quad \text{Ans. } \$9000.$$

6. What is the face of a draft that may be bought for \$500 at a discount of $1\frac{1}{2}$ %?

7. A merchant in New York bought a draft on Cincinnati at $\frac{1}{2}$ % premium for \$275. What was the face of the draft?

Exchange with Europe.

616. Exchange with Europe is effected chiefly through large business centres, as London, Paris, Hamburg, etc.

In computing foreign exchange, it is necessary to change the values expressed in the currency of one country to equivalent values expressed in the currency of another country.

On page 311 of the Appendix will be found a list of the monetary units of foreign countries, with their values in United States money, as proclaimed by the Secretary of the Treasury, Jan. 1, 1878; also on page 312, tables of English, French, and German money.

The rates of exchange between this country and the principal business centres are given from day to day in the newspapers. The following is an extract showing the exchange value of the pound sterling in United States money; the number of francs and centimes which equal a dollar; and the exchange value of 4 marks in cents:

"We quote bankers' 60-day bills on London at \$4.84 @ 4.84½, and short-sight bills at \$4.86, both in gold. On Paris, francs 5.15 per dollar for short sight, and 5.18½ for 60-day bills. Gossler & Co.'s rates on Hamburg for 60-day bills are 95, and short-sight bills 95½."

In making bills on foreign countries, it is customary to write two or more of the same tenor and date, the payment of either one of which cancels the other one or two. And to provide against accident in their transmission, it is customary to send two, at least, of a set, at different times, or by different modes of conveyance.

617. Examples for the Slate.

8. What was the cost of the following bill in U. S. money, the rate of exchange being \$4.86?

<p>£500.</p> <p><i>At sight of this first of Exchange, second and third unpaid, pay to Brown, Smith, & Co., or order, Five Hundred Pounds sterling, value received, and charge the same to the account of</i></p> <p>To Messrs. McCalmont Bros. & Co., 3 Crown Court, London.</p>	<p><i>Boston, Aug. 23, 1876.</i></p> <p><i>Hidder, Peabody & Co.</i></p>
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9. T. Van Horn, of New York, bought of R. J. Birney & Co. a set of exchange payable at sight for £1000 sterling on Brown, Shipley, & Co., of Liverpool, at \$4.84. What was the cost in gold?

10. What is the cost in gold of a set of exchange on Paris for 5500 francs, exchange being 5.15 per dollar?

11. What is the cost of the above-named sum in currency, gold being quoted at 102½?

12. What is the cost of a draft on Hamburg for 200 marks when the quotation is 95?

13. What is the cost in New Orleans of a bill on London for £75 10s, when exchange is there quoted at \$4.85½? (See Appendix, page 312.)

14. When exchange is \$4.86, what is the face of a bill on London which can be bought for \$9720?

15. What is the face of a draft on London which can be bought for \$1938.42, the rate of exchange being \$4.84?

United States Bonds.

618. Governments and corporations sometimes borrow money, giving, as evidence of the loans, certificates or notes payable at or within some definite time, with interest at stated periods. Such certificates or notes are called *bonds*.

619. Bonds sometimes have certificates attached, promising the holder certain sums of interest as they become due upon the bonds. These interest certificates are called *coupons*.

NOTE I. When the interest is paid, the coupons are cut off by the holder and given up as receipts.

NOTE II. The extraordinary expenses of the government of the United States during the civil war were met in part by the sale of bonds.

620. United States coupon bonds are issued in the denominations of \$50, \$100, \$500, and \$1000. Registered

bonds are issued in the same denominations, and also in denominations of \$ 5000 and \$ 10000.

Bonds are usually named according to the rate of interest they bear.

621. The following is a list of the more important United States bonds not redeemed in 1878 :

Names of Bonds.	Redeemable.	Payable.	Rate of Int.	Int. payable in gold.
6's of 1881.....	1881.....	1881	6 %	Semi-annually.
5-20's.....	{ In 5 years from } { date of issue... }	In 20 years	6 %	“
10-40's	1874.....	1904	5 %	“
5's of 1881.....	1881.....	1881	5 %	Quarterly.
4½'s.....	1891.....	1891	4½ %	“
4's.....	1907.....	1907	4 %	“

Bonds are bought and sold as other stocks. Their prices from day to day are quoted in the newspapers.

622. *The rules of percentage already illustrated apply to bonds.*

623. Examples for the Slate.

16. When U. S. 5-20's are sold at $108\frac{1}{2}$, what is received for eight \$ 500 bonds ?

17. When U. S. 5's are worth $106\frac{1}{2}$, what will \$ 850 in bonds cost ?

18. What amount in bonds shall I receive for \$ 2675 invested in U. S. 5's at 107 ?

19. What shall I pay a broker for a \$ 1000 U. S. 5-20 bond at $110\frac{1}{2}$, and two \$ 1000 U. S. 6's of 1881 at $113\frac{1}{2}$, with his brokerage of $\frac{1}{4}$ % ?

20. How much money must I remit to a broker that he may purchase for me three U. S. 10-40 bonds of \$ 1000 each, the bonds selling at 109, and his commission being $\frac{1}{4}$ % ?

21. When gold is at $102\frac{3}{4}$, what is my semi-annual income in currency from 12 U. S. 6's of \$1000 each?

NOTE. First find the semi-annual interest in gold.

22. If the premium on gold is 3%, what per cent do I make semi-annually in currency on a U. S. 5-20 bond purchased at 106?

23. Which yields the greater per cent semi-annually, and how much, U. S. 6's at 110, gold at $102\frac{1}{2}$, or a mortgage on real estate paying $3\frac{1}{2}\%$ semi-annually?

24. How much money must be invested in U. S. $4\frac{1}{2}$'s to yield a quarterly income of \$225 in gold, bonds selling at $105\frac{1}{4}$, gold at par?

624. General Review, No. 5.

25. By losing 3 cents a pound, I lose $12\frac{1}{2}\%$ of the cost of butter. If I had lost 4 cents a pound, what % should I have lost?

26. What is the simple interest of \$300 from May 5, 1876, to Feb. 2, 1878, at $1\frac{1}{2}\%$ a month?

27. I hold a note for \$500, which bore interest at 7% from May 10, 1875. Sept. 16, 1875, received \$140; July 28, 1877, received \$50. What remained due Sept. 3, 1877?

28. If I pay \$45 interest for the use of \$500 for 3 years, what is the rate per cent?

29. How long must \$204 be on interest at 7 per cent to amount to \$217.09?

30. What principal will gain \$9.20 in 4 mo. 18 d., at 4 per cent?

31. What sum, at 7 per cent, will amount to \$221.075 in 3 yrs. 4 mo.?

32. At 6%, what is the compound interest of \$600 for 1 yr. 4 mo., interest payable semi-annually?

33. What is the present worth of a note for \$488.50, due in 2 yrs. 5 mo. 15 d., at 9 per cent?

34. What is the true discount of \$105.71, due 4 yrs. hence, rate 6%?

35. What is the bank discount of \$450 for 30 days and grace at 5%?

36. What are the avails of a note of \$100 due in 27 days, and discounted at a bank at 6%?

37. For what must a 60-days' note be given, which, discounted at a bank at 6%, will yield \$1295?

38. A debtor owes \$200, $\frac{1}{2}$ due in 2 months, $\frac{1}{4}$ in 3 months, and the remainder in 5 months. What is the equated time for paying the whole?

39. A man about to travel in England bought a bill for 250 pounds sterling. Exchange being \$4.85 $\frac{1}{2}$ in gold, and gold being quoted at 103, what amount of currency did he pay for the bill?

625. Miscellaneous Examples.

40. What is 124% of 5 T. 300 lbs.?

41. What is the amount at 6%, simple interest, of \$38.75, from Aug. 5 to Nov. 10?

42. What is the amount of \$380.25, at 6% compound interest, for 2 yrs. 5 mo.?

43. How much ought a broker to charge me for 5 shares of stock purchased for me at 7% advance, shares having originally been \$500, his brokerage at $\frac{1}{2}$ % included?

44. What will be the length and breadth of a piece of cloth, originally 2 $\frac{1}{4}$ yards long by 1 yard wide, after sponging, if in that operation it shrinks 4% in length and 6% in width?

45. A commission merchant receives \$544; of this he is to invest such a portion as remains after deducting his commission of 2 $\frac{3}{4}$ % on the investment. What is his commission, and what will remain?

46. What is the cost of insuring \$2500 at \$17.50 on \$1000?

47. If $\frac{1}{3}$ of a sum of money be due in 2 months, $\frac{1}{4}$ in 4 months, $\frac{1}{8}$ in 3 months, and the remainder in 4 months, at what time might the whole be paid without loss to the debtor?

48. A dealer has 18 barrels of sound apples remaining in a lot of which 10% have decayed. If his lot cost him \$1.50 per

bbl., would he gain or lose on the lot, and what %, by selling the remainder at \$1.75 per bbl. ?

49. What will be the net loss to an insurance company in case of the loss by fire of a property insured for \$4500, on which the company had received 3% premium, no allowance for interest ?

50. What must be paid for a policy to cover \$2575 at a premium of $1\frac{1}{4}\%$?

51. What per cent of 1 bushel is 1 peck 2 quarts ?

52. In what time will a sum of money double at 2% simple interest ?

53. A person lent a certain sum for 1 yr. 6 mo. at 5%. The interest being \$9.30, what was the sum ?

54. What principal will amount to \$63.25 in 1 yr. 3 mo. at 8% ?

55. A merchant imports from Hamburg a bale of cloth, containing 12 pieces of 40 yards each; the cloth, with charges there, cost him \$480; he pays a duty here of 35 cts. per yd., freight \$28.50, and other charges \$7.11. At what must he sell the cloth per yd. to gain 10% above all charges ?

56. In the year 1872 the town of B voted to raise, by taxes, \$97290; $\frac{1}{4}$ of this was levied upon the polls; the valuation of the town was \$10134375. What was the tax on \$1, and what was the tax of a non-resident who owned a house in town valued at \$2000 ?

57. What must be the face of a note, which, discounted at a bank for 30 days and grace, would yield \$500 ?

58. On a note for \$2500, dated Sept. 5, 1875, were paid \$50 January 29, 1876, and \$500 July 1, 1877. The note being on interest at 6% from its date, what was due Sept. 5, 1877 ?

59. Paid \$18.77 for insuring my schooner at a premium of $\frac{1}{2}\%$. What was the sum covered ?

60. What is the par value of stock, which, selling at 25% above par, brings \$500 ?

\$ 150.25.

CHICAGO, Jan. 5, 1876.

On the fifteenth of May, 1876, I promise to pay to the order of B. F. Archer, One Hundred and Fifty $1\frac{5}{10}$ Dollars; value received.

D. D. CORWIN.

61. If the holder of the above note has it discounted at a bank Feb. 15, 1876, at 6%, what will he receive?

62. What is the true present worth of the above note at its date, rate 7%?

63. If the note above was unpaid when due, and drew interest at 6% from the time it became due, what would settle it Oct. 27, 1876?

64. Find the amount of the above note at compound interest at 5% from the time it became due till Oct. 27, 1881?

626. Questions for Review.

What is PERCENTAGE? What is the base? the amount? the remainder? rate per cent? Express $\frac{3}{4}$ and its complement decimally. How do you change a fraction to a per cent? a per cent to its lowest terms? How do you find a percentage of a number? the amount? the remainder? the base? the rate per cent? Give the formula for finding the percentage; the amount; remainder; base; rate per cent. Upon what is the percentage of PROFIT or LOSS reckoned? If goods cost 24 cents, for what must they be sold to gain $8\frac{1}{2}\%$? to lose $16\frac{2}{3}\%$? What per cent would be gained or lost by selling goods that cost 24 cents for 30 cents? for 21 cents? If 24 cents is 20% less than the value of goods, what is the value? If 24 cents is $33\frac{1}{3}\%$ more than the value of goods, what is the value? If 18 cents is 10% less than cost, for what would you sell goods to gain 10%? to lose 25%? If 10% of what you receive for goods is gain, what is your gain per cent?

What is COMMISSION? Who is the factor? the consignor? the consignee? What is meant by net proceeds?

What is a company? a corporation? capital STOCK? a certificate of stock? Who are stockholders? What is an assessment? a DIVIDEND? a stockbroker? BROKERAGE? When are stocks above par? below par? Upon what is the per cent of commission or brokerage estimated? How do you find what sum is to be expended when a remittance contains that sum together with the commission?

What is **INSURANCE**? a policy? a premium? Who are underwriters? What is expectation of life?

What is a **TAX**? a poll tax? real estate? personal property? Who are assessors? How do you find the tax to be assessed on a dollar in any town?

What are **CUSTOMS** or **DUTIES**? What is a specific duty? an *ad valorem* duty? gross weight? net weight? In estimating specific duties, what allowances are made?

What is **INTEREST**? what is the principal? the amount? What is meant by the *rate*, in interest? What is legal rate? usury? simple interest? Give your method of computing interest. How do you find accurate interest?

What is a promissory note? What is the face of a note? What are **PARTIAL PAYMENTS**? Where is the record of payments made? What is the United States rule for partial payments? What is the merchant's rule?

What three factors are used to find interest? The interest, principal, and rate being known, how do you find the time? The interest, principal, and time being known, how do you find the rate? The interest, rate, and time being known, how do you find the principal? What is the dividend in each case? The amount, rate, and time being known, how do you find the principal?

What is the **PRESENT WORTH** of a debt? What is discount? Give a rule for finding present worth. How do you find discount? How can you prove the work?

What is **BANK DISCOUNT**? What are days of grace? avails of a note?

Which is the larger, true or bank present worth? true or bank discount? Describe the process of getting a note discounted at a bank. What is indorsing a note? How do you find the face of a note, which, discounted at a bank, will yield a certain sum?

What is **COMPOUND INTEREST**? How often may interest be compounded? For how many periods of time will interest be compounded in 2 y. 9 mo., if it is compounded semi-annually? quarterly?

How do you find the average time for paying several bills due at different times?

What is **EXCHANGE**? what are drafts or bills of exchange? Who are the parties to a draft? who is the holder? When are drafts at a premium? at a discount? Where can you find rates of exchange?

What are **BONDS**? What is a coupon?

627. DRILL TABLE No. 8.

Examples.	A Principal.	B Interest.	C Time.	D Per	E cent.
1.	\$640.08	\$16.305	1 y. 6 m. 24 d.	5	$\frac{1}{3}$
2.	\$305.40	\$28.14	1 y. 2 m. 6 d.	8	$\frac{2}{3}$
3.	\$90.508	\$17.083	4 y. 11 m.	3	$\frac{2}{3}$
4.	\$705.38	\$78.90	3 y. 7 m. 27 d.	2	$\frac{2}{3}$
5.	\$4000.	\$100.	2 y. 3 m. 20 d.	7	$\frac{1}{2}$
6.	\$240.08	\$50.40	4 y. 9 m. 5 d.	4	$\frac{1}{2}$
7.	\$9.034	\$15.08	7 y. 5 m. 18 d.	11	$\frac{3}{8}$
8.	\$80.50	\$7.005	2 y. 11 m. 26 d.	1	$\frac{7}{8}$
9.	\$3050.	\$430.20	3 y. 10 m. 3 d.	10	$\frac{5}{8}$
10.	\$560.08	\$6.095	17 d.	9	$\frac{1}{8}$
11.	\$150.20	\$30.75	1 y. 3 m.	12	$\frac{1}{8}$
12.	\$5400.	\$175.60	1 y. 4 m. 25 d.	3	$\frac{3}{8}$
13.	\$690.40	\$290.14	5 y. 21 d.	2	$\frac{3}{8}$ of 1
14.	\$60.75	\$5.872	1 y. 9 d.	9	$\frac{1}{8}$ of 1
15.	\$850.06	\$25.642	10 m. 13 d.	5	$\frac{3}{8}$ of 1
16.	\$6.508	\$11.75	5 y. 7 m. 2 d.	11	$\frac{1}{8}$ of 1
17.	\$700.01	\$10.90	2 m. 28 d.	10	$\frac{1}{8}$ of 1
18.	\$38.20	\$3.956	2 y. 8 m. 19 d.	1	$\frac{3}{8}$ of 1
19.	\$590.04	\$105.20	3 m. 16 d.	4	0.4
20.	\$11.80	\$5.769	4 y. 8 m. 2 d.	8	0.1
21.	\$809.06	\$340.50	2 y. 15 d.	7	0.5
22.	\$654.09	\$75.80	4 y. 4 d.	12	0.0 $\frac{1}{2}$
23.	\$10000.	\$500.	4 m. 14 d.	50	0.06
24.	\$3600.	\$1640.	1 d.	100	0.08
25.	\$908.70	\$64.37	1 y. 7 m. 15 d.	7	0.3

628. Exercises upon the Table.

- | | |
|---|--|
| <p>216. Find D per cent of A.</p> <p>217. Find E per cent of A.</p> <p>218. Find $D + E$ per cent of B.</p> <p>219. A is D per cent of what sum?</p> <p>220. A is E per cent of what sum?</p> <p>221.* B is what per cent of A?</p> <p>222. Find the commission for collecting or investing A at $(D - E)\%$.</p> <p>223. If A includes both the commission and sum to be invested, what is the commission at D%?</p> <p>224. If A includes both the commission and sum to be invested, what is the sum to be invested, the commission being D%?</p> <p>225. Find the date, which is C years, months, and days after Nov. 27, 1871.</p> <p>226. Find the interest of \$1 at 6% for the time in C.</p> <p>227. Find the interest of \$1 at 1% for the time in C.</p> <p>228. Find the interest of \$1 at D% for the time in C.</p> <p>229. Find the interest of \$1 at E% for the time in C.</p> <p>230. Find the interest of \$1 at $(D + E)\%$ for the time in C.</p> <p>231. Find the interest of A at D% for the time in C.</p> <p>232. Find the interest of A at $(D + E)\%$ for the time in C.</p> <p>233. Find the amount of A at 6% for the time in C.</p> | <p>234. Find the compound interest of A at D% for 2 y. 9 mo. 18 d.</p> <p>235. Find the compound interest of A at D% for 1 y. and the months and days in C, interest payable semiannually.</p> <p>236. Find the amount of A at compound interest for 2 y. 6 mo. 15 d. at 6%.</p> <p>237. Find the rate, A, B, C being given.
(Let the fraction of the per cent be changed to tenths, and the answer be expressed thus: 8.3% +.)</p> <p>238. Find the time, A, B, and $(D + E)$ being given.</p> <p>239. Find the principal, B, C, D being given.</p> <p>240. Find the principal, A being the amount, C the time, and 6% the rate.</p> <p>241. Find the present worth of A, due in the time in C, at D%.</p> <p>242. Find the discount on A, due in the time in C, at D%.</p> <p>243. Find the discount on A, due in the time in C, at 6%.</p> <p>244. Find the bank discount on a note for A, payable in the months and days in C, at D%.</p> <p>245. Find the avails of a note for A, payable in the months and days in C, at D%.</p> <p>246. Find the face of a note, which, being discounted at a bank at 6% for the months and days in C, will yield A.</p> |
|---|--|

* See note after Exercise 237.

SECTION XVII.

RATIO AND PROPORTION.

SIMPLE RATIO.

629. Ten equals how many 2's. *Ans.* Five 2's.

In the above answer we express the relation of 10 to 2 by their quotient. The relation of two numbers expressed by their quotient is *ratio*.

630. Oral Exercises.

- a. What is the ratio of 8 to 2? of 2 to 8? of 9 to 3?
- b. What is the ratio of 6 to 2? of $\frac{6}{2}$ to $\frac{2}{2}$? of $\frac{3}{2}$ to $\frac{6}{2}$?
- c. What is the ratio of 5 to 2? of 0.5 to 0.2? of 2 lb. to 7 lb.?

631. The ratio of 10 to 2 is indicated thus, 10:2. The expression is read, "The ratio of ten to two."

- d. Indicate the ratio of 7 to 9; of 8 days to 15 days.
- e. Read the following expressions: 12:15; \$4:\$18.

632. The numbers whose ratio is to be found are the *terms* of the ratio. The two terms of a ratio form a *couplet*. The first term of a couplet is the *antecedent*; the second term is the *consequent*.

NOTE. The terms of a ratio must be numbers of the same denomination.

633. As the antecedent of a ratio is the dividend and the consequent the divisor, it follows that

When the antecedent is multiplied or
the consequent is divided, } the ratio is multiplied.

When the antecedent is divided or the
consequent is multiplied, } the ratio is divided.

When both terms of a ratio are multi-
plied or divided by the same number, } the value of the ratio is
not changed.

634. Examples for the Slate.

Find the ratios of the following couplets:

- (1.) 16 : 256. (4.) 45 : 990. (7.) \$ 9.00 : \$ 12.50.
 (2.) $8\frac{1}{2}$: 300. (5.) 28 : 910. (8.) \$ 0.87 $\frac{1}{2}$: \$ 0.12 $\frac{1}{2}$.
 (3.) 19 : 110 $\frac{1}{2}$. (6.) $6\frac{1}{2}$: 75. (9.) 100 lb. : 16 $\frac{3}{4}$ lb.

635. The ratio of two numbers is a *simple ratio*. A simple ratio has one antecedent and one consequent.

COMPOUND RATIO.

636. Find the ratio of 2 to 5, and of 3 to 4; and then find the product of these ratios. *Ans.* $\frac{2}{5}$ and $\frac{3}{4}$; product $\frac{6}{20}$.

The product of two or more simple ratios is a *compound ratio*.

637. The compound ratio given above is indicated thus:

$$\begin{array}{l} 2 : 5 \\ 3 : 4 \end{array} \left. \vphantom{\begin{array}{l} 2 : 5 \\ 3 : 4 \end{array}} \right\} \begin{array}{l} \text{The expression is read,} \\ \text{"The compound ratio of 2 to 5 and 3 to 4."} \end{array}$$

638. From Art. 636 it will be seen that when several general numbers form a compound ratio, *the value of the ratio may be found by dividing the product of the antecedents by the product of the consequents*.

639. Oral Exercises.

Find the value of the compound ratios indicated by each of the following expressions:

- a. $\begin{array}{l} 5 : 8 \\ 4 : 9 \end{array} \left. \vphantom{\begin{array}{l} 5 : 8 \\ 4 : 9 \end{array}} \right\} = ?$ c. $\begin{array}{l} 3 : 7 \\ 4 : 12 \end{array} \left. \vphantom{\begin{array}{l} 3 : 7 \\ 4 : 12 \end{array}} \right\} = ?$
 b. $\begin{array}{l} 8 : 1 \\ 7 : 4 \end{array} \left. \vphantom{\begin{array}{l} 8 : 1 \\ 7 : 4 \end{array}} \right\} = ?$ d. $\begin{array}{l} 7 \text{ men} : 5 \text{ men} \\ \$ 10.00 : \$ 8.00 \end{array} \left. \vphantom{\begin{array}{l} 7 \text{ men} : 5 \text{ men} \\ \$ 10.00 : \$ 8.00 \end{array}} \right\} = ?$

NOTE. The ratio of numbers is the same whether the numbers are denominate or general; hence, in finding the value of the ratio in the last example, the terms may be regarded as general numbers.

PROPORTION.

640. What is the ratio of 3 ft. to 6 ft. ? of \$5 to \$10 ?
These ratios are equal to each other.

An equality of ratios is a *proportion*.

641. The equality of the above-named ratios is expressed thus, 3 ft. : 6 ft. = \$5 : \$10.

This expression is read, "3 ft. is to 6 ft. as \$5 is to \$10."

642. Exercises.

Read the following:

a. $5 : 7 = 15 : 21$.

c. $40 : 10 = 15 \text{ min.} : 3\frac{1}{2} \text{ min.}$

b. $\frac{1}{2} : 3 = \$7 : \105 .

d. $9 : 6 = 6 : 4$.

643. The first and fourth terms of a proportion are the *extremes*, and the second and third are the *means*.

NOTE I. In Example *d* above, 6 is the consequent of the first couplet and the antecedent of the second; and so 6 is a *mean proportional* between 9 and 4.

NOTE II. Four quantities are *directly proportional* when the first is to the second as the third is to the fourth. Four quantities are *inversely proportional* when the first is to the second as the fourth is to the third; or when one ratio is direct and the other inverse. Thus, the *amount of work* done in any given time is *directly* proportional to the number of men employed; that is, the more men, the more work: but the *time* occupied in doing a certain work is *inversely* proportional to the number of men employed; that is, the more men, the less time.

To supply a Missing Term of a Proportion.

644. ILLUSTRATIVE EXAMPLE. Supply the missing term denoted by x in the proportion, $x : 5 = 4 : 10$.

WRITTEN WORK.

$$x : 5 = 4 : 10$$

$$\frac{x}{5} = \frac{4}{10}$$

$$\frac{x \times 10}{5 \times 10} = \frac{4 \times 5}{10 \times 5}$$

$$x \times 10 = 4 \times 5$$

$$\frac{4 \times 5}{10} = 2 \left\{ \begin{array}{l} \text{Missing} \\ \text{term.} \end{array} \right.$$

$$2 : 5 = 4 : 10$$

Explanation. — The ratios of the two couplets are $\frac{x}{5}$ and $\frac{4}{10}$; these changed to fractions having a common denominator are $\frac{x \times 10}{5 \times 10}$ and $\frac{4 \times 5}{10 \times 5}$.

As these fractions are equal, and their denominators the same, their numerators must be equal. But one numerator is the product of the means of the proportion, and the other

the product of the extremes. Therefore the missing extreme may be found by dividing the product of the means (4×5) by the given extreme (10). The missing term then is 2, and the proportion is $2 : 5 = 4 : 10$.

645. From the preceding illustration may be derived the following principles :

1. When four general numbers form a proportion, *the product of the means is equal to the product of the extremes.*
2. *A missing extreme may be found by dividing the product of the means by the given extreme.*
3. *A missing mean may be found by dividing the product of the extremes by the given mean.*

646. Oral Exercises.

Supply the missing terms represented by x in the following proportions :

a. $3 : 4 = 9 : x$.

d. $x : 7 = 8 : 9$.

b. $8 : 6 = x : 3$.

e. $27 : 3 = x : 1$.

c. $12 : x = 15 : 3$.

f. $x : 4 \text{ days} = \$5 : \15 .

NOTE. To find x in Example *f*, disregard the denominations, and proceed as if the terms were general numbers. $\frac{4 \times 5}{15} = 1\frac{1}{3}$. Then x equals $1\frac{1}{3}$ days.

647. Examples for the Slate.

Supply the missing terms in the following :

(10.) $2 : 100 = 17 : x$. (13.) $750 \text{ A.} : 3 \text{ A.} = x : 13 \text{ tons.}$

(11.) $9 : 150 = 105 : x$. (14.) $x : 200 \text{ hats} = \$87.50 : \500 .

(12.) $65 : x = \$75 : \850 . (15.) $\$800 : \$56 = \$390 : x$.

ANALYSIS AND PROPORTION.

648. ILLUSTRATIVE EXAMPLE. If 14 slates cost 98 cents, what will 10 slates cost ?

By Analysis.

WRITTEN WORK.

$$\begin{array}{r} 7 \\ 98 \times 10 \\ \hline 14 \\ \text{Ans. } 70 \text{ cents.} \end{array}$$

Explanation. — If 14 slates cost 98 cents, 1 slate will cost 1 fourteenth of 98 cents, and 10 slates will cost 10 times 1 fourteenth of 98 cents, which is 70 cents. *Ans.* 70 cents.

By Proportion.**WRITTEN WORK.**

$$14 : 10 = 98 : x.$$

$$\begin{array}{r} 7 \\ 98 \times 10 \\ \hline 14 \end{array} = 70.$$

Ans. 70 cents.

Explanation. — The ratio of 14 slates to 10 slates must be the same as the ratio of 98 cents, the cost of 14 slates, to the cost of 10 slates.

We may arrange the terms in any order which will express the equality of these ratios. For convenience, we make 98 cents the third term, and x , the unknown cost of 10 slates, the fourth

term. As the cost of 10 slates will be less than 98 cents, we make 10 the second term and 14 the first. Multiplying 98 by 10 and dividing the product by 14, we have for the fourth or missing term, 70.

Ans. 70 cents.

649. Rule.

To solve examples by simple proportion :

1. *Make the number that is of the same denomination as the required answer the third term.*
2. *Determine from the statement of the example whether the answer is to be greater or less than the third term.*
3. *Make the other two numbers in the example the first and second terms of the proportion, taking the greater number for the second term if the answer is to be greater than the third term, and the less number for the second term if the answer is to be less than the third term.*
4. *Multiply the third term by the second term, and divide the product by the first term.*

650. Examples for the Slate.

The following examples may be solved by analysis or by proportion, or by both methods, at the option of the teacher.

16. If 4 yards of velvet cost \$ 20, what will 14 yards cost ?
17. If 12 bushels of wheat cost \$ 8, what will 30 bushels cost ?
18. What will 250 sheep cost if 24 sheep cost \$ 72 ?
19. What will 75 pounds of cheese cost if 64 pounds cost \$ 6.08 ?

20. How many feet of plank will be required for a bridge 528 feet long, if 17280 feet of plank are required for 288 feet?

21. If 500 bushels of plaster were sufficient for the dressing of $3\frac{1}{2}$ acres of land, what would be required for $17\frac{1}{2}$ acres of the same kind of soil?

22. If a building 13 ft. high casts a shadow of 4 ft., what length of shadow will a church spire $346\frac{1}{4}$ ft. high cast at the same time?

23. If crackers can be sold at 10 cents a pound when flour is worth \$6.50 a barrel, for what can they be sold when flour is worth \$9.75 a barrel, the cost of making not being considered?

24. If a hind wheel, which is $8\frac{3}{4}$ feet in circumference, turns 800 times in a journey, how many times will the fore wheel, which is $6\frac{1}{2}$ feet in circumference, turn in the same journey?

25. If 400 bushels of potatoes were bought for \$350.90, and sold for \$425.50, what was gained on 25 bushels?

26. If a 10-cent loaf weighs 1 lb. 2 oz. when flour is worth \$7 $\frac{1}{2}$ per bbl., what should it weigh when flour is \$6 per bbl.?

27. If my friend lends me \$7000 for 15 days, for what time should I lend him \$4500 to requite the favor?

28. If my friend lends me money for 4 months when interest is 10 per cent, for what time should I lend him the same sum to requite the favor when interest is 7 per cent?

29. If 2 lbs. 5 oz. of wool make 1 yd. of cloth 32 inches wide, how much will make a yard of the same quality $1\frac{1}{4}$ yards wide?

30. How many yards of cambric 34 inches wide will be required to line $14\frac{1}{2}$ yards of silk which is 22 inches wide?

31. If 400 lbs. of coal are required to run an engine 12 hours, what number of tons will be required to run three similar engines for 30 days, day and night?

32. A deer, 150 rods before a hound, runs 30 rods a minute; the hound follows at the rate of 42 rods a minute. In what time will the deer be overtaken?

COMPOUND PROPORTION.

651. A *compound proportion* is a proportion in which one of the ratios is compound.

652. ILLUSTRATIVE EXAMPLE. If it takes a man 5 days of 9 hours each to earn \$15, how many days of 8 hours each will it take him to earn \$20?

By Analysis.

WRITTEN WORK.

$$\begin{array}{r} 5 \quad 3 \\ \$ \times 20 \times 9 \\ \hline 15 \times 8 \\ \$ \quad 2 \end{array} = 7\frac{1}{2}.$$

Ans. $7\frac{1}{2}$ days.

Explanation. — If it takes a man 5 days to earn \$15, it will take him 1 fifteenth of 5 days to earn \$1, and 20 times that to earn \$20. If it takes him this number of days when the days are 9 hours long, it will take him 9 times as many days when they are 1 hour long, and 1 eighth of that number when they are 8 hours long, which is $7\frac{1}{2}$ days. Ans. $7\frac{1}{2}$ days.

By Compound Proportion.

WRITTEN WORK.

$$\left. \begin{array}{l} 1 \quad 1 \\ \$ \quad 4 \\ 15 : 20 \\ \$: 9 \\ 2 \quad 3 \end{array} \right\} = 5 \text{ days} : x.$$

$$\frac{5 \times 3}{2} = 7\frac{1}{2}.$$

Ans. $7\frac{1}{2}$ days.

Explanation. — The number of days it will take depends, *first*, on the amount of money to be earned, and, *secondly*, on the number of hours a day the man works. We might get the answer by using two simple proportions. In the first we could find the number of days, so far as it depends on the amount of money to be earned; and then, taking this result as the third term of another proportion, we could find the number of days so far as it depends on the number of hours in a day's work. It will be more convenient, however, to combine the two proportions, thus forming a compound proportion.

To do this we make 5 days, which is a number of the same denomination as the required answer, the third term, and then consider the statements of the example in order.

(1.) As \$20 is a larger sum than \$15, it will take a larger number of days to earn it; that is, the answer, so far as it depends on the

amount of money to be earned, will be larger than the third term ; so we make 20 the second term and 15 the first term of the first ratio.

(2.) As it will take more days 8 hours long to earn this money than days 9 hours long, the answer, so far as it depends on the length of the days, will be larger than the third term ; so we make 9 the second term and 8 the first term of the second ratio.

We now have the compound proportion,

$$\left. \begin{array}{l} 15 : 20 \\ 8 : 9 \end{array} \right\} = 5 \text{ days} : x \text{ days.}$$

Multiplying 5 days by 20×9 , and dividing the product by 15×8 , gives $7\frac{1}{2}$ days. *Ans.* $7\frac{1}{2}$ days.

The work may be shortened, as shown in the written work, by cancelling.

653. Rule.

To solve examples by compound proportion :

1. *Make the number that is of the same denomination as the answer the third term.*

2. *Take the two numbers in each separate statement in the example, and consider whether the answer, so far as it depends on them alone, will be greater or less than the third term. Arrange these two numbers accordingly as terms of a ratio.*

3. *Multiply the third term by the product of the second terms and divide this product by the product of the first terms.*

654. Examples for the Slate.

33. If \$90 is paid for the work of 20 men 6 days, what should be paid for the work of 5 men 8 days ?

34. If in 84 days 75 men can earn \$68.75, in how many days can 90 men earn \$41.25 ?

35. If it costs \$30 to paint the outside of a house 40 ft. by 30 ft. and 25 ft. high, what will it cost to paint the outside of a house 50 ft. by 40 ft., and 20 ft. high ?

36. If 450 pounds of merchandise can be carried 26 miles for 30¢, how many miles can 3 tons be carried for \$4 ?

37. If 85 tons of coal are required to run 6 engines 17 hours a day for a certain number of days, how many tons will be required to run 25 engines 12 hours a day for the same number of days?

38. If 500 lbs. of wool worth 42¢ a pound are given for 75 yds. of cloth $1\frac{1}{2}$ yds. wide, how much wool worth 36¢ a pound should be given for 27 yds. that is $1\frac{1}{2}$ yds. wide?

39. If it costs \$135.00 to carry 855 pounds 64 miles, what will it cost to carry 1288 pounds $15\frac{1}{2}$ miles?

40. If 200 rods of wall can be built by 25 men in $9\frac{1}{2}$ days of 10 hours each, how many rods can be built by 12 men in 1 day of 12 hours?

41. If it costs a certain family \$700 a year to live in Brownville, and the cost of living is twice as great in Chicago as in Brownville, what will it cost to live 8 months in Chicago?

42. How many men will be required, working 12 hours a day for 250 days, to dig a ditch 750 ft. long, 4 ft. wide, and 3 ft. deep, if it requires 27 men, working 13 hours a day for 62 days, to dig a ditch 403 ft. long, 3 ft. wide, and 3 ft. deep?

43. If 9 men working 12 hours a day for 7 days can make 7 cases of boots, in how many days of 11 hours each can 3 men and 4 boys (one boy's work being equal to $\frac{3}{4}$ of the work of a man) make 33 cases of the same kind of boots?

44. Wishing to find the number of bricks in a wall 6 rods long, 4 feet high, and 13 inches thick, I found that a part of the wall $6\frac{1}{2}$ feet long, 2 feet high, and 13 inches thick, contained 330 bricks. How many bricks did the whole wall contain?

45. Wishing to find the weight of a block of marble 5 feet long, 2 feet wide, and $1\frac{1}{2}$ feet thick, I weighed a smaller block 6 inches long, 4 inches wide, and 2 inches thick, and found it weighed 4 lb. 5 oz. What was the weight of the larger block?

46. The weight of a cubical block of granite measuring 2 feet on each edge is 1352 pounds. What is the weight of a cubical block measuring 4 feet on each edge?

PARTNERSHIP.

655. ILLUSTRATIVE EXAMPLE. A and B associated themselves together in business for one year. A invested \$500 and B \$700, agreeing to share their gains or losses in proportion to their investments. They gained \$2700. What was each person's share?

WRITTEN WORK.

$$\frac{\$2700 \times 5}{12} = \$1125. \quad A's \text{ gain.}$$

$$\frac{\$2700 \times 7}{12} = \$1575. \quad B's \text{ gain.}$$

Explanation. — The whole investment was \$1200, of which A put in $\frac{5}{12}$ and B $\frac{7}{12}$. A should then have $\frac{5}{12}$ of \$2700, or \$1125, and B should have $\frac{7}{12}$ of \$2700, or \$1575.

Ans. A's, \$1125; B's, \$1575.

Each person's share of the gain can be found by proportion, thus :

$$\$1200 : \$500 = \$2700 : x. \quad x = \$1125 \text{ } A's \text{ gain.}$$

$$\$1200 : \$700 = \$2700 : x. \quad x = \$1575 \text{ } B's \text{ gain.}$$

656. *The gains or losses of a partnership are shared according to the agreement or contract of the partners.*

657. In the following examples, when no agreement or contract is mentioned, *divide the gains or losses in proportion to the capital invested by each partner and the time it is employed.*

NOTE. Some simple examples in partnership have already been given. See page 113, Example *m*, and page 114, Example 196.

658. Examples for the Slate.

47. Two men, A and B, formed a partnership, A putting in \$5000, and B \$3000. They gained \$3000. What was the share of each?

48. Blood and Searle shipped coal from Philadelphia to New York. Blood had on board 450 tons, and Searle 900 tons. It became necessary to throw overboard 250 tons. What was the loss to each person?

49. A bankrupt owed to M \$ 900, to N \$ 350, and to O and P \$ 500 each; his whole property was sold for \$1584.80, of which \$158.48 was used to pay the expenses of the sale. What was each person's share of the remainder?

50. Of a store valued at \$ 90000, A owned $\frac{1}{4}$, B owned $\frac{1}{3}$, and C the rest. The store was insured for $\frac{3}{4}$ of its value, and was entirely consumed by fire. What was the loss to each owner?

51. Divide \$ 1500 among three persons so that their shares shall be in the proportion of 3, 4, and 5.

52. Hinds, Bascom, and Ladd traded in company. Hinds put in \$2500 for 10 months, Bascom \$2300 for 11 months, and Ladd conducted the business, which was considered equal to \$2000 in trade for 12 months. They gained \$1486. What should each receive?

53. X and Y received \$857.50 for grading a road. X furnished 5 hands for 20 days, and 6 others for 15 days; Y furnished 10 hands for 12 days, and 9 others for 20 days. What was the share of each contractor?

54. Rand and Parker engaged in trade. Rand had in trade \$1000 from January 1 till April 1, when he withdrew \$550; July 1 he added \$700. Parker had in trade \$3000 from Feb. 1 to Oct. 1, when he added \$300; Nov. 1 he withdrew \$900. The net gain during the year was \$3500. What was the share of each?

659. Questions for Review.

What is a **RATIO**? Name the terms of a ratio. What is a simple ratio? a compound ratio?

What is a **PROPORTION**? Which are the means of a proportion? the extremes? What is a mean proportional between two numbers? When are four quantities inversely proportional? When four general quantities form a proportion, what two products are equal? How do you find a missing extreme? a missing mean? How do you solve examples by simple proportion? When is compound proportion used?

How are the gains or losses of a **PARTNERSHIP** shared?

SECTION XVIII.

POWERS AND ROOTS.

INVOLUTION.

660. Name some products made by using 3's only as factors. *Ans.* 9, which equals 3×3 ; 27, which equals $3 \times 3 \times 3$; 81, which equals $3 \times 3 \times 3 \times 3$.

A product made by using only equal factors is a *power*.

661. A power made by using two equal factors is a *second power*. A power made by using three equal factors is a *third power*. A power made by using four equal factors is a *fourth power*; and so on.

What is the second power of 2? the third? the fourth? the fifth?

662. The process of forming powers is *involution*.

NOTE. The process of forming any power of a number is sometimes called *raising* the number to that power; the process of forming the second power is called *squaring* the number; the process of forming the third power is called *cubing* the number.

663. The second power of 3 is indicated thus, 3^2 ; the expression is read, "The second power of three." The third power of 3 is indicated thus, 3^3 ; the expression is read, "The third power of three"; and so on.

NOTE. The names *square* and *cube* are often used for "second power" and "third power," because the contents of a square is found by raising to the second power the number of units in one of its sides, and the contents of a cube by raising to the third power the number of units in one of its edges.

664. The small figure above and at the right, which shows to what power a number is raised, is the *index* or *exponent* of the power.

665. Oral Exercises.

- a. Name the squares of the numbers from 1 to 10 inclusive.
- b. What is the cube of 1? of 2? of 3? of 4? of 5?
- c. What is the fourth power of 2? of 3? the fifth power of 2?
- d. What is the square of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of 0.5?
- e. What is the cube of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of 0.2?

666. Examples for the Slate.

1. Find and commit to memory the cubes of the integers from 1 to 10.

Find the powers indicated below.

- | | | | |
|---------------------------|---------------------------|---------------------------|-----------------------------|
| (2.) 17^2 . | (6.) $(1\frac{1}{2})^3$. | (10.) 11^4 . | (14.) $(16\frac{1}{2})^2$. |
| (3.) 28^2 . | (7.) 10.2^2 . | (11.) $(\frac{2}{3})^4$. | (15.) 2.4^2 . |
| (4.) $(\frac{4}{15})^2$. | (8.) 15^3 . | (12.) 0.5^3 . | (16.) 0.24^2 . |
| (5.) 0.17^2 . | (9.) 0.12^3 . | (13.) 1.9^2 . | (17.) 0.16^3 . |

EVOLUTION.

- 667.** Name one of the two equal factors of 4; of 9; of 25; of 36; of 64; of 81.

Name one of the three equal factors of 8; of 27; of 125; of 216.

One of the equal factors which produce a number is the *root* of that number.

668. One of the *two* equal factors of a number is its *second* or *square* root. One of the *three* equal factors of a number is its *third* or *cube* root. One of the *four* equal factors of a number is its *fourth* root, and so on.

669. The process of finding the root of a number is *evolution*.

NOTE. The process of finding the root of a number is sometimes called *extracting the root*.

670. The second or square root of 4 is indicated thus, $\sqrt{4}$; the third or cube root of 8 is indicated thus, $\sqrt[3]{8}$; the fourth root of 16, thus, $\sqrt[4]{16}$; and so on. These expressions are read, "The square root of four"; "The cube root of eight"; "The fourth root of sixteen."

The symbol $\sqrt{}$ is called the *radical sign*. The small figure at the left of the radical sign is called the *index* of the root.

The root of a number is also indicated by a fractional exponent. Thus, $4^{\frac{1}{2}}$ means the same as $\sqrt{4}$; $8^{\frac{1}{3}}$ means the same as $\sqrt[3]{8}$.

SQUARE ROOT.

671. Oral Exercises.

- a. What is one of the two equal factors of 25? of 49?
- b. What is the square root of 100? of 400? of 10000? of 1?
- c. What is the square root of $\frac{1}{4}$? of $\frac{1}{9}$? of $\frac{4}{9}$? of $\frac{9}{4}$?
- d. What is the square root of 0.01? of 0.09? of 0.81?
- e. $\sqrt{121} = ?$ g. $\sqrt{144} = ?$ i. $\sqrt{\frac{1}{9}} = ?$ k. $\sqrt{12\frac{1}{4}} = ?$
- f. $\sqrt{1.21} = ?$ h. $\sqrt{1.44} = ?$ j. $\sqrt{1\frac{1}{9}} = ?$ l. $\sqrt{30\frac{1}{4}} = ?$

To find the Number of Terms in the Square Root of a given Number.

672. By squaring 1 and 9, 10 and 99, 100 and 999, etc., we obtain the following results:

$$\begin{array}{lll} 1^2 = 1. & 10^2 = 100. & 100^2 = 10000. \\ 9^2 = 81. & 99^2 = 9801. & 999^2 = 998001. \end{array} \quad \text{etc.}$$

These results show that when a number has one term its square is expressed by one or two figures; when a number has two terms the square is expressed by three or four figures; when a number has three terms the square is expressed by five or six figures, and so on. Hence,

673. *If a numerical expression be separated into periods of two figures each, beginning with the units' figure, the number of periods will be the same as the number of terms in the square root.*

To find the Parts of a Second Power.

674. To find what parts a second power is made up of, we may take a number consisting of tens and units, 36 for example, and raise it to the second power.

$$\begin{array}{rcl}
 36 = & 30 + 6 = & 30 + 6 \\
 36 = & 30 + 6 = & 30 + 6 \\
 \hline
 216 & 180 + 36 & 30 \times 6 + 6^2 \\
 108 & 900 + 180 & 30^2 + 30 \times 6 \\
 \hline
 1296 = & 900 + 360 + 36 = & 30^2 + 2 \times (30 \times 6) + 6^2
 \end{array}$$

The written work above shows the partial products obtained by multiplying each term of the multiplicand by each term of the multiplier.

From this we see that the second power of a number consisting of tens and units is made up of three parts, —

- (1.) *The square of the tens.*
- (2.) *Twice the product of the tens by the units.*
- (3.) *The square of the units.*

These parts may be expressed by the formula,

$$\text{Tens}^2 + 2 (\text{tens} \times \text{units}) + \text{units}^2.$$

675. ILLUSTRATIVE EXAMPLE I. What is the square root of 1296? *

WRITTEN WORK.

$$\text{Tens}^2 + 2 (\text{tens} \times \text{units}) + \text{units}^2.$$

$$\begin{array}{rcl}
 & & 12'96 \text{ (36)} \\
 (3 \text{ tens})^2 = & & 9 \\
 3 \text{ tens} \times 2 = 6 \text{ tens) } & & 39 \\
 6 \text{ tens} \times 6 = & & 36 \\
 & & \hline
 & & 36 \\
 6^2 = & & 36 \\
 & & \hline
 & & 0
 \end{array}$$

Explanation. — We first find the number of terms in the root by separating the expression 1296 into periods of two figures each, beginning with the units' figure. The square root of this number will consist of tens and units. We know that this number must have in it the square of the tens of the root, plus twice the product of the tens by the units, plus the square of the units.

* For another method of finding square roots, see Appendix, page 312.

As the first part of the power, the *square of the tens*, is hundreds, the 12 hundreds of the given number must have in it the square of the tens of the root.

The greatest square in 12 (hundreds) is 9 (hundreds), the square root of which is 3 (tens). This we write as the first term, or tens, of the root.

Taking the square of 3 (tens), or 9 (hundreds), out of 12 (hundreds), there remain 3 (hundreds).

As the second part of the power, *twice the product of the tens by the units*, is tens, we unite the 9 tens of the given number with the 3 hundreds remaining, and have 39 tens.

This 39 tens has in it a product of which *twice the tens* of the root is one factor, and the *units* of the root the other factor. Dividing the 39 tens by twice 3 tens, 6 tens, we find 6 (units) to be the other factor, which we write as the second term or units of the root.

Multiplying the 6 (tens) by 6 (units), and taking the product 36 (tens) out of 39 (tens), we have 3 (tens) left.

As the third part of the power, *the square of the units*, is units, we unite the 6 units of the given number with the 3 tens remaining, and have 36 units. This 36 units has in it the square of the units of the root. Subtracting the square of 6 units, or 36, from 36, nothing remains.

Ans. 36.

676. ILLUSTRATIVE EXAMPLE II. What is the square root of 1159.4025?

WRITTEN WORK.

Formula, Tens² + 2 (tens × units) + units².

$$\begin{array}{r}
 11'59.40'25 \text{ (34.05)} \\
 \underline{9} \\
 3 \times 2 = 6) \ 25 \\
 6 \times 4 = \underline{24} \\
 19 \\
 4^2 = \underline{16} \\
 34 \times \frac{2}{2} = 68) \ 3402 \\
 680 \times 5 = \underline{3400} \\
 25 \\
 5^2 = \underline{25}
 \end{array}$$

Explanation.—To extract the square root of the integral part of the number 1159.4025, we proceed as in Illustrative Example I. Having found this part of the root, we consider it as tens in reference to the next term, double it for a new divisor, form a new dividend, and proceed as before.

So, whatever the number of terms in the root, having found a part of them, we consider the part found to

be the tens, double it for a new divisor, and proceed as before.

677. Rule.

To extract the square root of a number :

1. *Beginning with the units' figure, point off the expression into periods of two figures each.*
2. *Find the greatest square in the number expressed by the left-hand period, and write its square root as the first term of the root.*
3. *Subtract this square from the part of the number used, and with the remainder unite the next term of the given number for a dividend.*
4. *Double the part of the root already found for a divisor, and by this divide the dividend, writing the quotient as the next term of the root.**
5. *Multiply the divisor by this term, and subtract the product from the dividend. With the remainder unite the next term of the given number and subtract from the number thus formed the square of the term of the root last found.*
6. *If there are more terms of the root to be found, unite with the remainder the next term of the given number, take for a divisor double the part of the root now found, and proceed as before.*

NOTE I. *When, as in the work of Illustrative Example II., the divisor is not contained in the dividend, place a zero as the next figure in the root, place also a zero at the right of the divisor, and for a new dividend unite the next two terms of the given number with the previous dividend.*

NOTE II. *When there is a remainder after all the terms of the given number have been used, annex zeros to the remainder, and continue the work as far as desired.*

NOTE III. *The square root of a common fraction may be obtained by extracting the roots of both numerator and denominator, when they are perfect squares. When they are not perfect squares, first change the fraction to a decimal, and then extract its square root.*

* If this quotient should prove to be too large, make it less and repeat the work.

678. Examples for the Slate.

Roots of numbers not perfect squares may be found to thousandths.

Find the square root

- | | | |
|-----------------|-----------------|-------------------|
| 18. Of 18769. | 21. Of 66.7489. | 24. Of 54802.81. |
| 19. Of 811801. | 22. Of 55.2049. | 25. Of 5.004169. |
| 20. Of 8779.69. | 23. Of 0.1849. | 26. Of 3656.6209. |

Find the square root

- | | | | |
|--------------------------|-------------------------|--------------|---|
| 27. Of $\frac{4}{3}$. | 31. Of $\frac{1}{2}$. | 35. Of 37. | 39. $\sqrt{0.144} = ?$ |
| 28. Of $\frac{1}{100}$. | 32. Of $\frac{3}{4}$. | 36. Of 5.39. | 40. $\sqrt{\frac{1}{16} \times 40} = ?$ |
| 29. Of $\frac{1}{16}$. | 33. Of $7\frac{1}{2}$. | 37. Of 0.78. | 41. $\sqrt{272.25} = ?$ |
| 30. Of $\frac{7}{16}$. | 34. Of $8\frac{1}{2}$. | 38. Of 2. | 42. $\sqrt{4 \times 268} = ?$ |

679. Applications.

43. What is the length of one side of a square which contains 8836 square feet?

44. A body of troops consisting of 2401 men has an equal number in rank and file. How many are there in each?

45. Find the side of a square that will contain as much surface as a rectangle 280 feet long and 70 feet wide.

46. Find the mean proportional between 42 and 168.

NOTE. The mean proportional between two numbers is the square root of their product. (See p. 256, Note I.)

47. Find the mean proportional between 56 and 224.

48. How many rods of fence will be required to enclose a square lot of 3 acres?

49. On one side of a roof there are laid 5000 slates, the number in the length being twice the number in the breadth. What is the number each way?

50. A and B each own a 10-acre lot. A's lot is square, and B's is twice as long as it is wide. How much greater length of fence will B require to enclose his lot than A to enclose his?

51. There is a rectangular court paved with 1728 paving-stones each 15 inches square; the length of the court is to the width as 4 to 3. What is the length and width of the court?

52. A rectangular block of granite is 8 ft. high, square at the base, and contains 162 cubic feet. What is the length of one side of the base?

CUBE ROOT.

680. Oral Exercises.

a. What is one of the three equal factors of 8? of 27? of 64? of 125? of 216? of 1000? of 1728?

b. What is the cube root of 0.001? of 0.008? of 0.027? of 0.216? of 1.728?

c. $\sqrt[3]{\frac{1}{8}} = ?$ $\sqrt[3]{\frac{1}{64}} = ?$ $\sqrt[3]{\frac{1}{216}} = ?$ $\sqrt[3]{\frac{1}{1000}} = ?$ $\sqrt[3]{\frac{1}{1728}} = ?$ $\sqrt[3]{3\frac{3}{8}} = ?$

To find the Number of Terms in the Cube Root of a given Number.

681. By cubing 1 and 9, 10 and 99, 100 and 999, etc., we obtain the following results:

$$\begin{array}{lll} 1^3 = 1 & 10^3 = 1000 & 100^3 = 1000000 \\ 9^3 = 729 & 99^3 = 970299 & 999^3 = 997002999 \end{array}$$

These results show that when a number has one term, its cube is expressed by one, two, or three figures; when a number has two terms, its cube is expressed by four, five, or six figures; when a number has three terms, its cube is expressed by seven, eight, or nine figures, and so on. Hence,

682. *If a numerical expression be separated into periods of three figures each, beginning with the units' figure, the number of periods will be the same as the number of terms in the cube root.*

To find the Parts of a Third Power.

683. To find what parts a third power is made up of, we may take a number consisting of tens and units, 36, for example, and raise it to the third power.

$$\begin{array}{rcl} 36^3 = 1296 = & & 30^3 + 2 \times (30 \times 6) + 6^3 \\ & & \quad \quad \quad 30 + 6 \\ & & \quad \quad \quad \hline & & 7776 \\ & & \quad \quad \quad (30^2 \times 6) + 2 \times (30 \times 6^2) + 6^3 \\ & & \quad \quad \quad \hline & & 3888 \\ & & \quad \quad \quad 30^3 + 2 \times (30^2 \times 6) + \quad (30 \times 6^2) \\ & & \quad \quad \quad \hline 36^3 = 46656 = & & 30^3 + 3 \times (30^2 \times 6) + 3 \times (30 \times 6^2) + 6^3 \end{array}$$

The foregoing written work shows the partial products obtained by multiplying each term of the square of 36 by each term of 36.

From this we see that the third power of a number consisting of tens and units is made up of four parts :

- (1.) *The cube of the tens.*
- (2.) *Three times the product of the square of the tens by the units.*
- (3.) *Three times the product of the tens by the square of the units.*
- (4.) *The cube of the units.*

These parts may be expressed by the formula,

$$\text{Tens}^3 + 3 (\text{tens}^2 \times \text{units}) + 3 (\text{tens} \times \text{units}^2) + \text{units}^3.$$

684. ILLUSTRATIVE EXAMPLE I. What is the cube root of 262144 ? *

WRITTEN WORK.

$$\text{Tens}^3 + 3 (\text{tens}^2 \times \text{units}) + 3 (\text{tens} \times \text{units}^2) + \text{units}^3.$$

	262'144 (64
	216
(6 tens) ³ =	216
(6 tens) ² × 3 = 108 hund.)	461
108 hund. × 4 =	432
	294
6 tens × 4 ² × 3 =	288
	64
4 ³ =	64
	—

Explanation. — We first find the number of terms in the root by separating the expression 262144 into periods of three figures each, beginning with the units' figure. The cube root of this number will consist of tens and units. We know that this number must have in it the cube of the tens of the root, plus three times the product of

the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.

As the first part of the power, the cube of tens, is thousands, we find the greatest cube contained in 262 (thousands), which is 216 (thousands), and write its cube root 6 (tens) as the first term, or tens, of the root.

Taking the cube of 6 (tens), 216 (thousands), out of 262 (thousands), there remain 46 (thousands).

Now, as the second part of the power, three times the product of the square of the tens by the units, is hundreds, we unite the 1 hundred of the given number with the 46 thousands remaining, and have 461 hundreds.

* For another method of finding cube roots, see Appendix, page 313.

This 461 hundreds has in it a product of which *three times the square of the tens of the root* is one factor, and the *units* of the root is the other factor. Dividing 461 (hundreds) by three times the square of the tens, or 108 (hundreds), we find 4 (units) to be the other factor, which we write as the second term, or units of the root.

Multiplying 108 (hundreds) by 4 (units) and taking the product 432 (hundreds) out of 461 (hundreds), we have 29 (hundreds) left.

Now, as the third part of the power, *three times the product of the tens by the square of the units*, is tens, we unite 4 (tens) of the given number with the 29 (hundreds) remaining, and have 294 (tens).

We take out of this number three times the product of the tens by the square of the units, 288 (tens), and have 6 (tens) left.

Now, as the fourth part of the power, *the cube of the units*, is units, we unite the 4 units of the given number with the 6 tens remaining, and have 64 units.

We take the cube of the 4 units out of this number, and nothing remains. *Ans.* 64.

685. ILLUSTRATIVE EXAMPLE II. What is the cube root of 126732.947167?

WRITTEN WORK.

Formula,

$$\text{Tens}^3 + 3 (\text{tens}^2 \times \text{units}) + 3 (\text{tens} \times \text{units}^2) + \text{units}^3.$$

	126'732.947'167 (50.23
5 ³ =	125
50 ² × 3 = ⁷⁵ 7500	17329
7500 × 2 =	15000
	23294
50 × 2 ² × 3 =	600
	226947
2 ³ =	8
502 ² × 3 = 756012)	2269391
756012 × 3 =	2268036
	13556
502 × 3 ² × 3 =	13554
	27
3 ³ =	27

Explanation.—To extract the cube root of the integral part of the number 126732.947167, we proceed as in Illustrative Example I. Having found this part of the root, we consider it as tens in reference to the next term, take three times its square for a new divisor, form a new dividend, and proceed as before.

So, whatever the number of terms in the root, having found a part of them, we consider that part to be the tens, take three times its square for a new divisor, and proceed as before.

686. Rule.

To extract the cube root of a number :

1. *Beginning with the units' figure, point off the expression into periods of three figures each.*

2. *Find the greatest cube in the number expressed by the left-hand period, and write its cube root as the first term of the root.*

3. *Subtract this cube from the part of the number used, and with the remainder unite the next term of the given number for a dividend.*

4. *Take three times the square of the part of the root already found for a divisor, and by this divide the dividend, writing the quotient as the next term of the root.**

5. *Multiply the divisor by this term, and subtract the product from the dividend. With the remainder unite the next term of the given number.*

6. *Subtract from the number thus formed three times the product of the first term of the root by the square of the second. With this remainder unite the next term of the given number.*

7. *Subtract from the number thus formed the cube of the second term of the root.*

8. *If there are more terms of the root to be found, unite with the remainder the next term of the given number, take for a divisor three times the square of the part of the root now found, and proceed as before.*

NOTE I. *When, as in the work of Illustrative Example II., the divisor is not contained in the dividend, place a zero as the next figure in the root ; also place two zeros at the right of the divisor, and for a new dividend unite the next three terms of the given number with the previous dividend.*

* If this quotient should prove to be too large, make it less and repeat the work.

NOTE II. The cube root of a common fraction whose numerator and denominator are both perfect cubes may be found by taking the cube root of the numerator and of the denominator.

But when the numerator or the denominator of the fraction is not a perfect cube, change the fraction to a decimal and then extract its cube root.

NOTE III. When there is a remainder after all the terms of the given number have been used, annex zeros, and continue the operation as far as desired.

687. Examples for the Slate.

Roots of numbers not perfect cubes may be found to the fourth term.

What is the cube root

- | | | |
|------------------|---------------------|----------------|
| 53. Of 12167 ? | 57. Of 2924207 ? | 61. Of 832 ? |
| 54. Of 97336 ? | 58. Of 14172.488 ? | 62. Of 346 ? |
| 55. Of 405.224 ? | 59. Of 82881.856 ? | 63. Of 0.353 ? |
| 56. Of 941192 ? | 60. Of 817.400375 ? | 64. Of 0.80 ? |

What is the cube root

- | | | |
|--------------------------|--------------------------|---------------------------|
| 65. Of $1\frac{2}{3}$? | 68. Of $1\frac{2}{3}$? | 71. $\sqrt[3]{0.27} = ?$ |
| 66. Of $3\frac{2}{3}$? | 69. Of 0.87 ? | 72. $\sqrt[3]{0.918} = ?$ |
| 67. Of $42\frac{1}{2}$? | 70. Of $63\frac{1}{2}$? | 73. $\sqrt[3]{2} = ?$ |

688. Applications.

74. What is the length of one edge of a cube which contains 36594368 cubic inches ?

75. What is the length of a cubical pile of wood which contains 2 cords ?

76. What is the depth of a cubical cistern which contains 300 gallons ?

77. What is the depth of a cubical bin which will hold 65 bushels of grain ?

78. What will be the cost of lead, at $12\frac{1}{2}$ cents per lb., $1\frac{1}{2}$ lbs. to the square foot, to line a cubical box containing $15\frac{1}{8}$ cubic feet ?

689. Exercises upon Drill Table No. 2 (page 60).

247. Extract the square root of the numbers expressed in column B.
 248. Extract the cube root of the numbers expressed in column E.

SECTION XIX.

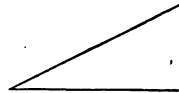
MENSURATION.

TRIANGLES.

690. A plane figure bounded by three straight lines is a *triangle*.

691. A triangle having a right angle in it is a *right triangle*.

Fig. 1.



Right Triangle.

Fig. 2.



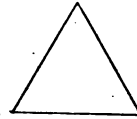
Isosceles Triangle.

NOTE. All other triangles are oblique triangles.

692. A triangle having two sides equal is an *isosceles triangle*.

693. A triangle having its three sides equal is an *equilateral triangle*.

Fig. 3.

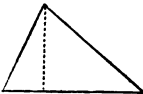


Equilateral Triangle.

694. Any side of a triangle may be taken to be the *base*. The length of a perpendicular

drawn from the opposite vertex to the base, or the base prolonged, is the *height*.

Fig. 4.



Base.

Fig. 5.



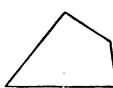
Base.

QUADRILATERALS.

695. A plane figure bounded by four straight lines is a *quadrilateral*.

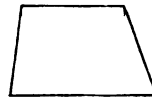
696. Two straight lines having the same direction are *parallel lines*.

Fig. 6.



Trapezium.

Fig. 7.



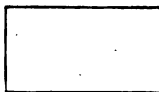
Trapezoid.

697. A quadrilateral having none of its sides parallel is a *trapezium*. (Fig. 6.)

698. A quadrilateral having two of its sides parallel is a *trapezoid*. (Fig. 7.)

699. A quadrilateral having its opposite sides parallel is a *parallelogram*. (Figs. 8, 9, 10, and 11.)

Fig. 8.



Rectangle.

Fig. 9.

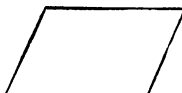


Square.

NOTE. A rectangle is a parallelogram whose angles are all right angles. A square is a rectangle whose sides are all equal.

A rhomboid is a parallelogram whose angles are not right angles. A rhombus is a parallelogram whose angles are not right angles and whose sides are all equal.

Fig. 10.



Rhomboid.

Fig. 11.



Rhombus.

Fig. 12.



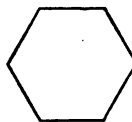
Base.

700. Any side of a parallelogram may be taken to be the *base*. The length of a perpendicular drawn from the opposite side to the base is the *height*.

POLYGONS.

701. A plane figure bounded by straight lines is a *polygon*. A polygon having all its sides equal and all its angles equal is a *regular polygon*.

Fig. 13.



Regular Polygon.

702. Exercises.

a. Which of the following named figures are regular polygons: square, rhombus, equilateral triangle, trapezium?

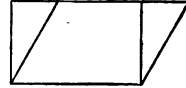
b. Draw a right triangle; an equilateral triangle; an isosceles triangle; a right-angled isosceles triangle.

c. Draw a rhombus; a rhomboid; a rectangle; a square; a trapezoid; a regular polygon of six sides.

AREAS OF POLYGONS.

703. The area of any PARALLELOGRAM is equal to that of a rectangle of the same base and height. For, if we cut off a right triangle from one end and put it on at the other end, as in Fig. 14, we change the form to a rectangle having the same base and height as the parallelogram, though we do not change the area. But the area of a rectangle is equal to the product of its base and height. Hence the

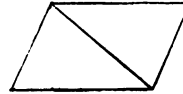
Fig. 14.

**Rule.**

To find the area of any parallelogram: *Multiply the base by the height.* (See Art. 313.)

704. A TRIANGLE is half a parallelogram of the same base and height. For two equal triangles may be placed, as shown in Fig. 15, so as to make a parallelogram. Hence the

Fig. 15.

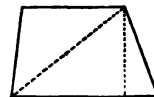
**Rule.**

To find the area of a triangle: *Multiply the base by the height, and divide the product by 2.*

To find the area of a triangle when the three sides are given: *Find half the sum of the three sides; from this subtract each side separately; multiply together the four results thus obtained, and extract the square root of the product.*

705. A TRAPEZOID may be divided into two triangles having the parallel sides of the trapezoid for bases and the distance between them for their height. Hence the

Fig. 16.



Rule.

To find the area of a trapezoid : *Multiply the sum of the parallel sides by the distance between them, and divide the product by 2.*

706. Any POLYGON may be divided into triangles. Hence for cases not met by the rules already given, we have the

Rule.

To find the area of a polygon : *Divide it into triangles, and find the sum of their areas.*

707. Examples for the Slate.

The pupil will be aided by drawing figures to illustrate the examples which follow.

1. How many square feet are there in a parallelogram whose base is 3 feet 4 inches, and height 1 foot 3 inches?
2. What is the area of a triangle, the base being 20 feet, and the height 10 feet 4 inches?
3. What is the area of a right triangle, the sides, which include the right angle, being 20 feet and 18 feet long respectively?
4. How many square feet are there in a building lot having two parallel sides respectively 140 feet and 116 feet long, and 83 feet apart?
5. How many acres are there in a four-sided field having two parallel sides, which are 60 rods apart and 40.05 rods and 64.08 rods long respectively?

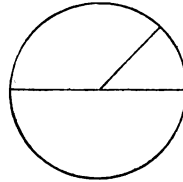
708. Exercises.

- a. Draw a triangle having a base 8 centimeters long; measure its height and find its area in square centimeters.
- b. Draw a polygon of five sides, letting one of them be a decimeter long; divide the polygon into triangles, and find the sum of their areas in square centimeters.

CIRCLES.

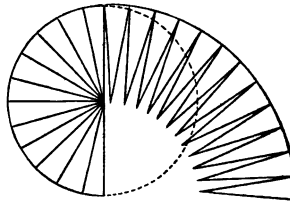
709. A line drawn from the centre of a circle to any point of the circumference is a *radius*. A line drawn through the centre of a circle terminated each way by the circumference is a *diameter*. A diameter is double the radius.

Fig. 17.



710. A circle may without sensible error be said to consist of triangles whose bases form the circumference and whose vertices are at the centre, the height of the triangles being equal to the radius of the circle. Hence the

Fig. 18.

**Rule.**

To find the area of a circle :
Multiply the circumference by the radius, and divide the product by 2.

711. If the circumference of a circle be divided by its diameter, the quotient, expressed to the nearest ten-thousandth, is 3.1416.* Hence the following

Formulas relating to the Circle.

1. Circumference = Diameter \times 3.1416.
2. Diameter = $\frac{\text{Circumference}}{3.1416}$.
3. Circumference = 2 Radius \times 3.1416.
4. Radius = $\frac{\text{Circumference}}{2 \times 3.1416}$

712. The rule in Art. 710 may be expressed thus :
Area = $\frac{\text{Circumference} \times \text{Radius}}{2}$. Putting in the place of the

* The number 3 $\frac{1}{2}$ is accurate enough for most purposes.

circumference its value as given in Formula 3, we have :

$$\text{Area} = \frac{\text{Radius} \times 2 \times 3.1416 \times \text{Radius}}{2}. \quad \text{Hence the formulas :}$$

$$5. \text{ Area} = \text{Radius}^2 \times 3.1416.$$

$$6. \text{ Radius} = \sqrt{\frac{\text{Area}}{3.1416}}$$

713. Examples for the Slate.

6. What is the distance round a circular pond which is 24 feet across the middle ?

7. If the radius of a circle is 4 feet, what is the circumference ?

8. How many feet is it across a circular grass-plot which measures 100 feet around ?

9. How many square feet of glass in a circular window 4 ft. 10 in. across ?

10. If a horse is tethered so that his head can reach $10\frac{1}{2}$ feet from the stake in any direction, over how many square feet can he graze ?

11. How many rods of fencing will be required to enclose a circular piece of ground which contains an acre ?

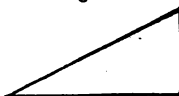
12. How many planks 2 inches thick can be sawed from a log 10 feet in circumference, allowing $\frac{1}{4}$ of an inch for each saw-cut, and 2 slabs, each at least 5 inches thick, to be cast aside ?

13. In the Yosemite Valley is the stump of a tree 32 feet across ; what is the circumference ? Allowing 3 square feet as standing-room for one person, how many persons can stand on the top of the stump ?

RIGHT TRIANGLES.

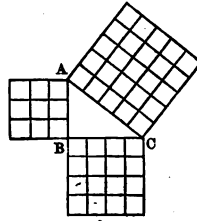
714. In a right triangle, the side opposite the right angle is the *hypotenuse*, one of the remaining sides is the *base*, and the other is the *perpendicular*.

Fig. 19.



715. Suppose the figure $A B C$ to be a right triangle, whose sides are 3, 4, and 5 feet respectively. A square formed upon the hypotenuse, $A C$, will contain 25 square feet; one formed upon the base, $B C$, will contain 16 square feet; and one formed upon the perpendicular, $A B$, will contain 9 square feet. Thus it appears that the square upon the line $A C$ is equal to the sum of the two squares upon $A B$ and $B C$.

Fig. 20.



716. It can be shown to be true in general that *the square upon the hypotenuse of a right triangle is equal to the sum of the squares upon the other two sides*. Hence the

Rules.

I. To find the hypotenuse, having the base and perpendicular given : *Square the base and the perpendicular, add the squares, and extract the square root of the sum.*

II. To find the base or perpendicular, having the hypotenuse and the other side given : *Square the hypotenuse and the given side, subtract the latter square from the former, and extract the square root of the remainder.*

717. Examples for the Slate.

14. The base of a right triangle being 18 feet, the perpendicular 24 feet, what is the hypotenuse?

15. The hypotenuse of a right triangle being 32.5 feet, the base 26 feet, what is the perpendicular?

16. If the foot of a ladder 20 feet long be put 8 feet from the base of a wall, how high will it reach?

17. Two boats started at the same point, and sailed one north 5280 feet and the other east 3960 feet. How far apart were they then?

18. Near Lexington battle-ground was an elm partially broken off and fallen across the street. If the height of the fracture was 15 feet from the ground, and the distance from the foot of the tree to the point where the top reached the ground was 66 feet, what was the entire height of the tree before it fell?

19. B lives 40 rods east of A, 72 west of C, and 30 north of the school-house. If A and C both call for B in going to school, how much farther does each travel than if he goes directly to school?

20. Four boys, Dwight, Brown, Chase, and Doane, live in different places, as follows: Dwight 40 rods north of the school-house; Brown 60 rods east; Chase 57 rods south; and Doane 36 rods west. What is the shortest distance one of the boys must travel to visit all the rest, and reach his own home?

21. What is the length of a diagonal, that is, the distance from one corner to the opposite corner of the floor of a room 15 feet square?

22. If the height of this room is 9 feet, what is the distance from one corner of the floor to the opposite upper corner of the ceiling?

NOTE. The diagonal of the floor is the base of the right triangle of which the required diagonal of the room is the hypotenuse. Hence *the square of the diagonal of a room, or of any rectangular solid, equals the sum of the squares of its three dimensions.*

23. What is the length of the longest rod that, without bending, can be put into a box 1 yard long, 1 foot wide, and 9 inches high, measured on the inside?

24. Find the height and area of an equilateral triangle whose sides are 10 feet long?

NOTE. The perpendicular MN divides the triangle into two equal right triangles. AN is then one half of AB .

25. Find the height and area of an isosceles triangle whose base is 20 feet long, the other sides being each 26 feet long?

Fig. 21.

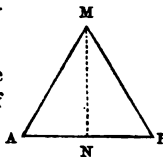
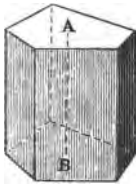


Fig. 22.



Prism.

SOLIDS.

Fig. 23.



Cube.

Fig. 24.



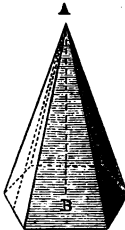
Rectangular Solid.

718. A *prism* is a solid bounded by two equal and parallel polygons and a number of parallelograms.

NOTE I. It will be seen that all rectangular solids are prisms.

NOTE II. The equal and parallel polygons are the *bases* of the prism, and the parallelograms taken together form its *convex surface*. When the bases are regular polygons and the parallelograms are perpendicular to the bases, the prism is a *regular prism*.

Fig. 25.



Pyramid.

Fig. 26.



Cylinder.

Fig. 27.



Cone.

719. A *pyramid* is a solid bounded by one polygon, which is the base, and a number of triangles which terminate in a common point called the vertex.

NOTE. The triangles form the *convex surface* of the pyramid. When the base of a pyramid is a regular polygon, and a line drawn from the vertex to the middle of the base is perpendicular to the base, the pyramid is a *regular pyramid*.

720. A solid formed by turning a rectangle about one of its sides as an axis is a *cylinder*.

NOTE. As the rectangle is turned, the side opposite the axis describes the *convex surface* of the cylinder, and the other two sides describe parallel circles, which are the bases of the cylinder.

721. A solid formed by turning a right triangle about one of its shorter sides is a *cone*.

NOTE. As the triangle is turned, the hypotenuse describes the *convex surface* of the cone, and the side perpendicular to the axis describes a circle, which is the base of the cone.

Fig. 28.

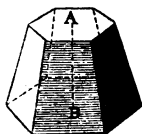
Frustum of
a Pyramid.

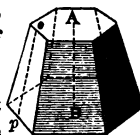
Fig. 29.

Frustum of
a Cone.

722. If the upper part of a pyramid or of a cone is cut off by a plane parallel to the base, the part that remains is a *frustum* of the pyramid or of the cone.

723. The *height* of any of the solids here defined is the perpendicular distance from the highest point above the base to the plane of the base. Thus, in Fig. 30, the line *A B* indicates the height of the solid.

Fig. 30.

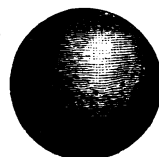


NOTE. In a regular pyramid or in a cone, the shortest distance from the vertex to the perimeter (boundary) of the base is the *slant height*.

In the frustum of a regular pyramid or of a cone, the shortest distance between the perimeters of the two bases is the *slant height*. Thus, in Fig. 30, the line *o p* indicates the slant height of the frustum.

Fig. 31.

724. A *globe*, or *sphere*, is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the centre.



Sphere.

NOTE. A circle which divides a sphere into two equal parts is a *great circle* of the sphere. A circumference, a diameter, or a radius of a great circle of a sphere is also a *circumference*, a *diameter*, or a *radius* of the sphere itself.

Volumes of Solids and Areas of their Convex Surfaces.

725. A PRISM or a CYLINDER 1 inch high contains as many cubic inches as there are square inches in the base. If the height be increased to 2, 3, or any number of inches, the volume will be increased in the same proportion. Hence the

Rule.

To find the volume of a prism or cylinder : *Multiply the area of the base by the height.*

726. If a PRISM or a CYLINDER is 1 inch high, its convex surface contains as many square inches as there are inches in the perimeter of the base. If the height be increased to 2, 3, or any number of inches, the convex surface will be increased in the same proportion. Hence the

Rule.

To find the convex surface of an upright prism or cylinder : *Multiply the perimeter of the base by the height.*

727. It can be proved that a PYRAMID or a CONE is equivalent to $\frac{1}{3}$ of a prism or a cylinder of the same base and height. Hence the

Rule.

To find the volume of a pyramid or cone : *Multiply the area of the base by the height, and divide the product by 3.*

728. The convex surface of a regular PYRAMID or CONE may be regarded as composed of triangles whose bases form the perimeter of the base of the solid, and whose height is the slant height of the solid. Hence the

Rule.

To find the convex surface of a regular pyramid or cone : *Multiply the perimeter of the base by the slant height and divide the product by 2.*

729. It can be proved that the FRUSTUM of a pyramid or cone is equivalent to the sum of three pyramids or cones, which have for their common height the height of the frustum, and whose bases are the lower base of the frustum, the upper base, and a mean proportional between them. Hence the

Rule.

To find the volume of a frustum of a pyramid or cone : *Multiply the sum of the areas of the two bases, plus the square root of their product, by the height, and divide the product by 3.*

730. The convex surface of the FRUSTUM of a regular pyramid or cone may be regarded as made up of trapezoids whose parallel sides form the perimeters of the bases, and whose height is the slant height of the frustum. Hence the

Rule.

To find the convex surface of a frustum of a regular pyramid or cone: *Multiply the sum of the perimeters of the two bases by the slant height and divide the product by 2.*

731. It can be proved that the surface of a SPHERE is equivalent to that of four great circles of the sphere. Hence the

Rule.

To find the surface of a sphere: *Find the area of a great circle of the sphere, and multiply it by 4.*

732. A SPHERE may be regarded as composed of pyramids whose bases taken together form the surface of the sphere and whose height is the radius. Hence the

Rule.

To find the volume of a sphere: *Multiply the convex surface by the radius and divide the product by 3.*

733. The last two rules may be expressed by the following formulas:

$$\text{Surface of a sphere} = 4 \text{ Radius}^2 \times 3.1416.$$

$$\text{— Volume of a sphere} = \frac{4}{3} \text{ Radius}^3 \times 3.1416.$$

NOTE. To find the capacity of a cask, see Appendix, page 314.

734. Examples for the Slate.

26. How many cubic inches are there in a prism whose base is 8 inches square, and whose height is 7 inches?

27. How many cubic feet in a prism 5 feet high and having for its base a triangle, each side of which is 10 feet long?

28. How many cubic feet in a pyramid 10 feet high and having for the base 1 square rod?

29. How many cubic inches in the frustum of a pyramid whose bases contain 12 and 108 square inches respectively, and whose height is 18 inches?

30. How many bushels of corn can be put into a corn-crib 9 feet square at the bottom, 12 feet square at the top, and 8 feet high? (See Art. 393, note.)

31. What is the convex surface of a prism, the perimeter of whose base is 7 yards 2 feet, and whose height is 5 yards 1 foot?

32. How many square feet in the surface of a four-sided pyramidal roof, the slant height being 18 feet and the house 20 feet square?

33. How many feet of boards will cover the sides of an eight-sided tower, the length of each side of the base being 2 feet 9 inches, that of each side of the top 1 foot 10 inches, and the slant height 12 feet?

34. How many gallons will a pail contain that measures on the inside 14 inches in depth and 11 inches across?

35. How many square feet of sheet-iron are there in a piece of stove-pipe 9 feet long and 6 inches in diameter, no allowance being made for lapping at the joints?

36. What is the height of a conical tent, if the diameter of the base is 15 feet and the slant height is $19\frac{1}{2}$ feet, and how many cubic feet will the tent contain?

37. How many square yards of canvas will be required to make this tent, no allowance being made for seams?

38. How many gallons will a circular vat contain that measures across the bottom 12 feet, across the top 15 feet, the depth being 6 feet?

39. How many square feet in the surface of a foot-ball 1 foot in diameter?

40. At 38¢ a square foot, what is the cost of painting a globe 6 feet in diameter?

41. How many cubic feet are there in this globe?

42. If the diameter of the earth is 7900 miles, and $73\frac{1}{2}\%$ of the surface of the earth is water, how many square miles of the surface is water?

SIMILAR SURFACES.

735. Figures which have the same shape are *similar figures*.

NOTE. The corresponding sides of similar figures are proportional.

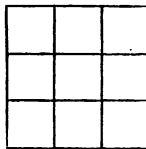
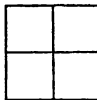


Fig. 32.

736. We see from the illustration above that a figure 1 inch square contains 1 square inch, a figure 2 inches square contains 4 square inches, and a figure 3 inches square contains 9 square inches, etc. In general,

The areas of similar figures are to each other as the squares of their corresponding dimensions.

737. ILLUSTRATIVE EXAMPLE I. The area of a certain triangle is 120 square feet and its base is 24 feet. What is the area of a similar triangle whose base is 96 feet?

WRITTEN WORK.

$$24^2 : 96^2 = 120 : x \quad \frac{120 \times \overset{4}{96} \times \overset{4}{96}}{\cancel{24} \times \cancel{24}} = 1920. \quad \text{Ans. } 1920 \text{ sq. ft.}$$

738. ILLUSTRATIVE EXAMPLE II. One side of a triangle is 40 feet long; what must be the length of a side of a similar triangle containing twice the area?

WRITTEN WORK.

$$1 : 2 = 40^2 : x^2 \quad \sqrt{40 \times 40 \times 2} = 56.568... \quad \text{Ans. } 56.568... \text{ feet.}$$

739. Examples for the Slate.

43. If a room 16 feet in length requires 22 yards of carpeting to cover the floor, how many yards of carpeting will be required for a room 20 feet long and of the same shape?

44. There is a public park 1320 feet long, containing 25 acres. What is the length of a park of the same shape containing 49 acres?

45. If a circular lot of land which is 10 rods in diameter contains 78.5398 square rods, what number of rods will a circular lot contain which is 5 rods in diameter?

46. If a pipe 2 inches in diameter discharges 20 gallons of water in a given time, how many gallons will a pipe 5 inches in diameter discharge in the same time, no allowance being made for friction?

47. If it costs 17¢ for tin to make a pail 6 inches high, what will it cost for tin to make a similar pail 14 inches high?

48. If it costs \$72 for material to paint a spire 50 feet high, what will it cost for material to paint a similar spire 75 feet high?

49. If the cost of plating a pitcher 6 inches high is \$1.75, what is the cost of plating a pitcher of the same shape 10 inches high?

50. What is the height of a pitcher similar to that described above, of which the cost of plating is \$4.00?

SIMILAR SOLIDS.

740. Solids which have the same shape are *similar solids*.

NOTE. The corresponding dimensions of similar solids are proportional.

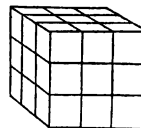


Fig. 33.

741. We see from the illustration above that a cube whose edge is 1 inch contains 1 cubic inch, a cube whose edge is 2 inches contains 8 cubic inches, a cube whose edge is 3 inches contains 27 cubic inches, etc. In general,

The volumes of similar solids are to each other as the cubes of their corresponding dimensions.

742. ILLUSTRATIVE EXAMPLE I. If a cube of lead whose edge is 3 inches weighs 12 pounds, what is the weight of a cube of lead whose edge is 2 inches?

WRITTEN WORK.

$$3^3 : 2^3 = 12 \text{ lb.} : x \quad \frac{2^3 \times 12}{3^3} = 3\frac{2}{3}. \quad \text{Ans. } 3\frac{2}{3} \text{ lb.}$$

743. ILLUSTRATIVE EXAMPLE II. A pyramid 9 feet high contains 48 cubic feet. What is the height of a similar pyramid that contains 100 cubic feet?

WRITTEN WORK.

$$48 : 100 = 9^3 : x^3 \quad \sqrt[3]{\frac{100 \times 9 \times 9 \times 9}{48}} = 11.49... \quad \text{Ans. } 11.49... \text{ feet.}$$

744. Examples for the Slate.

51. If an egg $2\frac{1}{4}$ inches in circumference weighs 1 ounce, what would another of the same form and consistency weigh whose circumference is 6 inches?

52. An ox measuring 7 feet in girth weighs 1500 pounds; what is the weight of an ox measuring 9 feet in girth?

53. If a bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep, what must be the diameter and depth of a half-bushel measure similar in form?

54. Estimating the mean diameter of the earth at 7912 miles, and that of the moon at 2160 miles, how many bodies of the size of the moon could be made from the bulk of the earth?

55. If the bulk of Saturn be 1000 times as great as that of the earth, what is the diameter of Saturn?

56. At what distance from the top must a cone 12 inches high be cut parallel with the base, that the cone may be divided into two equivalent parts?

57. Mr. Root has three stacks of hay of similar shape, the diameters of their bases being respectively 10, 12, and 14 feet; if the smallest stack contains $2\frac{1}{2}$ tons, what will each of the others contain?

745. General Review, No. 6.

58. Supply the 2d term in the proportion $3\frac{1}{2} : x = 8 : 25$.
59. What is the mean proportional between 0.8 and 0.72 ?
60. Divide \$1900 between two men, in the proportion of 3 to 5.
61. Divide \$45 among three boys, so that one shall have as much as the other two, whose shares are as 2 to 7.
62. How many pounds can 5 horses draw, if 6 horses can draw as much as 10 oxen, and 2 oxen can draw 2400 pounds ?
63. Smith and Lee formed a partnership. Smith put in \$1000 for 6 months and \$800 for 2 months. Lee put in \$600 for 8 months and was allowed \$800 for his services. They gained \$1435.50; what was each partner's share ?
64. What is the 5th power of 23 ? the cube of 96 ?
65. What is the largest number of men in a regiment of 1000 that can be arranged in a square; and how many men will remain ? How many men will there be on each side of the square ?
66. How many feet of fencing are required to enclose a square farm containing 15 acres ?
67. A ladder $27\frac{1}{2}$ feet long reaches a window $25\frac{1}{2}$ feet from the ground. How far does the foot of the ladder stand from the house ?
68. What is the diameter of a circle which contains $314\frac{1}{2}$ square feet ?
69. How many rods of fencing on both sides of a road which surrounds a circular park containing 15.708 acres, the road being 3 rods wide ?
70. What must be the depth of a pail that is 10 inches across, to contain 5 gallons, the sides being upright ?
71. How many square feet of canvas are required to construct a conical tent 14 feet across the bottom and $9\frac{1}{2}$ feet from the highest point to the ground ?
72. If a pipe $2\frac{1}{2}$ inches in diameter will fill a cistern in two hours, in what time will a pipe 5 inches in diameter fill the same ?

746. Questions for Review.

What is a power? a second power? a third? a fourth? What is INVOLUTION? What is squaring a number? cubing a number? Give the squares of the numbers from 1 to 12; the cubes of the numbers from 1 to 10.

What is the root of a number? What is EVOLUTION? Repeat the formula used in extracting the square root; the rule. What do you do when a term of the root proves too large? when the trial divisor is not contained in the dividend? How do you extract the square root of a fraction? of a mixed number? Repeat the formula used in extracting the cube root; the rule. How do you extract the cube root of a fraction? of a mixed number?

What is MENSURATION? Name and describe the different kinds of triangles given; the different kinds of quadrilaterals given. What is the base of a triangle? the height?

How do you find the area of a square? of a rectangle? of any parallelogram? of a triangle? of a trapezoid? of any polygon?

How do you find the circumference of a circle when the diameter is given? when the radius is given? How do you find the diameter when the circumference is given? How do you find the area of a circle when the radius is given? the radius of a circle when the area is given?

What is a cube? a prism? a pyramid? a cylinder? a cone? What is the frustum of a pyramid or a cone? What is a sphere? Draw or name something in the form of each of these solids. What is the height of any solid? the slant height of a pyramid or of a cone? the slant height of a frustum of a pyramid or of a cone?

How do you find the volume of a cube? of any rectangular solid? of a prism or a cylinder? of a pyramid or a cone? of the frustum of a pyramid or cone? How do you find the convex surface of each of these solids?

When the diameter and height are given, how do you find the volume of a cylinder? of a cone? of a frustum of a cone? How do you find the volume of a sphere? How do you find the convex surface of a cylinder? of a cone? of the frustum of a cone? of a sphere?

When are plane figures similar? What proportion is there between the areas of similar figures? When are solids similar? What proportion is there between the volumes of similar solids?

747. Miscellaneous Examples.

73. What is the weight of a bale of cloth containing 13 pieces, 42 yards to the piece, every 3 yards weighing $1\frac{1}{2}$ pounds?

74. The sum of three numbers is $55\frac{1}{2}$; two of them are $14\frac{1}{2}$ and $24\frac{1}{2}$; what is the third?

75. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ of a certain number increased by $3\frac{1}{2}$ equals 40. What is the number?

76. A trader bought apples at \$1.62 $\frac{1}{2}$ per barrel, and immediately sold them at \$2.25, making \$234.37 $\frac{1}{2}$. How many barrels were bought?

77. Suppose a dividend to be 241.3, and the quotient 0.127, what was the divisor?

78. The ridge-pole of a house is 46 feet from the ground, the eaves 38 feet, the rafters on each side of the roof to the eaves being 18 feet long. What is the width of the house?

79. When the ice upon a pond is 10 inches thick, what will be the value of the ice taken from one acre of the pond at $\frac{1}{4}$ of a cent a pound, 1 cubic foot of ice containing 58 $\frac{1}{2}$ pounds?

80. The city tax of Lincoln being $\frac{3}{4}\%$, and the State and county tax 0.15%; for what sum is James Otis taxed, who pays \$56.22, including \$1.50 poll-tax?

81. Of what number is $\frac{3}{4}$ the $\frac{1}{4}$ part?

82. A city collector received 0.8% for collecting taxes; he paid into the treasury \$94625.64 after deducting his commission; what was the whole sum collected?

83. A coal-dealer purchased 500 tons of coal at \$7.50 per long ton, paid \$1 per ton for freighting, and sold it for \$11 by the short ton. What per cent did he gain?

84. A can do a piece of work in $1\frac{1}{2}$ hours, A and B in 48 min.; in what time can B do it alone?

85. Crane Brothers & Co. purchased oil stocks to the amount of \$5714.25, including their commission of $\frac{1}{4}\%$; the stock, the par value of which was \$50 per share, was purchased at 95%. How many shares were purchased?

86. When butter is 25 cents a pound, and $\frac{3}{4}$ of a pound will pay for $\frac{1}{4}$ of a dozen eggs, how many eggs will be required to pay for 6 pounds of raisins, 7 pounds of which cost 98 cents?

87. What is the difference between the true and bank discount of \$700, due in 90 days, when the legal rate is 7%?

88. How much would you receive from a bank, June 12, 1878, for a note of \$820, dated April 12, 1878, payable 6 months after date, discount being 6%?

89. Write a note for 60 days, for which you should get \$300 at a bank, discount being 6%?

90. If \$1239 were paid for harvesting the wheat on a lot of land 400 rods long, 350 rods wide, what should be paid for harvesting the oats upon a lot 500 rods long, 450 rods wide, the cost of harvesting oats being $\frac{2}{3}$ as much as for harvesting wheat?

91. Bates and Henricks traded in hides for one year. Bates put in \$2000 at first; at the end of 3 months he withdrew \$700, and at the end of 7 months put in \$1000. Henricks put in \$1200 at first, and \$500 more in 4 months. At the end of 6 months he withdrew \$200. The gain for the year was \$2355.75, of which Henricks received \$1000 for conducting the business. What was the share of each?

92. The pyramid of Cheops, in Egypt, is said to contain 82111000 cubic feet of masonry, and to have been 480 feet high. Allowing 7000000 cubic feet, which are required to perfect its pyramidal form and to fill its chambers, what is the length of one side of its base, which is a square?

93. If you buy figs at the rate of 9 pounds for \$1.50, and sell them at the rate of 10 pounds for \$2, what per cent do you gain?

94. What is the average time for paying for \$200 worth of goods purchased May 17, 1877, on 4 months' credit; \$500 worth, purchased June 18, 1877, on 60 days' credit; and \$300 worth, purchased June 19, 1877, on 90 days' credit?

95. The captain of a ship at sea finds by his chronometer, at 12 o'clock, noon, that it is 45 m. past 8 o'clock in the evening at London. What is his longitude?

96. Pedrick & Closson sold at auction,					
2 mattresses,	at	\$16.00,	which cost	\$13.50.	
8 chairs,	at	4.62,	"	"	3.75.
1 rocker,	at	17.50,	"	"	17.00.
1 set furniture,	at	38.00,	"	"	42.00.
1 " "	at	83.50,	"	"	62.00.

They also sold on commission, at 10%, 5 chairs, at \$8; 12 chairs, at \$1.70; 1 bureau, at \$18; 1 table, at \$8; 1 lounge, at \$12; 1 stove, at \$17. What were their net proceeds from the above sales?

97. How many bricks 8 in. by 4 in. by 2 in. in the walls of a building 29 ft. long by 24 ft. wide and 20 ft. high, outside measurement, having 10 windows 6 ft. by 4 ft. and 2 doors 7 ft. by 4½ ft., the thickness of the walls being 1 foot, and ¼ of the entire wall being mortar?

NOTE. In estimating the number of bricks required, masons reckon by *outside* measurements, and make no allowance for corners.

98. The circular outlet to a cistern being 4 inches in diameter, what must be the width of a rectangular receiving-pipe, whose depth is 2 inches, that its capacity may be the same as that of the discharging-pipe?

99. When it is 10 A. M. in X, which is 44° 15' 2" W. long., what is the time in Y, which is 8° 4' 40" E. long.?

100. A, B, and C shipped goods by the same vessel. The value of A's goods was \$50000; of B's, \$40000; of C's, \$30000. During a storm half of A's goods and one fifth of B's were thrown overboard. What should be each man's share of the loss, and how much should be paid to A by C and by B to adjust the losses?

101. I sold 6 sewing-machines at \$72 each. On two of them I gained 20%, on two others 33⅓%, and on the rest I lost 25%. What was the balance of gain or loss?

102. A grocer imported 75 gallons of oil, which cost him \$2 a gallon and a duty of 10%. Suppose 5 gallons to leak out, for what must he sell the remainder per gallon to gain 10% on the money spent?

103. At 1 cent per cubic foot, what will be the cost of digging a ditch outside a square garden containing 12.75 square rods, the ditch to be 7 feet wide and 5 feet deep?

104. How many gallons in a cylindrical jar 2 feet across and 4 feet high?

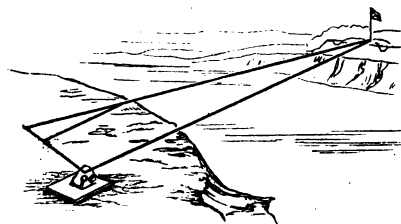
105. I found, on going to Gile & Walcott's dry-goods store, that they had that morning marked up their goods 15%. What did I save by purchasing the day before the following goods: 18 yds. blk. silk, at \$1.12; 13 yds. de laine, at \$0.27; 9 yds. cambric, at \$0.15; 3 yds. silesia, at \$0.25; 1 waterproof, at \$8?

106. A grocer paid 21 cents a gallon for a cask containing 27 gallons of kerosene, 10 % of which leaked out. If the remainder was sold 25% on the gallon higher than it cost, what was the gain or loss on the money invested?

107. Supposing a cubic foot of snow to weigh 21 lbs., what will be the pressure of a body of snow 9 inches deep upon a flat roof 100 ft. by 25 ft.?

108. If an elephant's tusk $9\frac{1}{2}$ feet long and 8 inches in diameter at the base weighs 214 pounds, what would be the dimensions of a similar tusk weighing 75 pounds?

109. An engineer, having placed a mortar near the bank of a river, wished to find its distance from a fort on the opposite



shore. To do this he marked off a line from the mortar towards the fort; went 8 rods up the river, where he drove a stake; and 6 feet farther on took his station. Then he told his assistant to

start from the stake and mark off a line parallel with the first line, till he came in range between him and the fort. This line measured 480 feet. What was the distance sought? (See Art. 735, note.)

For other miscellaneous examples, see Appendix, page 315.

APPENDIX.

Names of Numbers (Art. 2).

1. The only compound names of numbers that do not show plainly how they are made up are Eleven, Twelve, Twenty, and the other names ending in -ty.

Eleven (in Old English *endlif*, in Gothic *ainlif*) is a compound of *end* or *en*, meaning *one*, and *lif*, meaning *ten*. So eleven means *one and ten*.

Twelve (in Old English *twelf*, in Gothic *two-lif*) is a compound of *two*, meaning *two*, and *lif*, meaning *ten*. So twelve means *two and ten*.

Twenty (in Old English *twentig*) is a compound of *twen*, meaning *twain* or *two*, and *tig*, meaning *ten*. So twenty means *two tens*, thirty means *three tens*, and so on.

Roman Numerals (Art. 12).

2. The Roman Numerals are so called because they were used by the ancient Romans. They were in general use in Europe as late as the twelfth and thirteenth centuries for keeping accounts and other purposes of common life. They were not used as the Arabic numerals are, to make computations with, but merely to record the results. The computations were made mostly with counters.

By the Roman method of writing, seven letters are used to denote numbers, as follows:

I.	V.	X.	L.	C.	D.	M.
One.	Five.	Ten.	Fifty.	One hundred.	Five hundred.	One thousand.

The method of using these letters to denote numbers is shown in the following table:—

TABLE

I stands for 1.	XI stands for 11.	XXI stands for 21.	CI stands for 101.
II " " 2.	XII " " 12.	XXX " " 30.	CC " " 200.
III " " 3.	XIII " " 13.	XXXI " " 31.	CCC " " 300.
IV " " 4.	XIV " " 14.	XL " " 40.	CD " " 400.
V " " 5.	XV " " 15.	L " " 50.	D " " 500.
VI " " 6.	XVI " " 16.	LX " " 60.	DC " " 600.
VII " " 7.	XVII " " 17.	LXX " " 70.	DCC " " 700.
VIII " " 8.	XVIII " " 18.	LXXX " " 80.	DCCC " " 800.
IX " " 9.	XIX " " 19.	XC " " 90.	CM " " 900.
X " " 10.	XX " " 20.	C " " 100.	M " " 1000.

3. Names of Numbers higher than Trillions (Arts. 22, 23).

The names of the groups used to express numbers higher than trillions are, in their order from trillions, *quadrillions*, *quintillions*, *sextillions*, *septillions*, *octillions*, *nonillions*, *decillions*, *undecillions*, *duodecillions*, *tredecillions*, *quatuordecillions*, *quindecillions*, *sexdecillions*, *septendecillions*, *octodecillions*, *novemdecillions*, *vigintillions*, etc.

4. To Read Decimals (Art. 35).

The following method of reading decimals is recommended by its simplicity and its conformity to the method of reading whole numbers.

ILLUSTRATIVE EXAMPLE. Read the number 0.279036205.

Begin at the decimal point, and point off three figures at the right of the point for the thousandths' group, three more for the millionths' group, and so on; thus,

0.279,036,205.

Then read the number expressed in each group separately, pronouncing the name of the group; thus, "Two hundred seventy-nine thousandths, thirty-six millionths, two hundred five billionths."

The expression 0.36038 would be read, "Three hundred sixty thousandths, thirty-eight hundred thousandths."

5. Contractions in Multiplication (Art. 83).

To multiply by 9, 99, 999, etc.

Since 9 times a number is the same as 10 times the number less once the number, and 99 times a number is 100 times the number less once the number, and so on,

To multiply by any number whose terms are all 9's: *Annex as many zeros to the expression for the multiplicand as there are 9's in that*

of the multiplier, and from the number thus expressed subtract the multiplicand; thus, $27 \times 99 = 2700 - 27 = 2673$.

Examples for the Slate.

- | | |
|---------------------------|------------------------------|
| (1.) $36 \times 99 = ?$ | (4.) $36841 \times 9999 = ?$ |
| (2.) $264 \times 999 = ?$ | (5.) $7 \times 9999999 = ?$ |
| (3.) $58 \times 9999 = ?$ | (6.) $245 \times 998 = ?$ |

NOTE. In Example 6, multiply by 1000, and subtract twice the multiplier.

- | | |
|----------------------------|--------------------------------|
| (7.) $356 \times 9995 = ?$ | (8.) $54932 \times 999997 = ?$ |
|----------------------------|--------------------------------|

6. To multiply by a composite number.

Separate the multiplier into convenient factors, multiply the multiplicand by one of the factors, and that product by another factor, and so on, till all the factors have been used; the last product is the answer: thus, $41 \times 25 = 41 \times 5 \times 5$.

Examples for the Slate.

9. Multiply 368 by 72; by 36.
10. Multiply 4079 by 81; by 48.
11. Multiply 2145 by 108; by 144.
12. Multiply 50411 by 55; by 150.

7. To multiply by aliquot parts of 10, 100, 1000, etc.

Multiply by 10, 100, 1000, etc., as the case may require, and then find the required part; thus,

To multiply by 25, multiply by 100 and divide by 4.

By 125, multiply by 1000 and divide by 8.

By $33\frac{1}{3}$, multiply by 100 and divide by 3.

By $16\frac{2}{3}$, multiply by 100 and divide by 6.

By $12\frac{1}{2}$, multiply by 100 and divide by 8. (See Arts. 258 to 261.)

8. To multiply when the number of tens is the same in the multiplicand and multiplier, and the sum of the units is ten.

$\begin{array}{r} 27 \\ 23 \\ \hline 621 \end{array}$	<i>Multiply the number of tens by the number of tens plus one; write the product as hundreds; at the right express the product of the units by the units.</i>
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Examples.

- (13.) $23 \times 27 = ?$ (16.) $45 \times 45 = ?$ (19.) $55 \times 55 = ?$
 (14.) $28 \times 22 = ?$ (17.) $56 \times 54 = ?$ (20.) $85 \times 85 = ?$
 (15.) $31 \times 39 = ?$ (18.) $87 \times 83 = ?$ (21.) $105 \times 105 = ?$

9. To square a number consisting of an integer and $\frac{1}{2}$.

$$\begin{array}{r} 7\frac{1}{2} \\ 7\frac{1}{2} \\ \hline 56\frac{1}{4} \end{array} \quad \begin{array}{l} \text{Multiply the integer by the integer plus one, and to the} \\ \text{product add } \frac{1}{4}. \end{array}$$

Examples.

- (22.) $5\frac{1}{2} \times 5\frac{1}{2} = ?$ (24.) $11\frac{1}{2} \times 11\frac{1}{2} = ?$ (26.) $99\frac{1}{2} \times 99\frac{1}{2} = ?$
 (23.) $8\frac{1}{2} \times 8\frac{1}{2} = ?$ (25.) $13\frac{1}{2} \times 13\frac{1}{2} = ?$ (27.) $16\frac{1}{2} \times 16\frac{1}{2} = ?$

10. To square a number consisting of an integer and $\frac{1}{4}$.

$$\begin{array}{r} 12\frac{1}{4} \\ 12\frac{1}{4} \\ \hline 144 + 6 + \frac{1}{16} = 150\frac{1}{16} \end{array} \quad \begin{array}{l} \text{Square the integer; to this square add 1 half the} \\ \text{integer plus } \frac{1}{16}. \end{array}$$

Examples.

- (28.) $6\frac{1}{4} \times 6\frac{1}{4} = ?$ (30.) $7\frac{1}{4} \times 7\frac{1}{4} = ?$ (32.) $50\frac{1}{4} \times 50\frac{1}{4} = ?$
 (29.) $8\frac{1}{4} \times 8\frac{1}{4} = ?$ (31.) $30\frac{1}{4} \times 30\frac{1}{4} = ?$ (33.) $61\frac{1}{4} \times 61\frac{1}{4} = ?$

Contractions in Division (Art. 111).

11. To divide by aliquot parts of 10, 100, 1000, etc.

To divide by 5, divide by 10 and multiply the quotient by 2.

By 25, divide by 100 and multiply by 4.

By 125, divide by 1000 and multiply by 8.

By $33\frac{1}{3}$, divide by 100 and multiply by 3.

By $16\frac{2}{3}$, divide by 100 and multiply by 6.

By $166\frac{2}{3}$, divide by 1000 and multiply by 6, etc. (See Arts. 258 to 261.)

12. Analysis of Oral Examples (Art. 116).

ANALYSIS OF EXAMPLE a. If the car runs 69 miles in 3 hours, in 1 hour it will run $\frac{1}{3}$ of 69 miles, or 23 miles, and in 5 hours it will run 5 times 23 miles, or 115 miles. *Ans.* 115 miles.

EXAMPLE c. First find what \$1 will buy.

ANALYSIS OF EXAMPLE i. If a quantity of hay lasts 22 oxen 10 days, it will last 1 ox 22 times 10 days, or 220 days, and it will last 10 oxen (5 yoke) $\frac{1}{5}$ of 220 days, or 22 days. *Ans.* 22 days.

ANALYSIS OF EXAMPLE III. If the work can be done by 50 men in 4 weeks, it will require 4 times 50 men, or 200 men, to do it in one week. But 50 men are already employed. Then 200 men less 50 men, or 150 more men, must be employed to do it in a week.

In solving an example by analysis, we first take the number whose denomination is the same as that of the required answer to work upon, and then proceed according to the statements given in the example.

13. The Divisibility of Numbers (Art. 153).

1. Divisibility of numbers by 2, 4, 5, or 8.

Ten is divisible by 2, so any number of tens is divisible by 2. Hence a number is divisible by 2 if the number of its units is divisible by 2.

For a similar reason, a number is divisible by 5 if the number of its units is divisible by 5.

One hundred is divisible by 4, so any number of hundreds is divisible by 4. Hence a number is divisible by 4 if its tens and units together are divisible by 4.

One thousand is divisible by 8, so any number of thousands is divisible by 8. Hence a number is divisible by 8 if its hundreds, tens, and units together are divisible by 8.

2. Divisibility of numbers by 9.

ILLUSTRATION.

$$\begin{array}{rcll}
 & 3000 = & 333 \times 9 & + 3 \\
 3486 = & + 400 = & 44 \times 9 & + 4 \\
 & + 80 = & 8 \times 9 & + 8 \\
 & + 6 = & & 6 \\
 \hline
 3486 = & & (333 + 44 + 8) \times 9 + (3 + 4 + 8 + 6)
 \end{array}$$

The number 3846 is now separated into two parts, one of which is divisible by 9, and the other equals the sum of its digits. Any number can be so separated. Hence a number is divisible by 9 if the sum of its digits is divisible by 9.

3. Divisibility of numbers by 3. Any number that is divisible by 9 is divisible by 3. Therefore, when a number has been separated into two parts, as shown above, the first part is divisible by 3. The other part is equal to the sum of its digits. Hence a number is divisible by 3 if the sum of its digits is divisible by 3.

14. The Greatest Common Factor of Numbers (Art. 175).

The method of finding the g. c. f. of numbers, as given in Article 175, depends upon the following principles.

I. Any factor common to two numbers is also a factor of their sum and of their difference.

Thus 3, which is a common factor of 24 and 18, is a factor of 42 ($=24 + 18$) and of 6 ($=24 - 18$). Since 24 is equal to a certain number of 3's, and 18 to a certain other number of 3's, their sum must be a number of 3's, and their difference must be a number of 3's.

II. The greatest common factor of two numbers is equal to the greatest common factor of the smaller of them, and the remainder obtained by dividing one of them by the other.

52) 143 (2 Thus, the g. c. f. of 143 and 52 is equal to the g. c. f.
 104 of 52 and 39.
 — Since $39 = 143 - 52 - 52$, all factors common to 143
 39 and 52 are also factors of 39, and hence common factors

of 52 and 39. Therefore the g. c. f. of 143 and 52 is a *common factor* of 52 and 39, and cannot be greater than the g. c. f. of 52 and 39.

Again, since $143 = 52 + 52 + 39$, all factors common to 52 and 39 are also factors of 143, and hence common factors of 143 and 52. Therefore the g. c. f. of 52 and 39 is a *common factor* of 143 and 52, and cannot be greater than the g. c. f. of 143 and 52.

The g. c. f. of 143 and 52 and the g. c. f. of 52 and 39 are, then, two numbers, neither of which can be greater than the other; they are therefore equal.

15. Analysis of Illustrative Example (Art. 246).

WRITTEN WORK.

$$\frac{1 \times 3 \times 5}{2} = 7\frac{1}{2}.$$

Ans. $7\frac{1}{2}$ baskets.

Analysis. — If $\frac{2}{3}$ of a dollar will buy 1 basket of peaches, $\frac{1}{3}$ of a dollar will buy $\frac{1}{2}$ of a basket, and $\frac{2}{3}$ of a dollar, or 1 dollar, will buy $\frac{3}{2}$ of a basket. If 1 dollar will buy $\frac{3}{2}$ of a basket, 5 dollars will buy 5 times $\frac{3}{2}$ or $7\frac{1}{2}$ of a basket, which equals $7\frac{1}{2}$ baskets.

16. To Divide an Integral Number or a Fraction by a Fraction (Art. 248).

NOTE. The following methods of dividing by fractions are in common use. To understand either method, the learner must bear in mind that the smaller the divisor, the larger is the quotient, and the larger the divisor, the smaller is the quotient.

ILLUSTRATIVE EXAMPLE. What is the quotient of $\frac{4}{5} \div \frac{2}{3}$?

WRITTEN WORK.

$$\frac{\frac{4}{5} \times 3}{5 \times 2} = \frac{12}{10} = 1\frac{1}{5}.$$

Ans. $1\frac{1}{5}$.

Explanation 1. — The quotient of $\frac{4}{5}$ divided by 1 is $\frac{4}{5}$; of $\frac{4}{5}$ divided by 2 is $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{5 \times 2}$; of $\frac{4}{5}$ divided by $\frac{2}{3}$ (which is $\frac{1}{2}$ as large as 2) is 3 times $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4 \times 3}{5 \times 2}$, which equals $1\frac{1}{5}$.

Ans. $1\frac{1}{5}$.

Explanation 2. — The quotient of $\frac{4}{5}$ divided by 1, is $\frac{4}{5}$; of $\frac{4}{5}$ divided by $\frac{1}{2}$ (which is $\frac{1}{2}$ as large as 1), is 3 times $\frac{4}{5}$ or $\frac{4 \times 3}{5}$; and of $\frac{4}{5}$ divided by $\frac{2}{3}$, is $\frac{1}{2}$ of 3 times $\frac{4}{5}$ or $\frac{4 \times 3}{5 \times 2}$, which equals $1\frac{1}{5}$. *Ans.* $1\frac{1}{5}$.

17. To change a Circulating Decimal to a Common Fraction (Art. 287).

ILLUSTRATIVE EXAMPLE. Change $0.\dot{6}\dot{3}$ to a common fraction.

WRITTEN WORK.

$$0.\dot{6}\dot{3} \times 100 = 63.6363...$$

$$0.\dot{6}\dot{3} \times 1 = 0.6363...$$

$$0.\dot{6}\dot{3} \times 99 = 63.$$

$$0.\dot{6}\dot{3} = \frac{63}{99} = \frac{7}{11} \text{ Ans.}$$

Explanation. — As the repetend consists of two figures, we multiply the given circulate by 100, and find that the decimal part of the product is precisely the same as the given circulate. Hence, if we subtract the given circulate from this product

there will be no decimal fraction in the remainder. Thus we find that 99 times the given circulate equals 63; therefore once the given circulate is $\frac{63}{99}$, or $\frac{7}{11}$. Hence the rule given on page 129.

18. To change a Mixed Circulate to a Common Fraction (Art. 288).

ILLUSTRATIVE EXAMPLE. Change $0.2\dot{6}\dot{3}$ to a common fraction.

WRITTEN WORK.

$$0.2\dot{6}\dot{3} \times 100 = 26.3636...$$

$$0.2\dot{6}\dot{3} \times 1 = 0.2636...$$

$$0.2\dot{6}\dot{3} \times 99 = 26.1$$

$$0.2\dot{6}\dot{3} = \frac{26.1}{99} = \frac{261}{990} = \frac{29}{110}.$$

Ans. $\frac{29}{110}$.

Explanation. — As the repetend consists of two figures, we multiply the given mixed circulate by 100 and subtract from the product once the mixed circulate, and have 99 times the mixed circulate, equal to 26.1.

Then once the mixed circulate will equal $\frac{26.1}{99}$, or $\frac{261}{990}$. But 261 is the difference between 263, the mixed circulate regarded as an integer, and the finite part regarded as an integer. Hence the rule given on page 130.

19. Surveyors' and Mariners' Measures (Art. 302).

Surveyors, in measuring, use a chain called Gunter's chain (ch.), which is 4 rods or 66 feet long. The chain is divided into one hundred links (l.).

Surveyors' Long Measure.

7.92 in. = 1 link.

100 l. = 1 chain.

80 ch. = 1 mile.

NOTE. 25 links equal 1 rod.

Surveyors' Square Measure.

10 sq. ch. = 1 A.

640 A. = 1 sq. m.

1 sq. m. = 1 section.

36 sec. = 1 township.

Examples for the Slate.

34. A road was found to be 8 ch. 30 l. long, how many links in length was it?

35. Express 45 links as the decimal of a chain.

36. Express 11 ch. 56 l. in chains and decimals of a chain.

37. Change 18 ch. 5 l. to rods.

OPERATION: $18.05 \times 4 = 72.20$ rods.

38. Change 32 ch. 27 l. to rods and feet.

39. In a wall which measures 23 ch. 47 l., how many rods and how many feet over?

How many acres are there in a rectangular piece of land

(40.) 50 ch. long and 30 ch. wide?

(41.) 45 ch. long and 24 ch. wide?

(42.) 19.04 ch. long and 3.7 ch. wide?

(43.) 84 ch. $8\frac{1}{2}$ l. long and 13 ch. 24 l. wide?

20. Mariners, in measuring short distances at sea, use cable-lengths and fathoms.

6 feet = 1 fathom (used in measuring depths at sea).

120 fathoms = 1 cable-length.

$7\frac{1}{2}$ cable-lengths = 1 common mile.

Longer distances at sea are estimated in nautical or geographical miles (Art. 335). 3 nautical miles = 1 marine league.

21. Apothecaries' Weight (Art. 329, note II.).

In mixing medicines, apothecaries use the Troy pound divided into ounces (oz. or \mathfrak{z}), drams (dr. or \mathfrak{z}), scruples (sc or \mathfrak{d}), and grains. They also use the fluid ounce (f. \mathfrak{z}), fluid drachm (f. \mathfrak{z}), minims or drops (\mathfrak{m}).

Weights.

20 grains	= 1 scruple.
3 scruples	= 1 dram.
8 drams	= 1 ounce.
12 ounces	= 1 pound.

Liquid Measures.

60 minims	= 1 fluid drachm
8 fluid drachms	= 1 fluid ounce.
16 fluid ounces	= 1 pint (O).
8 pints	= 1 gallon (Cong)

22. Explanation of Leap Year (Art. 339).

The earth revolves around the sun in 365 days 5 hours 48 minutes and 50 seconds nearly, but we call 365 days a year. It will be seen that what we call a year is nearly 6 hours less than the true year, and 4 such years nearly one day less than 4 true years. To rectify this error, 366 days are allowed to every fourth year. The year of 366 days is called a *leap year*.

The addition of a day in every fourth year is too much by a number of minutes, which in one hundred years amounts to about three fourths of a day. To balance this error, only 365 days are allowed to the final year of a century in three centuries out of every four.

Hence any year is a leap year, *when the number denoting the year is divisible by 4 and not by 100, and when it is divisible by 400.*

23. Miscellaneous Tables.

Books.

A book formed of sheets folded

In 2 leaves is a folio.	In 16 leaves is a 16mo.
In 4 leaves is a quarto.	In 18 leaves is an 18mo.
In 8 leaves is an octavo.	In 24 leaves is a 24mo.
In 12 leaves is a duodecimo,	In 32 leaves is a 32mo.
or 12mo.	In 64 leaves is a 64mo.

Length.

3 in. = 1 palm.
9 in. = 1 span.
4 in. = 1 hand (used in measuring the height of horses).

Surface.

100 sq. ft. = 1 square	} used in estimating shingling, plastering, and other such work.

Capacity.		
1 barrel of flour		= 196 pounds.
1 barrel of beef, pork, or fish		= 200 "
1 cental of grain or 1 quintal of fish		= 100 "
1 cask of lime		= 240 "

Weights of Iron and Lead.

14 pounds	= 1 stone.
21½ stones	= 1 pig.
8 pigs	= 1 fother.

24. To compute Interest by Aliquot Parts (page 211).

ILLUSTRATIVE EXAMPLE. What is the interest of \$ 720 for 2 y. 7 mo. 29 d. at 8%?

WRITTEN WORK.

Principal, \$720

	0.08	
\$57.60	× 2 =	\$115.20 Int. for 2 y.
½ of 57.60	=	28.80 " " 6 mo.
¼ of 28.80	=	4.80 " " 1 mo.
0.8 of 4.80	=	3.84 " " 24 d.
⅓ of 4.80	=	0.80 " " 5 d.
		<u>\$153.44</u> " " 2 y. 7 m. 29 d.

To compute interest by aliquot parts: *Find the interest for one period of time, as 1 year or 1 month, and then find the interest for the balance of the time by taking conven-*

ient multiples or aliquot parts of this interest or of any of the results.

25. To compute Interest at 6 % by Aliquot Parts.

(First see Oral Exercises, Art. 541.)

At 6%, what part of the principal does the interest equal for

Months.	Ans.	Months.	Ans.	Months.	Ans.	Months.	Ans.
a. 2?	0.01.	d. 1?	0.00½.	g. 50?	½.	j. 66⅔?	⅓.
b. 20?	0.1.	e. 10?	0.0½.	h. 40?	⅓.	k. 33⅓?	⅓.
c. 200?	Prin.	f. 100?	½.	i. 25?	⅓.	l. 16⅔?	⅓.

At 6%, what part of the principal does the interest equal for

Days.	Ans.	Days.	Ans.	Days.	Ans.	Days.	Ans.
m. 60?	0.01.	o. 30?	0.00½.	q. 20?	0.00⅓.	s. 10?	0.00¼.
n. 6?	0.001.	p. 3?	0.000½.	r. 2?	0.000⅓.	t. 1?	0.000¼.

ILLUSTRATIVE EXAMPLE. Find the interest of \$700 for 10 y. 5 mo. 23 d. at 6%.

WRITTEN WORK.

Principal, \$700	
$\frac{1}{2}$ of \$700	= 350
$\frac{1}{4}$ of 350	= 87.50
$\frac{1}{8}$ of 7	= 2.33
$\frac{1}{2}$ of 0.70	= 0.35
<u>\$440.18</u>	

Int. for 100 mo.
 " " 25 mo.
 " " 20 d.
 " " 3 d.

Explanation. — 10 y. 5 mo. = 125 mo. We first find the interest for 100 mo. by taking $\frac{1}{2}$ of the principal, then for 25 mo. by taking $\frac{1}{4}$ of the interest for 100 mo. We then find the interest for 20 days

by taking $\frac{1}{8}$ of the interest for 60 days, or $\frac{1}{8}$ of 0.01 of \$700, and the interest for 3 days by taking $\frac{1}{2}$ of the interest for 6 days, or $\frac{1}{2}$ of 0.001 of \$700. The sum of these items equals \$440.18. *Ans.* \$440.18.

To compute interest at 6% by aliquot parts :

1. To find the interest for 200 months, take a sum equal to the principal; for 20 months, equal to $\frac{1}{10}$ of the principal; for 2 months, equal to $\frac{1}{50}$ of the principal; and for 6 days, equal to $\frac{1}{1000}$ of the principal.

2. For any other periods of time, take convenient multiples or aliquot parts of the interest for the times expressed above; and add the results.

26. Interest at any other % may be found by first finding the interest at 6%, as above, and then at any given %, as in Art. 542.

27. Annual Interest (page 220).

ILLUSTRATIVE EXAMPLE. What is the interest due on a note for \$1000, interest payable *annually* at 6%, if no payment be made till the expiration of 4 y. 6 mo. 12 d. ?

In some of the States the courts have sanctioned the taking of interest upon interest in cases like the above, where interest is not paid when it becomes due. Thus, interest is allowed on the above note

For the 4 y. 6 mo. 12 d. = \$272.00

Also on each year's interest (\$60) after it becomes due,

viz. :

On the 1st year's interest for	3 y. 6 mo. 12 d.
" " 2d " " "	2 y. 6 mo. 12 d.
" " 3d " " "	1 y. 6 mo. 12 d.
" " 4th " " "	6 mo. 12 d.

Which equals the interest of \$60 for 8 y. 1 mo. 18 d. = 29.28

Total interest . . . \$301.28

Simple interest, taken upon the principal, and upon each year's interest of the principal due and unpaid, is **annual interest**.

Rule.

To compute annual interest : *Compute simple interest on the principal for the time it is on interest. Also, on one year's simple interest for a period of time equal to the sum of the times for which the yearly interests severally remain unpaid. Add the results.*

Examples for the Slate.

44. What is the annual interest of \$334 for 3 y. 8 m. 10 d. ?
45. What is the annual interest of \$118.50 for 5 y. 3 m. 18 d. ?
46. What is the amount at annual interest of \$175 for 6 y. 2 m. 25 d. ?

28. Vermont Rule for Partial Payments.

1. *Compute annual interest upon the principal to the end of the first year in which any payments are made ; also compute interest upon the payment or payments from the time they are made to the end of the year.*
2. *Apply the amount of such payment or payments first to cancel any interests that may have accrued upon the yearly interests, then to cancel the yearly interests themselves, and then towards the payment of the principal.*
3. *Proceed in the same way with succeeding payments, computing, however, no interest beyond the time of settlement.*

29. The New Hampshire Rule

is the same as the foregoing, with the following provision :

If at the time of any payment no interest is due except what is accruing during the year, and the payment or payments are less than the interest due at the end of the year, deduct such payment or payments at the end of the year, without interest added.

30. Connecticut Rule for Partial Payments.

1. *When a year's interest or more has accrued at the time of a payment, and always in case of the last payment, follow the UNITED STATES RULE.*
2. *When LESS than a year's interest has accrued at the time of a payment, except it be the last payment, find the difference between the amount of the*

principal for an ENTIRE year, and the amount of the payment for the balance of a year after it is made; this difference will form the new principal.

8. If the interest which has arisen at the time of a payment exceeds the payment, compute interest upon the principal only.

31. Monetary Units of Foreign Countries (Art. 616).

The following table shows the par value in gold of the monetary units of different countries, as published by the Secretary of the Treasury of the United States, Jan. 1, 1878.

Countries.	Monetary Unit.	Standard.	Value in U. S. Money. Gold.
Austria	Florin	Silver	\$ 0.453
Belgium	Franc	Gold and silver...	0.193
Bogota	Peso	Gold	0.965
Bolivia	Dollar	Gold and silver...	0.965
Brazil	Milreis of 1000 reis	Gold	0.545
British Possessions in North America	Dollar	Gold	1.00
Central America ..	Dollar	Silver	0.918
Chili	Peso	Gold	0.912
Denmark	Crown	Gold	0.268
Ecuador	Dollar	Silver	0.918
Egypt	Pound of 100 piasters...	Gold	4.974
France	Franc	Gold and silver...	0.193
Great Britain	Pound sterling	Gold	4.866½
Greece	Drachma	Gold and silver...	0.193
German Empire	Mark	Gold	0.238
India	Rupree of 16 annas	Silver	0.436
Italy	Lira	Gold and silver...	0.193
Japan	Yen	Gold	0.997
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	0.998
Netherlands	Florin	Gold and silver...	0.385
Norway	Crown	Gold	0.268
Peru	Dollar	Silver	0.918
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks...	Silver	0.734
Sandwich Islands..	Dollar	Gold	1.00
Spain	Peseta of 100 centimes..	Gold and silver...	0.193.
Sweden	Crown	Gold	0.268
Switzerland	Franc	Gold and silver...	0.193
Tripoli	Mahbub of 20 piasters...	Silver	0.829
Tunis	Piaster of 16 caroubs...	Silver	0.118
Turkey	Piaster	Gold	0.043
U. S. of Colombia..	Peso	Silver	0.918

he 6 tens (the trial divisor), we have now the true divisor, which, multiplied by 6, completes the square. So the given number is a perfect square, and 36 is its square root.

36. Rule.

To extract the square root of a number :

1. *Beginning with the units' figure, point off the expression into periods of two figures each.*
2. *Find the greatest square in the number expressed by the left hand period, and write its square root as the first term of the root.*
3. *Subtract this square from the part of the number used, and with the remainder unite the next two terms of the given number for a dividend.*
4. *Double the part of the root already found for a trial divisor ; and by it divide the dividend (rejecting the lowest term of the dividend) and write the quotient as the next term of the root. Also write it at the right of the trial divisor to express the true divisor.*
5. *Multiply the true divisor by this term, and subtract the product from the dividend.*
6. *If there are more terms of the root to be found, unite with the remainder the next two terms of the given number, take for a trial divisor double the part of the root now found, and proceed as before.*

Cube Root (Art. 686).

37. The following method of extracting cube roots may be substituted for that given in the body of the book, if preferred. Let it be required to extract the cube root of 262144.

WRITTEN WORK.

Formula.

$$\text{Tens}^3 + (3 \times \text{tens}^2 + 3 \times \text{tens} \times \text{units} + \text{units}^3) \times \text{units}.$$

$$262'144 \text{ (64)}$$

$$(6 \text{ tens})^3 = \quad 216$$

$$\text{Trial divisor (6 tens)}^2 \times 3 = 108 \text{ hunds.} \quad 46144$$

$$6 \text{ tens} \times 3 \times 4 = 72 \text{ tens.}$$

$$4^2 = 16$$

$$\text{True divisor} \times 4 = 11536 \times 4 = 46144$$

Explanation. — The formula $\text{Tens}^3 + 3 (\text{tens}^2 \times \text{units}) + 3 (\text{tens} \times \text{units}^2) + \text{units}^3$ may be changed to the form, $\text{Tens}^3 + (3 \times \text{tens}^2 + 3 \times \text{tens} \times \text{units} + \text{units}^2) \times \text{units}$.

As the first part of the power, the cube of the tens, is thousands, we find the greatest cube contained in 262 (thousands), which is 216 (thousands), and write its cube root 6 (tens) as the first term, or tens, of the root.

Taking the cube of 6 (tens), 216 (thousands), out of 262 (thousands), there remain 46 (thousands), with which we unite the remaining part of the number, 144, making 46144, which must contain $(3 \times \text{tens}^2 + 3 \times \text{tens} \times \text{units} + \text{units}^2) \times \text{units}$.

If the number 46144 contained only $3 \times \text{tens}^2 \times \text{units}$, we should find the number of units by dividing 46144 by 3 times the square of the tens. So we make this number, 108 (hundreds), the trial divisor, and find that it is contained in 461 (hundreds) 4 times. Then 4 is probably the next term, or units, of the root.

Adding 3 times 6 tens \times 4 units, and the square of 4 units to the trial divisor, we have the true divisor, which multiplied by 4 completes the cube. So the given number is a perfect cube, and 64 is its cube root.

38. Rule.

To extract the cube root of a number :

1. *Beginning with the units' figure, point off the expression into periods of three figures each.*
2. *Find the greatest cube in the number expressed by the left-hand period, and write its cube root as the first term of the root.*
3. *Subtract the cube from the part of the number used, and with the remainder unite the next three terms of the given number for a dividend.*
4. *Take three times the square of the part of the root already found for a trial divisor, and by this divide the dividend (rejecting the lowest two terms of the dividend) and write the quotient as the next term of the root.*
5. *To the trial divisor (which is hundreds) add three times the first term of the root (tens) multiplied by the last term, also the square of the last term.*
6. *Multiply this sum by the last term of the root, and subtract the product from the dividend.*
7. *If there are more terms of the root to be found, unite with the remainder the next three terms of the given number, take for a trial divisor three times the square of the part of the root now found, and proceed as before.*

39. To find the Capacity of a Cask or Barrel in Gallons.

Add to the head diameter $\frac{2}{3}$ of the difference between the head and bung diameters (or, if the staves are but little curved, 0.6 of the difference). This will give the mean diameter.

Multiply the square of the number of inches in the mean diameter by the number of inches in length, and this product by 0.0034.

40. Miscellaneous Examples.

47. If I lose 10% by selling goods at 18 cents per yard, for what should they have been sold to gain 20%?

48. If 30 men, working 11 hours a day, can do a piece of work in a certain time, how many more men must be employed, when it is half done, to finish it in the same number of days, working 10 hours a day?

49. Two armies are in opposite directions from a certain point, one being 300 miles east and the other 250 miles west of it, and marching towards each other, the first at the rate of 15 and the other of 18 miles in a day. In how many days will they meet, and where?

50. If, by selling goods at 60 cents per lb., 20% is gained, what % would have been gained by selling them at 75 cents per lb.?

51. A broker purchases a lot of stocks at an average of 9% below par, and sells them at an average of $7\frac{3}{4}\%$ above par, and makes \$300. What was the par value of the stocks?

52. How many bushels of corn at 50 cents a bushel must be mixed with 30 bushels of grain at 80 cents a bushel, that the mixture may be worth 75 cents a bushel?

NOTE. Take such a quantity of corn as shall make the gain in selling it at 75 cents a bushel equal the loss in selling the grain at 75 cents a bushel.

53. How much water must be mixed with a barrel of ink (31 gals.), which cost \$34.10, that it may be sold at \$1.10 a gallon and 25% be gained?

54. 20% of a lot of barley, originally 5000 bushels, was destroyed by fire, the cost having been $\$1\frac{1}{2}$ per bushel. What per cent will be gained on the lot by selling the remainder at \$2 per bushel?

55. I sell $\frac{1}{3}$ of a lot of goods for \$9, and thereby lose 25%. For what must I sell the remainder to make $8\frac{1}{3}\%$ on the whole?

56. I sold 4 ploughs at \$24 each; on 2 of them I made 20%, and on 2 I lost 20%. What did I gain or lose on the whole?

57. Divide 52 into two such parts that $\frac{1}{3}$ of one part shall equal $\frac{2}{3}$ of the other.

58. How many cubic yards of earth must be removed for a cellar 10 feet deep and measuring inside the walls 27 feet long and 15 feet wide, the wall being 2 feet 6 inches thick?

59. If 10% is lost by selling boards at \$7.20 per M., what % would be gained by selling them at 90 cents per C.?

60. A person takes a note on 2 months for \$110 in payment for a watch. On getting the note discounted at a bank, he finds that he has lost 40% on the first cost of the watch. What was the cost?

61. What would be due May 1, 1878, on a note for \$1000, dated March 26, 1875, at 8% interest, on which \$200 were paid at the end of each year from the date of the note?

62. If I buy coal at \$4.12 per ton on 6 months' credit, for what must I sell it immediately to gain 10%?

63. What will a pine log weigh whose length is 18 ft., measuring 3 ft. across the larger end, and $2\frac{1}{2}$ ft. across the smaller, pine being 0.6 as heavy as water, which weighs $62\frac{1}{2}$ lbs. to a cubic foot?

64. Required the number of square feet in the surface of a ditch surrounding a circular garden which is 25 yards across, the ditch being $2\frac{1}{2}$ ft. wide.

65. An aeronaut ascends at the rate of $4\frac{1}{2}$ miles an hour for 40 minutes, after which he maintains the same elevation; if his balloon is driven east 7 miles during the first hour from the time of his starting, and in an opposite direction at the rate of 10 miles an hour for the remaining time, how far from his starting-point in a straight line is he at the end of 5 hours?

66. 13% is lost by selling a lot of land for \$783. What would it have brought if it had been sold at a loss of $8\frac{1}{2}$ %?

67. What will be the per cent of gain on the cost of a Gas Co.'s stock, the par value of shares being \$87.50, if it be bought at 15% below par, and sold at $19\frac{1}{2}$ % above par?

68. What is the length of the edge of the largest cube that can be sawed from a globe 9 inches in diameter?

69. Two boys tried their skill in running for pegs. 5 pegs were set up 6 feet apart, and the boys commenced running 6 feet from the first peg. How far did each run to place the pegs at his starting-point?

70. John Barnes bought, June 8, 1875, 10 bales of cotton cloth, 14 pieces in a bale, 43 yds. in a piece, at 8¢ per yd., for which he gave his note on interest at 6%. On the 4th of Nov., 1877, he sold 1 bale at 30¢ a yd., and with the proceeds made part payment of his note. On the 3d of May, 1878, he sold 1 bale at 40¢, and paid on his note the amount he received. On the 17th of Sept., 1878, he sold the remainder at 60¢, and settled the note. What did he gain by his speculation?

SEP 11 1879

FEB 7 1880

JAN 25 1881

APR 12 1881

JUL 8 1881

18

OCT 17 1882

MAY 12 1885

SEP 2 1888

